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Accounting for Productivity Growth When Technical Change is Biased

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When Technical Change is Biased

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Abstract: Solow (1957) decomposed labor productivity growth into two components that are independent under Hicks neutrality: input growth and the residual, representing technical change. However, when technical change is Hicks biased, input growth is no longer independent of technical change, leading to ambiguous interpretation. Using Solow's model, I decompose output per worker into globally independent sources and show that technical bias directly contributes to labor productivity growth above what is captured in the Solow residual. This contribution is sometimes large, generating rates of total technical change that substantially exceed the Solow residual, prompting a reinterpretation of some well-known studies.

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Introduction

During 1842, the cotton mills of Lowell, Massachusetts increased the number of looms assigned to each weaver from two to three, increasing labor productivity by about 50% in short order. How much of this increase can be attributed to technical change and how much to capital-labor substitution? This pattern of labor productivity growth accompanied by capital deepening appears frequently in economic development. The decomposition of labor productivity growth into sources helps us interpret the relative importance of technological change, which should generate technical change, and of capital accumulation, which should induce a substitution of capital for labor.

Solow (1957) provided a simple model that does such a decomposition. The famous Solow residual measures the component that corresponds to technical change and this decomposition has been widely used to answer questions of this sort. However, when applied to this change in Lowell, the Solow decomposition seems to provide a misleading result. The Solow decomposition attributes just over half of the increase in labor productivity to the residual and about half to "input growth," referring to the growth in the capital to labor ratio. This suggests that technical change and capital accumulation played roughly equal roles. But a detailed examination of the actual changes (Bessen 2008) shows that capital-labor substitution can only account for a small percentage of the labor productivity growth. How could this be?

The reason for this discrepancy is that the technical change in Lowell was strongly biased and the Solow residual can be easily misinterpreted in this case. In his 1957 paper, Solow assumed that technical change was Hicks neutral.[1] Solow provided some limited empirical evidence to support his assumption, however, subsequent empirical research tends to reject the hypothesis of Hicks neutrality.[2]

1 Solow (1957, p. 313) writes "The not-necessarily-neutral case is a bit more complicated, but basically similar." However, he provides no details.

2 Solow found no correlation between the annual growth of the residual and the annual capital-labor ratio. However, if learning is involved, then annual differences are unlikely to show such a correlation and might even show a negative

The literature seems confused as to whether the Solow residual is an appropriate and meaningful measure of technical change when technical change is not Hicks neutral. Some early theoretical treatments identified the Solow residual as a measure of technical change even in the non-neutral case (Ferguson 1968, Sato 1970). More recently, several authors have proposed alternative measures that are supposed to provide more appropriate decompositions (Antonelli and Quartraro 2008, Bailey et al. 2004, Nelson and Pack 1999, Rodrik 1997).

Mankiw, Romer and Weil (1992) take another approach to measure the relative contributions of technical change and capital accumulation. They use an alternative model that assumes that technical change is Harrod neutral (purely labor-augmenting) instead of Hicks neutral. Klenow and Rodríguez-Clare (1997) and Hall and Jones (1999) also used this assumption to obtain non-parametric estimates of the contribution of technical change. This approach is consistent with neoclassical growth theory (Solow 1956, Swan 1956), which often assumes Harrod neutrality in order to get models to converge to a steady state. Although the empirical evidence tends to reject Hicks neutrality, it also does not tend to support Harrod neutrality. There clearly might be some advantage to obtaining a measure of productivity growth that does not require a particular assumption about the bias of technical change.

More generally, this affects how growth is interpreted. Reviewing the history of U.S. economic growth, Abramovitz and David (Abramovitz 1993, Abramovitz and David 1973, 2001), argue that it is misleading to interpret the Solow residual as the only measure of technical change because of evidence that labor-saving/capital-using technical change promoted substantial capital-deepening. Nevertheless, Abramovitz and David interpret nineteenth century growth in labor productivity as arising primarily

correlation. Empirical studies that formally test and reject the hypothesis of Hicks neutrality include Antras (2004), Berndt and Khaled (1979), Binswanger (1974), Brown and De Cani (1963), Cain and Paterson (1981), David and van de Klundert (1965), Ferguson (1965), Jorgenson and Fraumeni (1981), Kalt (1978), Klump et al. (2007). May and Denny (1979), Moroney and Trapani (1981), Panik (1976), Sato (1970), Wilkinson (1968), Wills (1979) and Yuhn (1991). Berndt and Wood (1982) could not discriminate between different technical biases. Toevs (1980) could not reject the Hicks neutral hypothesis against a restricted alternative. Berndt et al. (1993) reject Harrod neutrality, but could not reject Hicks neutral technical change.

from capital accumulation. Similarly, Young (1995) and Kim and Lau (1994), use the Solow residual to argue that the dramatic growth of labor productivity in East Asia owes more to capital accumulation than to technical change. This leads Krugman (1994) to call the East Asian miracle a "myth" that he compares to Soviet investment-driven economic growth. Rodrik (1997) and Nelson and Pack (1999) challenge this view, arguing that it is based on an inappropriate measurement because technical change is biased.

These differences hinge on how one interprets the "input growth" term that comes out of the Solow decomposition relative to the residual term. Some observers interpret input growth as driven strictly by capital accumulation: as in neoclassical growth models, a high savings rate raises the relative wage of labor compared to capital, causing firms to substitute labor for capital, raising the amount of capital per worker. This movement along the production function raises output per worker. In this view, input growth is independent of technical change. However, this interpretation is based on inferences that cannot be supported in many cases, as I show below. If technical change is biased, then input growth is *not* independent of technical change and is *not* driven purely by capital accumulation.

In this paper, I argue that this confusion arises from a muddling of what is exogenous and what is endogenous in Solow's model. A growth decomposition exercise cannot unambiguously determine how much growth (e.g., in labor productivity) comes from any one component (e.g., technical change) unless the components of that growth are orthogonal exogenous variables. For example, if one decomposes inflation growth into a contribution from oil prices and another contribution from transportation prices, the real contribution from oil prices will be understated because these two factors are not independent. Surprisingly, Solow's decomposition is not orthogonal when technical change is biased.

In Solow's model, the firm (or production sector at the aggregate level) maximizes profits by adjusting the capital-labor ratio subject to two exogenous variables: relative factor prices and technical change. Solow decomposes labor productivity growth into two components, one representing technical change, the other representing capital deepening. When technical change is Hicks neutral, these two components are orthogonal and each corresponds to one of the exogenous variables (the residual corresponds to technical change and capital-deepening corresponds to changes in factor prices). However, when technical change is Hicks biased, I show that these components are no longer orthogonal according to the assumptions of Solow's model. In particular, the capital-labor ratio is an endogenous function of technical change as well as factor prices. This means that some input growth can be attributed to technical change and not to capital accumulation. Consequently, the Solow residual cannot be unambiguously interpreted as a complete measure of technical change in this case.

Of course, productivity economists have not ignored technical bias. The econometric studies listed in footnote 2 obviously accommodate biased technical change. Several other papers use non-parametric methods that accommodate a limited degree of technical bias. Caves et al. (1982), Diewert and Morrison (1986) and Kohli (1990) develop index number measures of productivity that are exact for translog production functions that permit some technical bias.[3] But my point here concerns the *interpretation* of the Solow residual. The interpretation is compromised when technical change is biased. It is inconsistent to assume that the level of technology influences the quantities of factor inputs and then to attempt to measure technical change holding factor inputs constant, as if this could be done. The Solow residual simply fails to capture the entire effect of technical change on labor productivity growth because it fails to capture the direct effect of technical change on factor proportions.

Fortunately, I show that this problem can be solved with a simple adjustment to the Solow residual. Where Solow took a partial derivative, I take a total derivative. With just this single additional step of analysis, Solow's model provides a decomposition into components that are orthogonal even

3 Thanks to Erwin Diewert for pointing me to this literature. The first order parameters of these translog functions are permitted to change over time, allowing for some bias in technical change. However, the second order parameters are fixed. This means, for example, that these production functions cannot accommodate factor augmenting technical change that is biased.

when technical change is biased. There are three terms contributing to labor productivity growth: "neutral" technical change (the Solow residual), "biased" technical change, and factor substitution.

With this decomposition, it is straightforward to calculate both the magnitude of the technical bias and a generalized measure of technical change that is unambiguously distinguished from the effect of price-driven changes in input factors. Because the total rate of technical change includes both the neutral Solow residual plus a term associated with the technical bias, the total rate of technical change generally differs from the Solow residual.

This generalized residual provides a more meaningful measure of technical change when that change is biased. For example, the change at Lowell in 1842 was hardly a story of capital accumulation. Relative factor prices had not changed much at that time, there was little factor substitution (the production function for weaving was highly inelastic) and the capital-labor ratio increased not because of greater investment (capital accumulation), but because the mills used the same machines with less labor, thanks to better technical knowledge and skills (Bessen 2008). Consistent with these facts, the generalized residual attributes almost all of the growth in labor productivity to technical change and very little to factor substitution.

Of course, one might dispute the assumptions of the Solow model, for example, the assumption that technical change is exogenous. Nelson (1973) raises this point. Indeed, if technical change is endogenous it might well be influenced by relative factor prices in the long run. Below I show that my generalized residual is also consistent with a wide range of endogenous growth models.

Solow's Model Restated

Decomposing labor productivity growth

Solow's model assumes a neoclassical production function with constant returns to scale, profit maximization in competitive factor markets (factors are paid their marginal product) and Hicks neutral technical change.[4] The production function could pertain to an individual firm or it could be an aggregate production function.[5] I begin by restating the model without the assumption of Hicks neutrality.

For the moment, assume a two-factor neoclassical production function (with all the usual assumptions) that is linear homogenous in K and L, capital and labor:

$$Y = F(K, L;\ A(t)) \tag{1}$$

where Y is output and A is an index of technology (broadly construed to include organization and technical knowledge), which is an exogenous function of time, t. Below I will consider more than two input factors, but two factors make the exposition simpler and this also more closely matches Solow's exposition.

Given the constant returns to scale, this can also be expressed in intensive form,

$$y = \frac{Y}{L} = f(k;\ A(t)), \qquad k \equiv \frac{K}{L} \tag{2}$$

where y represents labor productivity and k is the capital-labor ratio. Given this, the instantaneous growth in labor productivity can be written (not showing function arguments)

$$\hat{y} \equiv \frac{d \ln f}{dt} = \frac{f_k}{f}\frac{dk}{dt} + \frac{f_A}{f}\frac{dA}{dt} \tag{3}$$

where a "hat" represents the instantaneous growth rate and subscripts denote partial derivatives.

Let

$$w(t) \equiv \frac{w^L}{w^K}$$

4 Hicks (1932, p. 121) classified technical change depending on whether the initial effect was "to increase, leave unchanged, or diminish the ratio of the marginal product of capital to that of labour." Hicks neutral change leaves this ratio unchanged. Under constant returns to scale, this also means that factor proportions remain unchanged.

5 Felipe and McCombie (2006) raise concerns about growth accounting using aggregate production functions that are separate from the analysis here.

where w is the ratio of the labor wage, w^L, to the capital rental rate, w^K. The model assumes that firms maximize profits by adjusting the capital-labor ratio instantaneously. That is, w and A change over time and both are exogenous to the profit maximization problem of the firm. The profit-maximizing capital-labor ratio is a function of these exogenous variables:

$$(4)\qquad k^*(w(t),\ A(t)) = k \quad \text{such that} \quad \frac{F_L}{F_K} = w(t)$$

Given this assumption, equation (3) can be re-written

$$(5)\qquad \hat{y} = s\,\frac{d\ln k^*}{dt} + \frac{f_A}{f}\,\frac{dA}{dt}, \qquad s(w(t), A(t)) \equiv \frac{f_k\,k^*}{f} = \frac{F_K K}{F}$$

where s is the output share of capital. Solow identified the first term as the increase in labor productivity growth that can be associated with growth in the capital-labor ratio and the second term with technical change. Accordingly, he defined the following residual as a measure of technical change:

$$(6)\qquad R_{Solow} = \hat{y} - s\hat{k}^*$$

However, note that the first term on the right hand side of (5) is endogenous: k^* is a function of both w and A. Consequently, the Solow residual does not decompose labor productivity growth into independent sources. As we shall see, it *does* do this under the assumption of Hicks neutrality, which is, after all, the condition that Solow imposed. However, in the general case, when technical change might be biased, the two terms on the right hand side of (5) are not independent.

Fortunately, (5) can be further decomposed by expanding the total derivative of k^*:

$$(7)\qquad \begin{aligned} \hat{y} &= s\left(\frac{\partial\ln k^*}{\partial w}\,\frac{dw}{dt} + \frac{\partial\ln k^*}{\partial A}\,\frac{dA}{dt}\right) + \frac{f_A}{f}\,\frac{dA}{dt} \\ &= s\sigma\,\frac{d\ln w}{dt} + s\,\frac{\partial\ln k^*}{\partial A}\,\frac{dA}{dt} + \frac{f_A}{f}\,\frac{dA}{dt} \\ &= s\sigma\,\hat{w} + s\,\frac{\partial\ln k^*}{\partial\ln A}\,\hat{A} + \frac{\partial\ln f}{\partial\ln A}\,\hat{A}, \qquad \sigma \equiv \frac{\partial\ln k^*}{\partial\ln w} \end{aligned}$$

where σ is the elasticity of substitution. This decomposes labor productivity growth into orthogonal terms based on exogenous variables. The first term varies with growth in *w*; the last two terms vary with growth in *A*. The first term captures the effect of price-driven factor substitution on labor productivity, the second term captures the effect of technical change-driven capital deepening and the third term, which is the Solow residual, can be thought of as the neutral component of technical change.

The second term is, in fact, directly related to the technical bias. This can be seen as follows. In my notation, the Hicks bias can be defined as (see, for example, Ferguson 1968)

$$\hat{B} \equiv \frac{\partial \ln F_K / F_L}{\partial \ln A} \hat{A}$$

Technical change is Hicks labor-saving, Hicks neutral, or Hicks capital saving depending on whether the bias is positive, zero or negative. Then it can be shown (see Appendix) that (7) becomes

(8) $$\hat{y} = s\sigma \hat{w} + s\sigma \hat{B} + \frac{\partial \ln f}{\partial \ln A} \hat{A}$$

Two intuitions follow from (7) and (8). First, technical bias itself contributes to labor productivity growth. Part of the change in input factors has nothing to do with factor accumulation, arising, instead, from technical bias. Second, the Solow residual, which equals the last term, does not capture the entire effect of technical change on labor productivity growth when the bias is non-zero. It is therefore misleading to interpret the Solow residual as the rate of technical change except under Hicks neutrality.

Given this decomposition, a generalized residual can be constructed as

(9) $$R = \hat{y} - s\sigma \hat{w}$$

This measure captures the entire effect of technical change independently of the effect of price-driven factor accumulation.

The Solow residual is sometimes described as the difference between output growth and growth in a Divisia (share-weighted) index of input quantities (see equation 6). In comparison, (9) appears as the difference between output per capita growth and a share-and-elasticity-of-substitution weighted index of factor *prices*. In effect, this is a weighted index of input factors, adjusted to include only the effects of factor prices, not the effects of technical bias.

Calculating technical bias and the generalized residual

This generalized residual and the technical bias can be calculated using the Solow residual. However, these calculations require two quantities not included in the normal growth accounting: the elasticity of substitution and the rate of growth of relative prices. The latter can, however, be obtained from quantities used in the normal growth accounting. Given constant returns to scale,

$$\frac{1-s}{s} = \frac{L\,w^L}{K\,w^K} = \frac{w}{k}$$

Taking the derivative of this equation with respect to *t* and using straightforward calculation, (9) can then be rewritten

$$R = \hat{y} - s\sigma\left(\hat{k}^* - \frac{\hat{s}}{1-s}\right) \tag{10}$$

This does still require estimation of the elasticity of substitution and, as Diamond et al. (1978) emphasize, when technical change is biased, this might require the imposition of some assumptions. Nevertheless, beginning with David and van de Klundert (1965), an empirical literature has derived estimates of the elasticity of substitution of aggregate production functions under assumptions of biased technical change at a constant rate of growth. Also, engineering production function studies have obtained bottoms-up estimates of the elasticity for specific technologies (see Wibe, 1984, for a review). In any case, even if estimates of the elasticity of substitution are not available, this equation provides a framework for assessing the sensitivity of technical change estimates variation in the elasticity of substitution.

Comparing (10) and (6), the generalized residual can be calculated as an adjustment to the Solow residual:

$$(11) \qquad R = R_{Solow} + s\sigma\hat{B} = R_{Solow} + s(1-\sigma)\hat{k}^{*} + \frac{s\sigma}{1-s}\hat{s}$$

Note that if factor shares are constant—as is assumed in many studies and is found in other many studies as a first order approximation—then the last term drops out. Also, if the production function is Cobb-Douglas (elasticity of substitution equal to one), then the last two terms drop out and the generalized residual equals the Solow residual.

Finally, the technical bias can be calculated as

$$(12) \qquad \hat{B} = \frac{R - R_{Solow}}{s\sigma} = \frac{1-\sigma}{\sigma}\hat{k}^{*} + \frac{\hat{s}}{1-s}$$

Interpreting technical change

The Hicks Neutral Case

When technical change is Hicks neutral, Solow's residual corresponds to the rate of technical change. It is well-known (Uzawa 1961, p. 120) that if technical change is Hicks neutral, the production function can be separated as

$$(13) \qquad F(K, L;\ A(t)) = A(t)\,G(K,\ L)$$

Inspecting (4), this means that k^* will be independent of A, that is, $k^*(w, A) = k^*(w)$ in this case. Given this independence, the second term in (7) will drop out, leaving the Solow residual equal to the generalized residual. More generally, by inspecting (12), since s and σ are strictly positive, it follows that

Proposition 1. Technical bias.

a.) If $\hat{B} > 0$ (Hicks labor-saving bias), $R > R_{Solow}$.

b.) If $\hat{B} = 0$ (Hicks neutral change), $R = R_{Solow}$.

c.) If $\hat{B} < 0$ (Hicks capital-saving bias), $R < R_{Solow}$.

Given that k^* is independent of A in the Hicks neutral case, there is a one-to-one correspondence between w and k. That is, k is an alternative exogenous variable. Consequently it is appropriate that some people describe technical change as the increase in labor productivity that could be achieved while holding k fixed. This is an appropriate description under Hicks neutrality, but since k is endogenous in the more general case, this description is only apt in the Hicks neutral case.

Over discrete time intervals, the Solow residual can be approximated by a Tornqvist index. The change in log labor productivity from t=0 to t=1 can be obtained by integrating (5),

$$\text{(14)} \qquad \Delta \ln y = \int_0^1 s \frac{d \ln k^*}{dt} \, dt + \int_0^1 \frac{f_A}{f} \frac{dA}{dt} \, dt \, .$$

Using the trapezoidal rule to approximate the first integral,

$$\Delta \ln y \approx \bar{s} \; \Delta \ln k + \int_0^1 \frac{f_A}{f} \frac{dA}{dt} \, dt \, , \qquad \bar{s} \; \equiv \; \frac{s(w(0), A(0)) + s(w(1), A(1))}{2}$$

so that

$$\text{(15)} \qquad R_{Solow} \approx \; \Delta \ln y \; - \; \bar{s} \, \Delta \ln k$$

An example of such a change is illustrated in Figure 1. The solid curve represents the initial production function and at t=0 production takes place at point A. The dashed curve represents the production function at t=1.[6] In this example, $w(1) > w(0)$, so production at t=1 takes place at point B, with a higher k^* as well as a more productive technology.

Solow described the decomposition in his model as a distinction between "movements along the

6 The functions, G, shown are CES functions with an elasticity of substitution of .10. Technical change increases A by about 50% and price changes increase k also by 50%.

production function" and "shifts of the production function." This interpretation holds in the Hicks neutral case. In Figure 1, the dashed curve appears as just a vertical displacement of the solid curve. Because technical change is multiplicative in (13), it appears as a vertical shift of the production function when displayed on a logarithmic scale. That the second term in (15) is equivalent to a movement along the production function can be seen from the following:

Proposition 2. When technical change is Hicks neutral,

(16a) $s(w(t), A(0)) = s(w(t), A(t))$, and,

(16b) $$R_{Solow} = \Delta \ln y - \int_0^1 \frac{d \ln f(k, A(0))}{d \ln k} \frac{d \ln k}{dt} dt \approx \Delta \ln y - \frac{s(w(0), A(0)) + s(w(1), A(0))}{2} \Delta \ln k$$

Proof: Under Hicks neutral change, the output share of capital is independent of *A*. This can be seen as follows. By definition, Hicks neutral change leaves the ratio of the marginal products of capital to labor unchanged, hence at constant factor prices it leaves the capital labor ratio unchanged, so the ratio of the capital share of output to the labor share of output is unchanged. Under constant returns to scale, this ratio is $s/(1-s)$. If this is constant, it must be true that s is also unchanged as in (16a). Substituting $s(w(t), A(0))$ for $s(w(t), A(t))$ in (15) yields (16b). QED.

The second term on the right hand side of the expression in (16b) represents the change in ln y brought about by movement from point A in Figure 1 to point C along the original production function. This is represented by segment CD. Segment BD represents $\Delta \ln y$, leaving the residual represented by segment BC. Solow's description of the residual as measuring shifts in the production function as distinct from movements along the production function holds in the Hicks neutral case. However, this description and Proposition 2 do not hold in general because in general neither k nor s are independent of A.

The Hicks Biased Case

Figure 2 shows a different technical change without any change in relative factor prices. The solid line is the same production function as in Figure 1. However, the dashed curve now represents this production function after a labor-augmenting technical change.[7] As can be seen, technical change can no longer be described as a simple "shift" in the production function; the multiplicative property of (12) no longer holds.

Nor is there a comparable "movement along the production function" in this case, since relative factor prices remain unchanged by assumption. That is, $w(0) = w(1)$ and point B has the same relative factor prices as does point A. There can be no movement induced by changing prices. The first term in (7) is zero and the generalized residual captures the entire increase in labor productivity represented by segment BD.

Of course, one might imagine a counterfactual hypothetical where relative prices increased prior to the technical change so that production would move from point A to point C. The idea would be that one might hope to then measure the increase in labor productivity from point C to point B, holding k constant. The problem with this, of course, is that now k is *endogenous* and it cannot be held constant while A changes in a Hicks biased manner. The only way to get from point C to point B is if the technical change were accompanied by a decrease in factor prices that restores them to their original level. It is not clear that such a change, if it could be measured, would reveal anything meaningful about the contribution of technical change.

But even if we wanted to measure the change from C to B, the Solow residual does not do this any longer. Instead, it measures the change from point E to point B. This is because now the capital share of output at point C no longer equals the capital share of output at point B, $s(w(1), A(0)) \neq s(w(1), A(1))$, so (15) no longer yields the same expression as (16b). This example is an

7 Labor was augmented 50%.

instance of a well-known property of the Solow residual when technical change is not Hicks neutral.[8]

Several researchers have noted that the Solow residual does not take account of the effect of technical change on factor shares (Antonelli and Quartraro 2008, Bailey et al. 2004, Nelson and Pack 1999, Rodrik 1997). These researchers[9] have proposed an alternative index that uses only the initial factor share:

$$\hat{y} - s(w(0), A(0))\,\hat{k}^*$$

In Figure 2 the corresponding residual is represented by the segment BF. This provides a small improvement over the Solow residual, but fails to capture the full segment BC, let alone BD. The reason for this is that $s(w(0), A(0)) \neq s(w(1), A(0))$. In general, researchers cannot easily measure $s(w(1), A(0))$ because production does not take place at point C—this point, after all, is only significant for a counterfactual hypothetical that has little relevance for interpreting actual technical change.

In short, when technical change is Hicks biased, the Solow residual no longer provides a meaningful measure of the portion of labor productivity growth that can be attributed to technical change, nor do the proposed alternative measures. The problem with these measures is that k is no longer exogenous in the Solow model, so the effects of technical change and capital accumulation are muddled. But w is exogenous and, for this reason, the generalized residual can cleanly distinguish the effects of technical change from capital accumulation mediated by factor prices changes, at least as far as the assumptions of Solow's model go.

The bottom line is that the change depicted in Figure 2 can be attributed entirely to technical change: factor prices remained unchanged and so factor substitution cannot realistically account for any of the growth in labor productivity in this example. A further increase in the relative wage, as in Figure

8 The first integral in (14) is equivalent to a Divisia index of inputs. When technical change is Hicks neutral, this integral can be evaluated independently of the path of $A(t)$ (see for example, Hillinger 1970). However, when technical change is biased, this line integral depends on the path of technical change because both s and k^* vary with A. See Hulten (1973) and Samuelson and Swamy (1974, p. 578-88).

9 Bailey et al. (2004) use an econometric approach as an alternative.

1, would move point B further along the dashed line representing the production function at time 2.

As a practical matter, the generalized residual over a discrete interval can be approximated in the same manner as the Solow residual, above,

$$R \approx \Delta \ln y - \tilde{s}\, \Delta \ln w, \qquad \tilde{s} \equiv \frac{s(w(0),\, A(0))\, \sigma(w(0),\, A(0)) + s(w(1),\, A(1))\, \sigma(w(1),\, A(1))}{2}$$

A similar approximation can be applied to (11).

Reinterpreting Multi-factor Productivity Estimates

How does this new residual measure compare to the Solow residual in practice? In Tables 1 through 3, I explore the impact of this alternative decomposition on a variety of multi-factor productivity estimates from the literature. In these calculations, I use (11) to obtain the generalized residual. This still requires an estimate of the elasticity of substitution, so I use examples where that elasticity has been estimated or where I can impose a hypothetical value.

Nineteenth Century Estimates

Table 1 compares calculations of these two decompositions for the nineteenth century. The first calculations return to the example of the 19th century cotton mills of Lowell. Using an engineering production function I previously obtained an estimate of the elasticity of substitution of 0.14 (Bessen 2008). This value is low, however, it is supported by similar estimates obtained econometrically for the same mill and from the cotton textile industry as a whole (Asher 1972). In this example, the generalized residual is dramatically greater than the Solow residual. In the generalized decomposition, only a small part of the growth in labor productivity can be attributed to factor substitution. Technical change accounts for almost all of the growth in labor productivity, in accord with my detailed analysis of changes at that mill.

This example may be atypical. Mechanized technologies might be particularly biased towards labor-saving innovation and there is reason to expect that the elasticities of substitution for these technologies are low. My analysis of the 19th century cotton industry provides a rationale for these differences. More generally, Abramovitz and David (Abramovitz 1993, Abramovitz and David 1973, 2001) suggest that technical change might have been particularly biased during the 19th century, noting the larger role of capital deepening then compared to the 20th century.

However, Abramovitz and David, using a Solow decomposition, nevertheless conclude that, "the pace of increase of the real gross domestic product was accounted for largely by that of the traditional, conventionally defined factors of production." They suggest that biased technical change kept the returns to capital from decreasing much as a high savings rate increased capital intensity and they criticize earlier economic historians who placed greater emphasis on technical change. Abramovitz and David's estimate (2001) of the Solow residual for the period from 1855 through 1890 is only 0.37% per year, while output per manhour grew at 1.06% per year. The remainder they attribute to the growth of conventionally defined factors of production, a contribution of 0.69% per year. Thus one might be tempted to conclude that growth in labor productivity was largely driven by growth in capital intensity that was itself driven by a high savings rate.

However, given that Abramovitz and David conclude that technical change was significantly biased during this period, perhaps an alternative calculation might present a somewhat different picture. This, however, requires an estimate of the elasticity of substitution between capital and labor. Abramovitz and David conclude that the aggregate elasticity of substitution between capital and labor is likely less than unity, but they do not provide an estimate. The modern empirical literature presents a range of estimates, however, it is important to recall that the results depend on the assumptions one makes regarding the bias of technical change (Diamond et al. 1978). Under assumptions of Hicks neutrality and constant elasticity of substitution, Berndt (1976) found an elasticity of substitution of about one between capital and labor. But estimates using translog production functions, which allow variable elasticities of substitution, typically reject the Cobb-Douglas restrictions (see for example Berndt and Christensen 1973 or Griffin and Gregory 1976). And estimates that assume a constant elasticity of substitution but allow factor-augmenting technical change also reject the Cobb-Douglas form, finding elasticities of substitution between capital and labor well below one (David and van de Klundert 1965 and Antras 2004). Antras performs a careful analysis that takes account of a wide variety of possible econometric issues. He finds that allowing for biased technical changes makes a critical difference and concludes that "the elasticity of substitution between capital and labor is likely to be considerably below one, and may even be lower than 0.5." Klump et al. (2007) obtain an elasticity of 0.55.

Supposing that the elasticity of substitution between capital and labor is 0.50, I calculate the alternative decomposition in Table 1. Under this assumption, technical change accounts for growth in output per manhour of 0.72% during this period, substantially larger than the contribution from conventional factor inputs and a majority of the total growth in labor productivity. A similar reversal occurs in the estimates for the earlier period from 1800 through 1855. Moreover, although 0.72% is a low rate of technical change by modern standards, this is not really so low considering that the Civil War occurred during this period. Field (2008) shows the large effect of the war on productivity over this interval and finds high rates of residual growth from the 1870s on.

This analysis suggests that the Solow decomposition might understate the significance of technical change for labor productivity growth both at the micro and macro levels.

Twentieth Century US Estimates

Table 2 shows two sets of well-known estimates reinterpreted using my decomposition. David and Van de Klundert (1965) estimated the elasticity of substitution in a model that also allowed biased technical change. The value they obtained was 0.3165. Table 2 shows the calculated residuals for their data (using two alternative estimates of capital shares that they use), which run from 1899 through 1960. Here the difference between the residuals is not large, although the generalized residual is still larger, consistent with their finding of a labor-saving bias.

Christensen and Jorgenson (1970) perform a non-parametric calculation of the Solow residual for the US private domestic economy from 1929 through 1967. Several previous studies had found that the Solow residual accounted for most of the growth in output and in output per manhour. Christensen and Jorgenson (see also Jorgenson and Griliches 1967) questioned whether those findings resulted from measurement error and, in their 1970 paper, sought to provide a more robust estimate. Their estimate of the Solow residual was 1.13% per annum, about one third of the rate of growth of GDP and about two thirds of the rate of growth of GDP per manhour.

Christensen and Jorgenson also obtained a rough estimate of the elasticity of substitution between capital and labor of 0.79.[10] Using this figure, I find that my alternative decomposition leaves the residual almost unchanged.

Thus although the role of technical bias seems to suggest a reinterpretation of nineteenth century productivity growth in the US, it does not seem to make much difference for twentieth century estimates.

World estimates

The same is not true for developing countries in the twentieth century, however. Table 3 shows growth accounting for regions of the world from 1960 through 2000 based on data from Bosworth and Collins (2003). The Solow decomposition shows the data supporting the view that the East Asian productivity miracle is a "myth." East Asia has a Solow residual of 1.0%—an average value—but a

10 They obtained this as the ratio of the average rate of growth of capital per unit labor to the rate of growth of the wage relative to the capital service cost. If technical change is biased, however, this calculation yields a biased estimate of the elasticity of substitution.

high rate of output growth per capita (3.9%), the difference attributed to growth in capital per worker. This makes it seem that East Asian growth might be driven mainly by the high savings rate of these countries, in line with the predictions of the transitional dynamics of the neoclassical growth model.

However, this impression is highly sensitive to the elasticity of substitution and the implicit assumption of Hicks neutral technical change. With an elasticity of substitution of 0.50, the generalized residual more than doubles to 2.3% per annum; this is larger than the contribution of factor substitution to labor productivity growth.[11] Moreover, using the alternative decomposition, the residual exceeds the contribution from factor substitution in all regions except Africa and Latin America, the two regions with the lowest labor productivity growth rates.

Thus factor substitution, perhaps driven by savings along the lines of the neoclassical growth model, does contribute to growth in output per capita in all regions. But under a reasonable assumption about the elasticity of substitution, the contribution of technical change is *larger* in all regions of the world experiencing labor productivity growth in excess of two percent per year. The conclusion that savings and capital accumulation is the main driver of growth in East Asia or anywhere else is not supported by the empirical evidence.

Extensions

Multiple factors

The generalized residual can be readily derived for production functions with more than two input factors. Let F be a neoclassical production function with n factors of production, X_i, in addition to labor, L, so that

$$Y = F(X_1,\ X_2,\ ...,\ X_n, L,\ A(t))$$

11 For these calculations, I have assumed that all changes in the quality of labor have arisen from relative factor price changes and not at all from any technology-skill bias.

Assume that F is linear homogenous in input factors so that it can be written in intensive form,

$$y = f(x_1, x_2, ..., x_n, A(t)), \quad x_i \equiv \frac{X_i}{L}$$

Also assume that factor markets are competitive and the firm optimizes, so that, for all i

$$x_i^*(w_1(t), w_2(t), ..., A(t)) = x_i \text{ such that } \frac{F_i}{F_L} = v_i(t) \equiv \frac{w_i}{w_L}$$

for factor prices w_i. Then, following as above (see Appendix for details),

$$R = R_{Solow} + \sum_{j=1}^{n} s_j\left[\left(1 + \sum_{i=1}^{n} s_i(\sigma_{ij} - \sigma_{Lj})\right)\hat{x}_j - \sum_{i=1}^{n} s_i(\sigma_{ij} - \sigma_{Lj})(\hat{s}_j - \hat{s}_L)\right] \tag{17}$$

where σ_{ij} is the Allen partial elasticity between factor i and and factor price j.

Endogenous Technical Change

The Solow model and the analysis in this paper so far assume that technical change is exogenous. However, factor prices might very well affect the development of new technology. There is a large literature on "induced innovation" or "directed technical change" from Hicks (1932) through Acemoglu (2003, 2007). If technical change responds to factor prices, then the decomposition into independent factors is no longer accurate. As noted in the introduction, this point has been raised as a criticism of the Solow residual and the same criticism applies to the generalized residual I introduce here.

However, my residual might still be meaningful even when technical change is endogenous for the following reason: the development of new technology in response to changing factor prices most often occurs only after a significant lag, as is well-known. For example, the cases studies of Jewkes et al. (1959) show lags from invention to commercial product ranging from three years to over 100 years. As long as productivity growth is measured over an interval shorter than this lag (e.g., one year), then it can still be decomposed into orthogonal components. In this case, the technical change observed during this interval will indeed be independent of the factor prices changes observed during this interval. Of course, the observed technical change might be a response to earlier factor price changes, so the measured technical change is not necessarily exogenous. Nevertheless, this growth accounting serves to distinguish labor productivity growth that arises from technical change from growth that arises from *concurrent* changes in factor prices.

Conclusion

I have shown how technical bias influences the growth in output per worker and I have shown how this influence can be calculated. I argue that technical bias is critical to understanding the full impact of technical change on output per worker. Because the Solow residual does not include the influence of technical bias, it is too easily misinterpreted as a complete measure of the rate of technical change. With labor-saving technical change, this tends to overestimate the role of capital accumulation in economic growth.

At the very least, this analysis suggests reasons for caution when attempting to draw inferences about the sources of productivity growth from productivity calculations. Moreover, under plausible assumptions, some well-known studies need to be reinterpreted. If the elasticity of substitution between capital and labor is significantly less than one—as the latest econometric estimates suggest—then neither nineteenth century US output per capita nor recent East Asian output per capita can be primarily attributed to capital accumulation. Although investment driven growth was important in both instances, technical change was even more important in both.

There are other examples where the role of technical bias might be important. Many economists attribute growing wage inequality in the U.S. to skill-biased technical change. If so, then what might be the effect of this technical bias on the growth of output per capita? The productivity growth slowdown of the 1970s or the computer "productivity paradox" might appear to have very different significance once the influence of technical bias is taken into account. A skill-using technical bias combined with substantial growth of skilled labor might mean, all else equal, that the total rate of technical change significantly exceeds the Solow residual.

More generally, this paper highlights the importance of research about the sign and magnitude of technical biases and about the possible endogenous origins of these biases, including "induced innovation" and Acemoglu's (2003, 2007) model of "directed technical change."

Appendix

Derivation of (8)

A useful corollary of the Euler theorem is that under constant returns to scale,

(A1) $F_{KK} K + F_{KL} L = 0$ and $F_{KL} K + F_{LL} L = 0$.

Assumption (4) can be restated as

(A2) $$H = \frac{1}{w} - \frac{F_K}{F_L} = 0 \cdot$$

Totally differentiating and using (A1) to simplify,

(A3) $$\begin{aligned} dH = 0 &= \frac{\partial H}{\partial \ln K} d\ln K + \frac{\partial H}{\partial \ln L} d\ln L + \frac{\partial H}{\partial \ln A} d\ln A \\ &= \frac{F_{LK} F}{F_L^2} d\ln K - \frac{F_{LK} F}{F_L^2} d\ln L - \frac{\partial F_K / F_L}{\partial \ln A} d\ln A \end{aligned}$$

so that

$$\frac{\partial \ln k^*}{\partial \ln A} = \frac{F_L^2}{F_{LK} F} \frac{\partial F_K / F_L}{\partial \ln A}$$

Then the second term of (7) can be written, using a standard expression for the elasticity of substitution,

(A4) $$s \frac{\partial \ln k^*}{\partial \ln A} \hat{A} = s \frac{F_L^2}{F_{LK} F} \frac{F_K}{F_L} \hat{B} = s \sigma \hat{B}$$

Substituting this expression into (7) yields (8).

Derivation of (17)

As in the two factor case, begin by taking the total derivative of *Y* with respect to *t*,

$$\hat{Y} = \frac{1}{Y}\sum_{i=1}^{n}\frac{\partial F}{\partial x_i}\frac{dx_i^*}{dt} + \frac{\partial \ln F}{\partial \ln A}\hat{A} = \sum_{i=1}^{n} s_i \hat{x}_i^* + \frac{\partial \ln F}{\partial \ln A}\hat{A}$$

$$= \sum_{i,j=1}^{n} s_i s_j (\sigma_{ij} - \sigma_{Lj})\hat{v}_j + \sum_{i=1}^{n} s_i \frac{\partial \ln x_i^*}{\partial \ln A}\hat{A} + \frac{\partial \ln F}{\partial \ln A}\hat{A}, \qquad \sigma_{ij} = \frac{1}{s_j}\frac{\partial \ln X_i^*}{\partial \ln w_j},\ s_i \equiv \frac{w_i X_i}{Y}$$

where the σ_{ij} are Allen partial elasticities of substitution. The last two terms correspond to technical change so that

(A5) $$R = \hat{y} - \sum_{i,j=1}^{n} s_i s_j (\sigma_{ij} - \sigma_{Lj})\hat{v}_j .$$

Taking the time derivative of the log of s_i,

$$\hat{s}_i = \hat{w}_i + \hat{X}_i - \hat{Y}$$

so, with a little manipulation,

$$\hat{v}_i = \hat{s}_i - \hat{s}_L - \hat{x}_i$$

Substituting this in (A5)

$$R = \hat{y} - \sum_{i,j=1}^{n} s_i s_j (\sigma_{ij} - \sigma_{Lj})(\hat{s}_j - \hat{s}_L - \hat{x}_j)$$

Given the Solow residual,

$$R_{Solow} = \hat{Y} - \sum_{j=1}^{n} s_j \hat{x}_j ,$$

(17) follows.

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Table 1. Nineteenth Century Growth of Output per Manhour, alternative decompositions
(annual growth rates)

		Solow Decomposition		Generalized Decomposition	
	Output / manhour	Residual	Input / manhour	Residual	Factor Substitution
Lawrence Co., Mill No. 2					
1835 – 1855	2.92%	1.67%	1.25%	2.71%	0.21%
US Private Domestic Economy					
1800 - 1855	0.39%	0.20%	0.19%	0.30%	0.09%
1855 - 1890	1.06%	0.37%	0.69%	0.72%	0.34%
1890 - 1927	2.01%	1.24%	0.77%	1.55%	0.46%

Note: The Lawrence Company decomposition is done using an estimated elasticity of substitution of 0.14 (see Bessen 2008). The economy-wide decompositions assume an elasticity of substitution of 0.50. Sources: Bessen (2008), Abramovitz and David (2001)

Table 2. Twentieth Century Growth of Output per Manhour, US Private Economy, alternative decompositions
(annual growth rates)

		Solow Decomposition		Generalized Decomposition	
	Output / manhour	Residual	Input / manhour	Residual	Factor Substitution
US Private Business Sector, 1899–1960					
using NIPA shares	2.38%	1.85%	0.53%	2.14%	0.24%
using Kendrick shares	2.38%	2.13%	0.25%	2.21%	0.17%
US Private Domestic Sector					
1929 - 1967	1.67%	1.13%	0.54%	1.12%	0.55%

Note: David and Van de Klundert (1965) estimate the elasticity of substitution at 0.3165; Christensen and Jorgenson (1970) estimate it at 0.79. Sources: David and Van de Klundert (1965), Christensen and Jorgenson (1970)

Table 3. World Growth of Output per Manhour, 1960 – 2000, alternative decompositions (annual growth rates)

		Solow Decomposition		Generalized Decomposition	
	Output / manhour	Residual	Input / manhour	Residual	Factor Substitution
World	2.3%	0.9%	1.4%	1.5%	0.8%
Industrial countries	2.2%	1.0%	1.2%	1.5%	0.8%
China	4.8%	2.6%	2.2%	3.6%	1.3%
East Asia exc. China	3.9%	1.0%	2.9%	2.3%	1.7%
Latin America	1.1%	0.2%	0.9%	0.4%	0.7%
South Asia	2.3%	1.0%	1.3%	1.5%	0.8%
Africa	0.6%	-0.1%	0.7%	0.1%	0.6%
Middle East	2.1%	0.5%	1.6%	1.2%	1.0%

Note: Source of data is Bosworth and Collins (2003). Alternative decomposition assumes an elasticity of substitution of 0.50.

Figure 1. Hicks Neutral Case

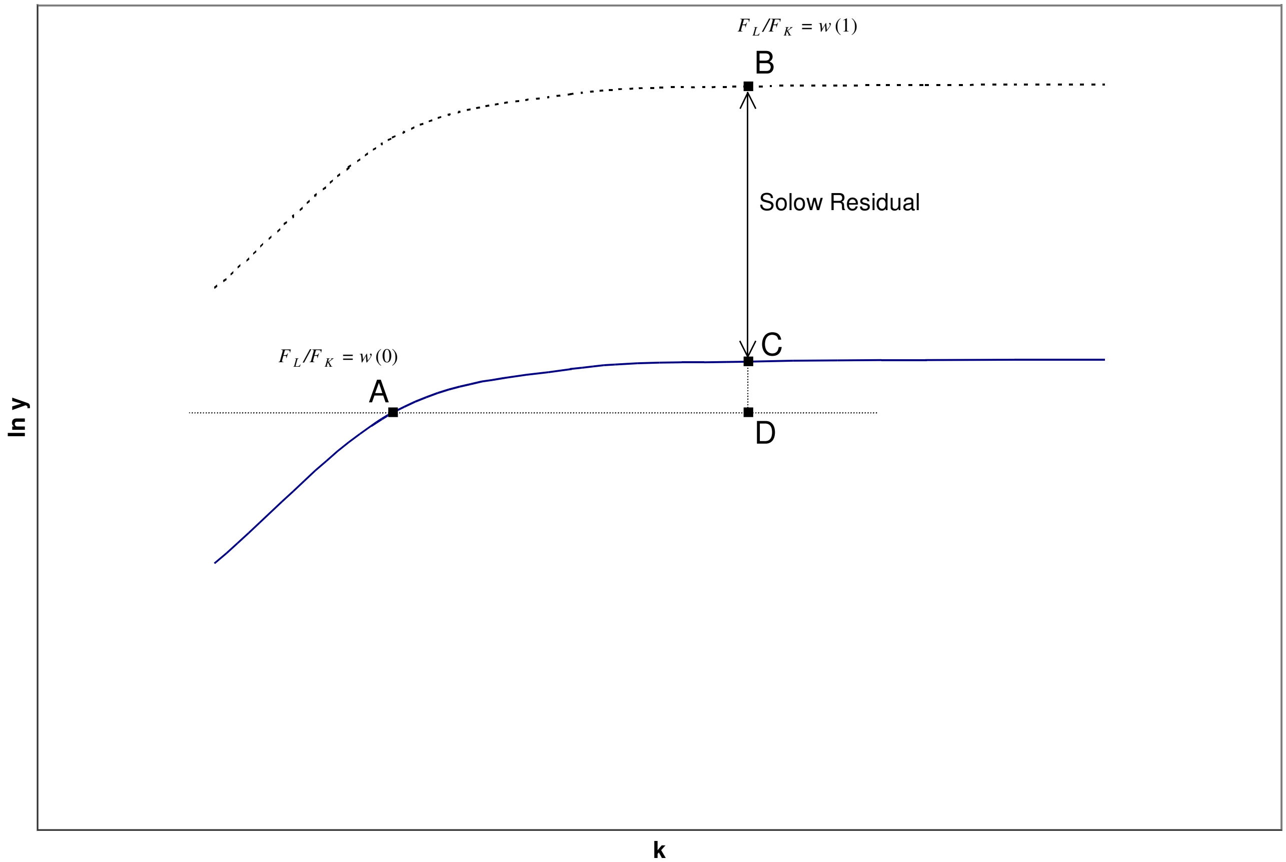

Figure 2. Labor Augmenting Technical Change, No Factor Substitution