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James Bessen

BU School of Law

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Abstract: The value of patent rents is an important quantity for policy analysis. However, estimates in the literature based on patent renewals might be understated. Market value regressions could provide validation, but they have not had clear theoretical foundations for estimating patent rents. I develop a simple model to make upper bound estimates of patent rents using regressions on Tobin's Q. I test this on a sample of US firms and find it robust to a variety of considerations. Estimates from market value regressions correspond well with estimates based on patentee behavior generally, but renewal estimates might be understated for pharmaceuticals.

Keywords: Technology, patents, innovation, patent value, firm value

JEL codes: O34, O38, K1

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Introduction

Patents are intended to provide an economic incentive for invention by granting the patent holder an exclusive right for a limited period. This right to exclude allows a patent-holding firm to become a monopolist or, perhaps more often, to achieve some lesser degree of market power, either in product markets or in the market for technology licenses. This market power, in turn, is supposed to permit the firm to earn supra-normal profits, "rents," that are an economic incentive to invent.

The value of patent rents is thus an important quantity for evaluating the performance of the patent system and also for understanding firm value. Some researchers have used the observed behavior of patent owners to estimate the private value of patents, which should equal the discounted value of patent rents.[1] Beginning with Pakes and Schankerman (1984), these studies have imputed patent value from observed decisions to pay maintenance fees (see Bessen 2008 for estimates using this method for the US),[2] decisions to file patents in multiple countries (Putnam 1996), and decisions to sell (re-assign) patents (Serrano 2006).

But these approaches share an important limitation: they do not directly reflect the value of the most valuable patents and, given the skewed distribution of patent values, most of the aggregate value of patents is determined by the relatively small number of highly valuable patents. These studies typically assume a distributional form, such as a log-normal distribution. They then fit that distribution to the observed data and extrapolate to the upper tail. However, if the upper tail diverges significantly from the assumed distribution, then estimates of mean patent value might be too large or too small (although estimates of median patent value obtained from these methods are accurate). In the worst case, the upper tail might be so "heavy" that the actual distribution has an infinite mean as with the Pareto distribution (Scherer and Harhoff 2000). Then estimates of the mean would be unstable and would not converge even at asymptotically large sample sizes.

An alternative might be to use firm market value to estimate patent value, that is, to decompose firm value into its component parts including that part attributed to patents. This way, investor behavior, rather than the behavior of patent owners, might reveal patent value. At the very least, estimates based on firm market value might serve as an important check on the values obtained from

1 Other researchers have used surveys to assess inventors' view of patent values (Harhoff et al. 2003a, Gambardella et al. 2008). However, these studies obtain a measure of patent value that is equivalent to the value of patent rents plus the value of the invention realized by other means (see Harhoff et al. 2003a). See section 4.4 below.

2 See Lanjouw et al. 1998 for a review of this literature. Recent studies also include Baudry and Dumont (2006) and Gustafsson (2005).

data on the behavior of patent owners.

A large number of researchers have run regressions that use firm market value (or Tobin's *Q*, which is firm market value divided by the replacement value of firm assets) as the dependent variable and some measure of patents as an independent variable.[3] Surprisingly, however, the inclusion of patent measures in these studies has been *ad hoc*. None of these papers has sought to formally model the role of patent rents in determining firm value. And, for this reason, none have reported any rigorous estimates of patent rents (or patent value), aside from a few casual observations.[4]

The main contribution of this paper is that I provide a simple model that relates patent rents to firm value, allowing me to make rigorous inferences about these rents from my own estimates and also to re-interpret previous regressions to obtain estimates of patent rents.

One major difficulty, which has been noted in the literature, is that patent rents cannot be fully identified. This is because patents proxy for other unmeasured variables that are plausibly correlated with firm value. For example, firms might obtain more patents if their research investments turn out to be successful. In this case, the quality of R&D (R&D "success") would be an omitted variable that might bias estimates of patent rents upwards. However, although a coefficient corresponding to patent rents cannot be fully identified, my model shows that an approximate *upper bound* on mean rents per patent *can* be estimated.

I also show that estimates based on this model are robust to a variety of considerations including firm-specific differences in appropriability conditions, other firm characteristics, different specifications and stability over time. I further test whether these estimates appear to be stable in light of the skewed distribution of invention values (Scherer 1965, Scherer and Harhoff 2000, 2003b, Silverberg and Verspagen 2004). I find that my estimates of mean patent value show definite evidence of convergence to the mean, suggesting that the distribution of patent values does not have an infinite mean (n.b., *invention* values might be different).

I compare upper bound estimates of patent value to estimates obtained from a variety of other sources and using a variety of different methods. These include estimates based on the renewal behavior of US and European patentees and estimates based on the choices of US patentees regarding re-assignment and international filing. I also compare my estimates of annual patent rents to several

3 See Hall (2000) for a review of this literature. Some recent additions to this literature include Bosworth and Rogers (2001), Toivanen et al. (2002), Hall (2007), Hall et al. (2005) and Griffiths et al. (2005).

4 Griliches's initial paper (1981) noted that "a successful patent is worth about $200,000," but this was more of an informal observation than a rigorous inference. Cockburn and Griliches (1988) mention tentative values implied by their coefficient estimates. Hall (2005) and Hall and MacGarvie (2006) compare coefficients for different sub-samples and infer that higher values imply more valuable patents on average.

rough benchmarks including the net income of large pharmaceutical companies and patent licensing revenues of IBM and US universities.

This exercise demonstrates that although only limited inferences can be drawn from market value regressions, by providing upper bound estimates, they can play a role in evaluating estimates obtained by other methods, possibly confirming those estimates or possibly calling them into question. In a sister paper (Bessen 2008), I use the renewal method to estimate patent value for US patents. In Section 4 of the current paper, I compare those estimates (and other estimates based on patentee behavior) to estimates obtained from market value regressions. In Bessen and Meurer (2008), we use all of these estimates of patent rents and patent value, along with some other considerations, to evaluate patent policy.

1. A Market Value Model of Patent Rents

1.1 Market value regressions using patent data

In theory, the value of patents derives from the rents they generate. Patents provide their owners a degree of market power—either in product markets or in markets for technology—that affects the demand for the owner's products, allowing them to charge prices that exceed those they could charge in a perfectly competitive market. These supra-normal prices generate supra-normal profits, or rents, and these should contribute to the value of the firm that owns the patents.

I wish to explore the extent to which market value regressions can be used to measure the magnitude of the mean rents earned per patent. There is a significant literature that performs market value regressions, however, that research line began with a different objective: it sought to measure the "knowledge stock" of firms and patent terms have been included in these regressions as *ad hoc* proxies of R&D quality. Because of this different objective, the models used in this literature do not explicitly include patent rents, making any inferences about rents difficult.

These studies mostly build on Griliches's (1981) "hedonic" model of the firm, where investors are assumed to value a firm based on a combination of its characteristics, including the firm's "knowledge stock."[5] Researchers have included a variety of knowledge stock quality characteristics on the right hand side of market value regressions, including R&D spending and stocks, patent counts, patent stocks, citation counts, and citations stocks, as well as counts of trademark and design applications. Hall (1993, 2007), using a model of Hayashi and Inoue (1991), provides a rigorous

5 This follows the literature on hedonic product pricing (Court 1939, Griliches 1961) where, for example, the value of an automobile is modeled as a combination of its characteristics, each independently valued.

treatment of the R&D coefficient in these regressions. Hall, Jaffe and Trajtenberg (2005) add patent terms to Hall's model in order to capture some measure of "R&D success."

However, the models used in these regressions do not lend themselves to inferences regarding patent rents and, for the most part, researchers have not tried to draw such inferences. For one thing, it is well known that the coefficients of hedonic regressions are difficult to interpret unless one makes some strong assumptions (Rosen 1974). Second, although Hall, Jaffe and Trajtenberg (2005) provide a rigorous interpretation of the R&D coefficient, their model, based on Hayashi and Inoue (1991), assumes competitive markets. It is therefore *inconsistent* with the occurrence of patent rents.

My approach is to explicitly consider the contribution of rents to firm market value but to also incorporate the insights of Hall, Jaffe and Trajtenberg regarding R&D and the possible effect of R&D success on patenting.

1.2 Patent rents

Hayashi (1982) developed a formal model for firms with market power that relates Tobin's Q to the value of rents. Under assumptions of constant returns to scale and profits as a function of an aggregate capital stock in nominal dollars, K, for the jth firm at time t,

$$V_{jt} = q_t (K_{jt} + W_{jt}) \tag{1}$$

where V is firm market value, W is the present discounted value of firm rents (Hayashi derives this as a function of the product demand elasticity), and q is "marginal q" which reflects short term disequilibrium in capital markets.[6] Marginal q is assumed to be approximately equal to 1 and, given competitive capital markets, it is equal across firms at any given time.

This simple equation captures two intuitions. One intuition is Tobin's original insight that the market value of a firm is related to the replacement cost of its assets. In a competitive market with no market power ($W = 0$), firms will add capacity (either new entrants or existing firms) at the replacement cost of capital, driving prices down until, in long run equilibrium, the discounted stream of expected future profits (market value) equals the cost of those assets.

The second intuition is that firms with market power earn sustained supra-normal profits reflected in a market value that exceeds that of a competitive firm. Firms with market power can charge prices above the competitive price, thus they earn supra-normal profits, and this stream of profits increases the value of the firm above the competitive level.

6 I will use the term "rents" to refer to the expected present value of the stream of future rents.

The rents, W, consist of rents earned from patents and rents earned by other means, including rents on technical knowledge realized through first-mover advantage, trade secrecy, etc. as well as other sorts of rents, e.g., from barriers to entry. If one assumes that rents from other sources are scaled by firm size (K), then we can write $W = W(P, K)$, where P is the firm's patent stock. This function can be approximated with a Taylor series. Assuming that $W(0,0) = 0$ and that W is linear homogeneous, a first order approximation is

(2) $$W_{jt} = u \cdot P_{jt} + \mu_j K_{jt}$$

where u is the mean rent per patent, the mean taken as the expected present value of the future rent flows from all of the firm's patented inventions, and μ is the markup for rents that the firm earns on its assets through other means, again, as an expected present value. Equation (2) posits a linear relationship; below I consider higher order terms in the Taylor series to capture possible non-linearity and I find that the simple linear relationship in (2) serves as a good approximation.

This means that equations (1) and (2) provide a simple and direct path to estimate mean patent rents from firm market value. For the case where there are no sources of rents other than patents, for instance, substituting (2) into (1) yields

(1b) $$V_{jt} = q_t(K_{jt} + u \cdot P_{jt}).$$

Note that although this form looks similar to equations used in Hall, Jaffe and Trajtenberg (e.g., their equation (1)) and in the hedonic literature, it is different in one critical aspect. In Hall's analysis (based on Hayashi and Inoue, 1991), K is treated as a weighted sum of different types of capital where the weights are the relative marginal productivities of each type. However, the coefficient u is *not* the marginal productivity of patents, but is, instead, the *mean* rent per patent. This is because patents differ from the assets that make up K in a subtle, but important way. Because patents confer market power, they alter the revenue function of the firm—that is, they change the inverse demand curve faced by the firm after taking competitors' behavior into account, generating rents.[7] The same is true for other sorts of rent-generating assets such as "brand capital." However, the assets that enter K are assumed, in Hall's analysis, to have *no* effect on the demand curve and hence they generate no rents. These assets contribute to the value of the firm by generating output; they enter the production function and, in Hall's model, they do so under some rather stringent assumptions. Patents, on the other hand, do not directly contribute to firm output; instead, they contribute to firm value by increasing the *price* of that

7 In a perfectly competitive market, the inverse demand curve facing the firm is perfectly elastic, so that the level of the firm's output has no effect on the market price. With market power, this is no longer true, so the firm can charge a supra-normal price.

output. This is an important difference.[8] Because my study aims to estimate patent rents, I explicitly treat patents as a rent-generating asset and this implies a somewhat different specification and a somewhat different interpretation of coefficients than might be inferred from the earlier literature.

1.3 R&D quality and the capital stock

However, any attempt to estimate a regression like (1b) runs into a well-known econometric difficulty: the patent stock might be correlated with unmeasured quality of the R&D stock and therefore estimates of u might be biased. Hall, Jaffe and Trajtenberg (2005) note that patents might proxy for R&D "success" and this notion is implicit in Griliches's "knowledge stock" regressions.

It is helpful to formally model this notion. The aggregate capital stock in (1) includes both the stock of tangible assets and the "knowledge stock." Typically, the knowledge stock is estimated as a sum of depreciated R&D investments. However, unlike tangible capital, the value of the firm's knowledge stock will vary considerably depending on the ex post outcome of R&D. That is, a successful R&D investment will generate knowledge that is far more valuable than an unsuccessful investment. Following Hall (1993, 2007), under some assumptions, the aggregate capital stock can be represented as a weighted sum of the tangible and intangible stocks, where the weights are determined by the marginal productivities of the different capital types.[9] Then we can write the current value of aggregate capital as

(3) $$K_{jt} = A_{jt} + s_{jt} R_{jt}$$

where A is conventional tangible assets in current dollars, R is the R&D stock in current dollars (calculated by applying the declining balance method to the stream of deflated R&D investments), and s is a normalized quality adjustment dependent on the firm's ex post success rate. This success rate reflects the greater marginal productivity of successful R&D and the quantity sR can be thought of as a quality-adjusted measure of the knowledge stock. If it is true on average that an additional dollar of R&D generates as much profit as an additional dollar invested in tangible assets (as it should in long run equilibrium), then s will be approximately equal to one on average.

Of course, s cannot be observed. However, the insight from the literature is that s might be

8 In theory, Hall's model could be extended to include patents as a component of the aggregate capital stock. However, the model requires that the profit function be a linear homogeneous function of an aggregate stock that is a linear combination of capital types. It is difficult if not impossible to reconcile this requirement with the idea that patent rents derive from the effect of patents on the demand curve.

9 This aggregation remains valid as long as the profits of the firm exhibit constant returns to scale and this will be true if the production function and patent propensity both exhibit constant returns. Below I discuss the significance of departures from the assumption of constant returns.

positively correlated with patenting rates. The idea is that firms might obtain more patents relative to their R&D if that R&D is successful because they have more valuable knowledge to protect. In other words, patent propensity increases with R&D success. Although one could model a complete patent propensity function (see, for example, Arora et al. 2003), for my purposes it is sufficient to use a simple approximation,

$$(4) \qquad s_{jt} = \alpha + \beta \frac{P_{jt}}{R_{jt}}, \qquad \alpha = \alpha(\mu_j,\ u,\ c),\ \ \beta = \beta(\mu_j,\ u,\ c)$$

where c is the cost of patenting. I consider higher order terms below.

This equation formally captures the intuition that patenting is related to R&D success or R&D quality. The functional definitions on the right underline that the parameters α and β will, in general, change along with changes in the value of patents, the cost of patents and other factors that affect patent propensity. This means that differences in patent propensity between groups or over time will likely correspond to changes in the relationship between patents and firm market value, thus limiting what inferences can be safely drawn from comparisons of market value regressions between groups or over time.

Equation (4) is quite general. The literature holds that β is non-negative on theoretical grounds, meaning that firms might acquire more (but not fewer) patents on commercially successful R&D projects. However, there is no guarantee that β is significantly positive. For example, in the pharmaceutical industry, patents might be obtained when an R&D project achieves technical success, e.g., when clinical trials are approved.[10] However, the value of the knowledge stock increases when *commercial* success is achieved, and that might occur years after the patents are acquired, if at all. The correlation between patents and the economic value of the knowledge stock might be quite weak in that case, implying a small β. I do not assume that β is positive and my results hold as long as β is not substantially negative.

Equation (4) could be further enhanced to include some measure of patent citations. Hall et al. (2005) argue that patent citations add significant informational content in a market value regression and patent citations have been frequently used to capture notions of patent quality or invention quality. Below I also estimate an alternative specification where the number of citations received per patent are assumed to capture the "quality" of the patent.

Finally, assuming that the second term of (4) is small implies that α approximately equals one,

10 Thanks to Mike Scherer for pointing out this distinction.

given the normalization of s above. In practice, as Hall (1993) shows, the coefficient on R&D can vary from year to year as technologies obsolesce at different rates. This implies that α and β may vary year to year as well. As we shall see below, estimates obtained over an extended time period (to average out short term fluctuations in the productivity of R&D) do, indeed, show values of α close to one and relatively small values for the second term in (4). Below I check the robustness of my estimates to variation in α.

1.4 Estimable specifications

Substituting (2) – (4) into (1), I obtain (see Appendix),

$$\text{(5)} \qquad \ln \frac{V_{jt}}{A_{jt}} = \ln q_t + \ln(1+\mu_j) + \ln\left(1 + \alpha \frac{R_{jt}}{A_{jt}} + \gamma \frac{P_{jt}}{A_{jt}}\right), \quad \gamma \equiv \frac{u}{1+\mu_j} + \beta \quad .$$

Assuming that all firms have the same rents from other sources (or, alternatively, that variation in μ is uncorrelated with other right hand side variables), and adding a stochastic error term, then this equation can be estimated using Non-Linear Least Squares:

$$\text{(6)} \qquad \ln \frac{V_{jt}}{A_{jt}} = \ln q_t + \ln(1+\mu) + \ln\left(1 + \alpha \frac{R_{jt}}{A_{jt}} + \gamma \frac{P_{jt}}{A_{jt}}\right) + \epsilon_{jt}, \quad \gamma \equiv \frac{u}{1+\mu} + \beta, \ \ \mu_j \equiv \mu$$

This is one specification I use below. Clearly, it is quite similar to the specification used by Hall, Jaffe, and Trajtenberg (2005).

However, firm specific effects may be an important source of heterogeneity. Moreover, depending on the nature of the patent propensity equation, μ may be correlated with the patent stock. For instance, firms that earn substantial non-patent rents may be less likely to patent successful R&D projects, all else equal. So it is helpful to also have a specification that incorporates firm effects.

Assuming that α approximately equals one and $\gamma P << K$ (both assumptions supported by the NLLS estimation), and using a Taylor series approximation, I derive (see Appendix)

$$\text{(7)} \qquad \ln \frac{V_{jt}}{A_{jt}+R_{jt}} = \ln q_t + \delta_j + \gamma \frac{P_{jt}}{A_{jt}+R_{jt}} - \frac{\gamma^2}{2}\left(\frac{P_{jt}}{A_{jt}+R_{jt}}\right)^2 \ldots + \epsilon_{jt} .$$

This specification uses a modified version of Tobin's Q as the dependent variable, but it can be estimated using fixed effects, random effects, first differences, or longer differences.

The functional forms of (6) and (7) differ slightly from specifications used in previous research. Below I derive estimates of γ from statistics reported in several other papers by transforming the equations used in those papers to one of these specifications.

1.5 Identification and alternatives

It comes as no surprise that in (5) – (7), mean rents per patent, u, is not identified. Two unknown parameters remain, μ and β. However, all hope is not lost because we know something about these parameters.

First, on theoretical grounds and much anecdotal evidence, β should be positive—that is, there should be a positive (or at least non-negative) relationship between R&D success and patenting as discussed above. Second, I show below that μ cannot be greater than ten or twenty percent. This means that

$$1.2 * \hat{\gamma} \geq u$$

In other words, an estimate of γ serves as an "almost upper bound" on u; a definite upper bound is within ten or twenty percent of the estimate. This means that although we cannot derive precise estimates of mean rents per patent from these market value regressions, it is still possible to use these regressions to check whether estimates based on patentee behavior are substantially understated. For that inquiry, errors of ten or twenty percent are not material—instead, we want to check merely whether γ is two times or so larger than the estimates based on patentee behavior. Given the differences in samples used and the precision of the estimates, we can only make rough comparisons in any case. This is what I do below. Note further that if the link between commercial success and patenting is weak (e.g., because of long delays between technical success and commercial success), then γ will be a less biased estimate of patent rents.

The above derivation made several strong assumptions, so the model needs to be checked against some alternative specifications. In (2) and (4) I used only the first order term in the Taylor series expansion, but higher order terms might come into play, for example, if the size of a firm's total stock of patents affected the marginal patent propensity. Suppose that the total patent rents of a firm were a concave function, $h(P)$, such that $h(0)=0$. In this case, h could be expanded into a Taylor series,

$$h(P) \approx h_1 P + \frac{1}{2} h_2 P^2 + \ldots .$$

But since (7) includes higher order terms in P, γ would then be an upper bound estimate of h_1, the first order coefficient in the Taylor expansion. And because h is concave, h_1 itself is an upper bound estimator of the mean rents per patent, so γ is still an upper bound estimator of mean patent rents.[11]

11 For a concave function, the mean rent per patent, $h(P)/P$ is less than or equal to $h'(0)$ if $h(0)=0$. But Limit[$h'(P)$, $P \to 0$] = h_1, thus $h(P)/P \leq h_1$.

The model above also assumed constant returns to scale in production. The Hayashi 1982 model in (1) uses this assumption to derive the relationship between firm value and the value of the aggregate capital stock. However, this derivation can be readily adapted to permit non-constant returns to scale (Chirinko and Fazzari 1994, Galeotti and Schiantarelli 1991). These changes would not alter estimates of γ.[12] The assumption of constant returns was also used in the Hall-Hayashi-Inoue construction of the aggregate capital stock from separate stocks for tangible and knowledge capital. Departures from this assumption might mean that I mismeasure the aggregate capital stock. Below I run some tests on the composition of the capital stock using fixed values of α. I find that the estimates of γ are not sensitive to these changes. The "true" capital aggregate might also include higher order terms in A and R, but it seems unlikely that these would make much difference to estimates of γ if the first order terms do not.

2. Data

2.1 Sample

The data come from two sources, Compustat and a database of patent information from 1969 through 2002. The patent data come from the US Patent and Trademark Office (USPTO) and have been supplemented with information on citations compiled by Bronwyn Hall.[13]

To match patent data to firms in Compustat, I used a matching program initially developed for another project (Bessen and Meurer 2005). The USPTO provides an assignee name for every assigned patent after 1969. To match the USPTO assignee name to the Compustat firm name, we began with the match file provided by the NBER (Hall et al. 2001). To this we added matches on subsidiaries developed by Bessen and Hunt (2007), we manually matched names for large patenters and R&D-performers, and we matched a large number of additional firms using a name-matching program.[14] In addition, using data on mergers and acquisitions from SDC, we tracked patent assignees to their acquiring firms. Since a public firm may be acquired, yet still receive patents as a subsidiary of its acquirer, we matched patents assigned to an acquired entity in a given year to the firm that owned that entity in that year.[15] Finally, using a software program, we identified a group of Compustat firms that had unique names that could not be found in the USPTO list of assignees. These were classified as

12 The effect is to add a quasi-rent term to μ, which leaves γ unchanged.

13 Downloaded from http://emlab.berkeley.edu/users/bhhall/bhdata.html.

14 A software program determined matches between the two files by identifying firm names that matched after taking into account spelling errors, abbreviations and common alternatives for legal forms of organization.

15 This dynamic matching process is different from that used in the original NBER data set which statically matched a patent assignee to a Compustat firm. These data were developed with the help of Megan MacGarvie, to whom I am indebted.

definite non-matches.

This group of firms with match information (either a match or a definite non-match) includes 10,736 patent assignees matched to one of 8,444 owning firms in Compustat, with as many as five different owners matched to each assignee. This group accounts for 96% of the R&D performed by all US Compustat firms, 77% of all R&D-reporting firms listed in Compustat and 62% of all patents issued to domestic non-governmental organizations during the sample period. Sample statistics show that this sample is broadly representative of the entire Compustat sample, although it is slightly weighted toward larger and incumbent firms. Testing our match against a sample of 131 semiconductor industry firms that had been manually matched, we correctly matched 90% of the firms that accounted for 99.5% of the patents acquired by this group.[16]

From this group with matching information, I excluded firms that did not have at least four years of non-missing data on key variables and firms that did not perform significant R&D.[17] I kept observations from 1979 through 1997, using the first ten years of data to build stock variables for patents and R&D and eliminating later years because of possible truncation bias in patent application data (see below). Finally, because (6) and (7) involves ratios and consequently any measurement error may be greatly exaggerated in the tails of the distributions of the variables, I trimmed the sample of the 1% tails of Tobin's Q. I also experimented with other screens. I obtained similar coefficient estimates with these screens, but these methods seemed more arbitrary.

This left me with 25,861 observations of 3,451 firms. Sample statistics are shown in Table 1. As can be seen, the sample is broadly representative of R&D-performing firms, including a large portion of small and newly public firms. 85% of the observations are for firms whose primary business is in the manufacturing sector.

2.2 Variables

Key variables are defined as follows:

- The market value of the firm, V, consists of the sum of all the claims on the firm, namely, the sum of the value of the common stock, the preferred stock (valued by dividing the preferred dividend by Moody's Index of Medium Risk Preferred Stock Yields), long term debt adjusted for inflation (see Hall 1990 and Brainard et al. 1980), and short term debt net of current assets.
- The value of assets, A, is the sum of the net book value of plant and equipment,

16 Thanks to Rosemarie Ziedonis, who originally compiled this data, for sharing it with me.
17 Firms were included if they performed at least $2000 of R&D for three years.

inventories, accounting intangibles, and investments in unconsolidated subsidiaries all adjusted for inflation using the method of Lewellen and Badrinath (1997).[18]

- The R&D stock, R, is calculated assuming a 15% annual depreciation rate and an 8% pre-sample growth rate (Hall 1990). I use Bronwyn Hall's R&D deflator to obtain the current value of R from the stream of past investments. .
- The patent stock of the firm, P, is based on the number of patent applications each year that resulted in a grant of a patent by 2002. Since there is a lag of possibly several years between the application and grant of a patent (see Hall et al. 2005), I only use data through 1997. I calculate the patent stock using a 15% depreciation rate. I also calculate patent citation stocks (stocks of citations received through 2002), using a 15% depreciation and adjusting for truncation using the method described in Hall et al. (2005). In order to interpret the coefficient γ in constant ($92) dollars, I multiply the associated variable (P/A in equation 6 and $P/(A+R)$ in equation 7) by the GDP deflator.
- To explore possible strategic interaction, I also develop a measure of rival firms' patent stocks. I do this using a technology distance measure similar to one developed by Jaffe (1986). I calculate the technology distance between two firms as follows. For each firm I construct a vector of the share of its patents that falls into each USPTO technology class (there are over 400 in the 1999 classification I use; note that Jaffe used only 48 classes). The distance measure is then the uncentered correlation between these two vectors (the vector product divided by the product of the standard deviations of each vector). This measure is 1 if the firms distribute their patents identically across classes and zero if they share no patent classes (hence this distance measure might be more appropriately termed a "nearness" measure). I calculate this measure using pooled patent data over three periods from 1979 through 1999. I calculate the measure of rivals' patents as the sum of the patent stocks of all other firms weighted by the other firms' distances.

3. Empirical Results

3.1 Regression estimates

Column 1 of Table 2 uses the nonlinear specification in (6), which ignores firm specific effects.

18 Thanks to Bronwyn Hall for providing Stata code to compute this. The code was developed by Bronwyn and Daehwan Kim.

The estimate of γ is \$370,000[19] and the estimate of α is just about equal to one, as predicted. This estimation is made over a 19 year period. Below I explore the possibility that these parameters may shift over time. I also tested the sensitivity of the estimates of γ to changes in α. I ran a series of regressions (not shown) using different, fixed values for α. I found that the resulting changes in γ were small for both the specification in (6) and the one in (7). This suggests that the assumption that α approximately equals one is a safe assumption, at least for a sufficiently long sample period. It also suggests that my estimates are not sensitive to the specification of the aggregate capital stock.

The time dummies for this regression (not shown in the table) correspond to the term

$$\ln q_t + \ln(1+\mu)$$

in equation (5). The mean of these time dummies is 0.24. This allows us to make some rough inferences about the magnitude of μ. We know that q is likely greater than unity in most years for at least two reasons. First, the capital stocks of the majority of firms in our sample are growing fairly rapidly. The mean annual growth rate is 17% (median 9%). Given the quasi-fixed nature of capital—capital stocks do not adjust instantly and costlessly—this means that q should be greater than 1 during most years.[20] Second, I have not included all intangibles in my capital calculations, for example, I have not included "brand capital" that might be associated with advertising and marketing investments. Some economists argue that these intangibles significantly elevate q (Villalonga 2004). Since q is generally greater than one, this implies that μ must be less than $e^{.24}$- 1 = .27, perhaps not larger than 0.10 or 0.20.[21] As above, this, in turn, implies that γ can be used as a rough upper bound estimate on mean rents per patent.

The remaining columns in Table 2 explore specification (7), which permits firm heterogeneity. Column 2 is estimated using simple Ordinary Least Square without firm effects and Column 3 is estimated using firm fixed effects. The differences are significant and a good deal of the variance is explained by the fixed effects. I also ran the same regressions using random effects. A Hausman test rejected the random effects specification, however. These results indicate that firm heterogeneity is important.

These regressions included the first three terms of the Taylor series expansion used in (7). The coefficients have the predicted signs, but third order terms are not significant, both economically and

19 Here and in the results and comparisons below, all dollar figures are measured in 1992 dollars.

20 The sample period begins in a recession, when q would have theoretically been less than 1, and ends in a stock market bubble, when q would have theoretically been greater than 1.

21 Since $q > 1$ on average, the mean of $\ln q > 0$. This means that $\ln(1+\mu) < .27$ or $\mu < e^{.27} - 1$.

statistically, so I only use the first two order terms in subsequent regressions. Note, however, that the second order term is quite influential and should not be ignored, as has sometimes been done in the literature. This may seem surprising because this term is small for most of the sample; at the mean it is only about .01. However, because some observations have rather high values of $P/(R+A)$, the second order term is significant for these and estimates of γ show a downward bias if this term is not included.

The estimate of γ in the fixed effects regression is much smaller than the estimate in the simple OLS regression. This may be because of the role of firm fixed effects or it may be because of attenuation—the fixed effects estimate may suffer from the well-known problem of errors in panel data (Griliches and Hausman 1985). Column 4 shows an estimate using four-year differences rather than fixed effects. These estimates should suffer less from problems of errors in the panel data, although at the cost of a smaller sample size. I tested several different lags and found little change in the estimates after a four year lag. This estimate, using four year differences, falls between the OLS and fixed effects estimates and is quite close to the Nonlinear Least Squares estimate in column 1.

Hall et al. (2005) suggest that patent citation data contain additional information about patent or invention quality beyond what is captured in patent count data alone. Unfortunately, current research provides little guidance about how patent citations might affect patent propensity and patent propensity provides the rationale for including patents in a Tobin's Q regression in my model. If one supposes that patent citations reflect patent quality (and that this, in turn, affects patent propensity) then citations can be included in my model as follows: in (4) replace P by

$$P_{jt}\left(1 + \delta \frac{C_{jt}}{P_{jt}}\right)$$

where C is the patent citation stock. This leads to a regression as in (7) with the addition of a term $C/(A+R)$ and, possibly, higher order terms. Column 5 of Table 2 shows a specification with just the first order term. In this specification, the citation stock term is not statistically significant and including this term reduces the estimate of γ a bit. Although patent citations may be useful in revealing information about invention quality, these data do not appear to be particularly important to the task I address here.

Table 3 conducts some additional robustness checks. Hall (1993) found that the productivity of R&D capital exhibited short term variation relative to other capital assets. This might imply changes in α and β over time. Columns 1 and 2 show separate regressions for the first and second halves of my panel using the nonlinear specification. Although the estimates of α change, the estimates of γ do not change significantly.[22] Column 3 shows a similar test using the four-year differenced specification and variables interacted with a time period dummy. The later period shows a lower estimate for γ, although the difference is only significant at the 10% level.

Because these different estimates of γ may reflect changes in u and/or changes in β, they do not imply that patent value was necessarily lower during the 1990s than during the 1980s. Nevertheless, these results do make it seem unlikely that patent value *increased* substantially during the 1990s. This might seem to contradict the notion that patents have been getting "stronger."[23] While patents may have gotten stronger during the 1980s, my results are consistent with the view that patent value has not increased substantially during the 1990s and may have even decreased then, although, for reasons mentioned, this evidence is not conclusive.

Column 4 explores the possible role of strategic interaction. Patent rents are realized when patents deliver a degree of market power. This implies, in turn, that other firms *lose* a degree of market power if they remain in the market. Other firms' patents might influence the magnitude of patent rents and also their interpretation. In column 4, I add a variable that measures the technology distance-weighted size of other firms' patent portfolios. This has a large and significant (at the 2% level) coefficient—at the sample mean, other firms' patents are associated with a reduction in firm value of 13%. Megna and Klock (1993) also found a negative relationship in a similar regression for the semiconductor industry.[24] However, this regression shows that the estimate of γ is little changed by the inclusion of rivals' patents in the regression. So the consideration of rival patents does not seem to affect the magnitude of the private rents received, although it may suggest that non-patent rents may be adversely affected by rival's patents, offsetting the incentive that patents provide to perform R&D.

Finally, I consider the rents that can be attributed to US patents specifically; this will be useful for some of the comparisons below. Although the patent stock measure I use is a stock of US patents, firms realize value from sister patents obtained in other countries. Since these other patent counts are not explicitly included in the regressions, the estimates of patent rents implicitly include the rents earned in other countries. Putnam (1996) finds that firms patent valuable inventions in multiple countries, so worldwide patent stocks will be correlated with US patent stocks. This means, in effect,

22 The change in α is consistent with Hall's (1993) finding that the marginal profitability of R&D relative to that of physical assets declined during the 1980s. Hall interprets this as evidence of underinvestment in R&D during the early 1980s, perhaps due to accelerated obsolescence, that was corrected during the late 1980s.

23 Evidence based on court decisions suggests a pro-patentee shift after the creation of the Court of Appeals for the Federal Circuit in 1982, but the evidence for the 1990s does not indicate a further pro-patentee shift. Instead, some scholars see a modest anti-patent shift during the 1990s (Lunney 2004, Henry and Turner 2006).

24 McGahan and Silverman (2006) explore a much richer set of potential interactions.

that the patent stock variable I use serves as a proxy for worldwide patent stocks and my estimates of patent rents should be interpreted as rents on worldwide patents. In column 5 of Table 3, I estimate the domestic share of γ by using a modified regression. I divide the variable $P/(A+R)$ by the portion of the firm's profits that derive from domestic operations. Assuming that the domestic share of patent rents roughly equals the domestic share of profits, the coefficient on this term should represent the domestic share of γ. Since not all firms report domestic and foreign profits separately, I also include the original variable $P/(A+R)$ for those observations. The estimate of domestic γ I obtain is $70,000, about a fifth of the estimate of worldwide patent rents. This number might be low because of measurement error: using Putnam's data below, I find that domestic patents account for about one third of worldwide patent rents.

The regressions explored so far suggest that estimates of γ are reasonably robust to a variety of considerations.

3.2 Industry differences

These estimates all use data samples that cover all technologies and industries. Yet some evidence suggests that patent value might vary substantially across technologies or industries. In surveys, research managers in the chemical and pharmaceutical industries rate patents as much more effective at appropriating returns from inventions than do managers in other industries (Levin et al. 1987, Cohen et al. 2000). Bessen (2008), using the renewal method, finds that patents on chemical entities are about 6 times more valuable than the average patent.[25]

Table 4 shows separate regressions on different industry groups using the same specification as in Table 2, Column 4.[26] Here, the "Large pharmaceutical" category includes firms in SIC 2834 ("Pharmaceutical preparations"), that employ more than 500 employees and that are not primarily manufacturers of generic drugs. The table breaks out chemical firms, large pharmaceutical firms and firms in other industries; estimates for the latter are broken out in greater detail, but at a loss of some precision.

The estimates of γ show an order of magnitude difference between chemical firms, especially large pharmaceutical firms, and other firms. These differences are similar to those in estimates from the renewal data, but even larger. These differences in value correspond to differences that industry

25 Schankerman (1998) and Lanjouw (1998) using European renewal data, however, find pharmaceutical patents to have low or modest value relative to other patents. However, in these countries, pharmaceutical products were subject to price controls.

26 A similar pooled regression with industry interaction terms had qualitatively similar results and the null hypothesis that the coefficients on these interaction terms were zero was strongly rejected.

managers report in surveys (Taylor and Silberston 1973, Mansfield 1986, Levin et al. 1987, Cohen et al. 2000) as well as differences in propensity to patent (Scherer 1983).

As above, there is no guarantee that β is constant across industries so these inter-industry differences not directly reflect differences in patent rents. Nevertheless, such large differences seem likely to reflect underlying differences in patent rents rather than differences in marginal patent propensity.

This is significant for two reasons. First, it suggests that much of aggregate patent value is highly concentrated among a relatively small number of firms and industries. Chemical firms account for a large portion of aggregate patent assets, especially two dozen or so large pharmaceutical companies. The fifth column of Table 4 shows the implied share of aggregate patent value. Chemical and pharmaceutical firms account for over 80%. Even though this percentage may be somewhat overstated, it does suggest that, economically speaking, there appear to be two distinct patent systems: one for chemical entities and one for other technologies and it is the former that receives most of the benefit.

Moreover, these differences do not arise just because some industries spend more on R&D. The last column of Table 4 shows the ratio of implied annual patent rents (assuming a 15% rate of return applied to the aggregate discounted patent rents) to associated R&D spending. Schankerman (1998) calls this ratio the "equivalent subsidy rate," arguing that it represents an upper bound on the subsidy that patents provide to perform R&D. Clearly, this ratio is much higher for chemical and pharmaceutical firms.

Second, industry/technology is an important determinant of patent rents and this variable may affect estimates. The bottom row of Table 4 shows the mean value of γ using the separate industry estimates in the table. Clearly, the mean value of $798,000 is substantially larger than the estimates obtained in Table 2 using pooled observations. This difference is largely driven by the large pharmaceutical firms that account for 7% of the aggregate patent stock, but over half the aggregate patent value. The influence of these observations (only 2% of the observations) is apparently lost in the pooled regressions of Table 2, but show up here albeit without a high degree of precision.

3.3 Do estimates of mean patent value converge?

Scherer (1965) first pointed out that coefficients of patent stocks in market value regressions might be substantially understated if the distribution of patent values is highly skewed. This is because the coefficient essentially represents an average patent value and if the distribution is highly skewed, then the average value might converge to the true mean only slowly or perhaps not at all. In extreme cases, such as the Pareto distribution, the true mean is infinite, so averages calculated over finite samples will not be representative.

Harhoff et al. (2003b) study the distribution of invention values and conclude that the lognormal distribution, which does have a finite mean and variance, fits the data well. On the other hand, Silverberg and Verspagen (2005) argue that a Pareto distribution provides a better fit for the upper tail. But these results concern the distribution of *invention* values, not the values of patents. Patents might follow a different distribution because firms may tend to obtain more patents on highly valuable inventions, tending to compress the distribution and thin the upper tail (see Bessen 2008).

Our market value regressions provide a simple test of convergence to the mean for patent values. If patent values do converge to the mean, the variance of the stochastic error term should decrease with the size of a firm's patent portfolio, all else equal. This is because the sampling variance of the mean patent value is proportional to the inverse of the patent stock. If the distribution of patent values has an infinite mean, then the regression variance should not vary with the size of the patent stock after controlling for other size-related variables.

Table 5 shows regressions where the dependent variable is the square of the residuals from the regression in Table 2, column 4. The first independent variable is the inverse of the patent stock (coded to zero for observations with zero patents). The second variable is a dummy flag for zero patents. The second column adds controls for the log of employment and the log of deflated R&D stock. The statistically significant coefficient on the inverse of the patent stock in both regressions suggests that regression error does decrease with the size of the patent stock, rejecting the hypothesis of no convergence to the mean.

4. Comparing the Estimates

Although the theoretical analysis indicates that estimates of patent rents obtained from market value regressions are upper bound estimates, it is nevertheless informative to compare these estimates to other estimates of patent value. If the biases in market value regressions are not too great, then one would expect the estimates to be roughly similar. These other estimates might be called into question if they were much smaller than the estimates obtained from market value regressions. On the other hand, if the other estimates were roughly similar to the market value estimates, then this would suggest that these other estimates were not substantially understated.

4.1 Comparison to implied estimates from previous market value regressions

I begin by looking at previous studies using market value regressions. Although my regression equations differ from specifications used in the literature, some rough computations can be used to compare previous results to my estimates of γ. Table 6 shows some approximate calculations of equivalent estimates of γ. The details used to calculate these estimates from the original reported coefficients are described in the Appendix.

The three studies cited all use data from US publicly-listed manufacturing firms. The estimates range from $119,000 to $343,000. One study, Megna and Klock (1993), is for semiconductor firms only. Its estimate of $343,000 is not too different my estimate for the electronics industry of $407,000. The other estimates pool data from different industries and are thus most comparable to my preferred estimate from Table 2, Column 4. My estimate is a bit higher, at $351,000, than the previous estimates, but overall these estimates are reasonably consistent.

4.2 Comparison to estimates based on patentee behavior

How, then, do they compare to estimates obtained based on observations of patentee behavior? There are at least two important considerations the need to be taken into account when making such a comparison. First, as I noted above, the estimates from most of the market value regressions are implicitly estimates of the value of *worldwide* patent rights, while most of the estimates based on patentee behavior (Putnam 1996 is the exception), calculate the value of domestic rights only. Second, patents held by public firms are a select sample. Estimates based on renewal studies suggest that patents held by public firms are worth substantially more, maybe 50% more or so, than other patents.[27] Most of the estimates based on patentee behavior use samples of all patents, but market value estimates are based only on public firms.

Table 7 compares various estimates using data pooled over all industries. Putnam (1996), using data on international patent applications, estimated the value of worldwide patent rights for patents that were applied for in 1974 in more than one country. Using his figures (see Appendix for calculations), I calculate that the worldwide value for inventions associated with US patent applications in 1974 is about $230,000. Considering that this estimate is for all patents, not just patents owned by publicly listed firms, a comparable estimate (based on a 30-50% premium) for publicly traded manufacturing firm might be $300,000 – 350,000. This suggests that after adjusting for differences in samples, the

27 In Bessen (2008) the mean value for all patents granted to domestic patentees in 1991 is $78,000; the mean value for patents granted to public manufacturing firms 1985-91 is $113,000, 45% more. This is not surprising for several reasons. For one, public firms may have greater complementary assets with which to utilize patents.

values of γ (using data pooled from different industries) are quite close to the value of patents implied by Putnam's estimates.

The studies based on renewal and re-assignment data produce estimates for the value of US patents only, not the associated worldwide patent rights. These estimates are \$62,000 for patents granted in 1986 (Barney 2002), \$47,000 for patents held by small patentees (Serrano 2006), \$78,000 for all patents granted in 1991 to domestic patentees (Bessen 2008) and \$113,000 for patents granted to publicly listed manufacturing firms from 1985 to 1991 (Bessen 2008).

I compare these numbers to the estimates of γ in two ways. First, in the regression shown in Table 3, column 5, I controlled for the domestic share of patent rents using the domestic share of profits and obtained an estimate of \$70,000 as an estimate of the domestic rents from patents, reasonably close to the renewal-based estimates (although, as noted above, these are for public firms, so they should be somewhat higher).

Second, using Putnam's data (see appendix), I estimate that the value of domestic US patents runs about 32% of the value of the associated worldwide patent rights. This implies, for example, that the estimate of γ from Table 2, column 4 corresponds to a domestic patent value upper bound of about \$112,000 for publicly listed firms, quite close to the most comparable renewal estimate of \$113,000. For the other studies at the top of Table 6, this thumbnail estimate corresponds to mean domestic patent values from \$38,000 to \$110,000, quite similar to the estimates derived from renewal data.

Table 8 compares estimates for technology classes (from Bessen 2008) with estimates in this paper by industry from Table 4. These classifications are different, nevertheless, one might expect industry and technology groupings to yield broadly similar results. The estimates outside of pharmaceuticals are quite similar, however, the market value estimates for large pharmaceutical firms are an order of magnitude larger than estimates for drug and medical patents.[28] It is possible that renewal value estimates for major drug patents are understated because the drug approval process might affect the way patent value changes over time in a way that conflicts with the renewal models.[29] In any case, these comparisons taken together suggest that renewal estimates may be reasonably accurate outside of the pharmaceutical industry, but that renewal estimates appear to be substantially understated within this industry.

28 Lanjouw (1998) and Schankerman (1998) also find relatively low estimates for the value of German and French pharmaceutical patents, however, as Schankerman notes, these countries had price controls on medicines.

29 See the discussion of depreciation and option value in Bessen (2008). Mike Scherer notes that patent renewals for pharmaceuticals are very much a function of the regulatory approval process. Molecules that seem promising from in vitro tests can be patented, however, as clinical trials proceed, most are abandoned. These will not be renewed and this rapid form of obsolescence might make it difficult to model patent renewals for such patents.

The ratios of patent rents to associated R&D shown in Table 4 provide another way to compare estimates from market value regressions to estimates obtained using renewal data. Lanjouw et al. (1998) summarize this ratio from renewal studies that use European patent data but pro-rate worldwide R&D.[30] They find that the estimates fall between 10% and 15% for most studies. Arora et al. (2003) estimate a ratio of 17% using a model employing survey data for the US. The ratio for the total sample in Table 4 is 18%. Note that the same calculation made using the specification in Table 2, column 1 with pooled data ($370,000), yields an equivalent subsidy ratio of 9%. Thus, these numbers roughly correspond to the range of earlier estimates.

4.3 Comparison to benchmarks

Another way of checking the market value estimates is to compute the implied annual flow of rents from the estimated value of *u*, which is the discounted value of the future stream of rents. That is, assuming a return on investment of 15%, then 0.15 * γ * *patent stock* gives a rough, upper-bound measure of the flow of patent rents. I compare this to the profits of large pharmaceutical firms, and to the patent licensing revenues of IBM and of universities.

It is widely held that pharmaceutical profits depend heavily on patent rents. For the years 1990-97, using the estimate in Table 4, I find that estimated patent rents accounted for 62% of the deflated net income of large pharmaceutical firms. If one assumes that firms earn "normal" profits equal to 5% of revenues, then the estimated patent rents accounted for 93% of the total rents of large pharmaceutical firms.[31] Of course, pharmaceutical firms earn rents from other sources in addition to patents: they earn rents on large marketing expenditures and rents from industry regulation (generic pharmaceutical companies also make above-average profit margins[32]). Nevertheless, this suggests that the estimate in Table 4 for large pharmaceutical firms is in the right ballpark.

Another benchmark is IBM's vaunted patent licensing program, which has been taken as a model of how firms can aggressively extract value from their patents. Beginning in 2000, IBM began reporting licensing and royalty fees. The patent licensing program has earned between $3,700 and $7,600 per year per patent in force.[33] This figure is gross of the costs of the several hundred patent

30 There is a difference in the way this ratio is calculated. The renewal studies calculate the flow of the value of patent grants to pro-rated R&D used to produce these patents. I use the ratio of the flow of patent rents (on the entire patent stock) to entire R&D stock. These should be equivalent in equilibrium.

31 On average, this group of firms had net income of $14.7 billion per year, flows of patent rents of $9.1 billion per year and total rents (net income less 5% of book value) of $9.8 billion per year all in 1992 dollars.

32 The mean pretax margin on sales during 1990-97 was 19.7% for large pharmaceutical firms and was 13.9% for generic pharmaceutical firms in my sample. For the entire sample, the pretax margin was 8.0%.

33 Measured in 1992 dollars. The figures in IBM's annual reports have been mis-represented, including the much-hyped

lawyers that IBM employs. These patent licensing revenues account for 2-3% of IBM's income before extraordinary items from 1999-2003. In comparison, estimated patent rents account for 8-13% of IBM's income, roughly four times larger.[34] Considering that IBM earns rents from its ability to exclude rivals from product markets in addition to rents in the form of licensing revenues, this seems to be reasonably consistent. For 1999, the estimated patent rents account for 19-26% of IBM's total rents, again calculated assuming normal profits corresponding to a 5% net margin. Considering that IBM operates in industries known to appropriate substantial returns to innovation by means other than patents, this figure, too, seems quite reasonable.

As another rough check, I compare estimated patent rents to the gross royalties earned on university patents, which include some patents of high social value such as the Cohen-Boyer patents. In 2003, universities realized gross licensing revenues of about $1.1 billion in 1992 dollars (AUTM 2003). This excludes the cost of running technology transfer offices, which apparently consume a substantial portion of this revenue, with some universities making net profits and others losing money (Thursby and Thursby 2003). Assuming that university patents are of comparable value to the patents of public companies (university patents are more concentrated in biotech and pharma), my estimates of rents from university patents in 2003 range from $1.0 billion to $2.2 billion.[35] These estimates, too, appear to be in the right ballpark, although perhaps a bit high.

4.4 Comparison to a survey estimate of value

Another comparison can be made to the recent PATVAL survey in Europe where inventors were asked to value their patents (Gambardella et al. 2008). Using a random sample of patents granted by the European Patent Office (EPO), inventors were asked at what value they would be willing to sell their patents to rival companies.[36] Assuming a log-normal distribution (Table 8), the authors estimate the mean value of patent European inventions was $2.8 – 3.3 million.

These estimates might seem to conflict with almost all of the estimates cited above. However,

"$1.7 billion in licensing revenues." This figure actually included many other things, including the value of IP in divisions that were spun off and fees from custom software development. The annual reports list "licensing/royalty-based fees." According to sources at IBM about 60% of this comes from technology licensing and about 40% from the pure patent licensing program. I use 40% of this figure.

34 I obtained these estimates using the "Other Industries" patent value of $260,000 from Table 4 and the estimate of $351,000 in Table 2, Column 4 applied to IBM's patent stock and income from 1995-99.

35 I obtained the lower estimate using the value of $351,000 from Table 2 and the higher estimate from the value of $798,000 in Table 4.

36 The actual question asked was "Suppose that on the day in which this patent was granted, the applicant had all the information about the value of the patent that is available today. In case a potential competitor of the applicant was interested in buying the patent, what would be the minimum price (in Euro) the applicant should demand?'

these figures are not directly comparable for two main reasons. First, EPO patents are likely several times more valuable than their corresponding US patents because of stricter standards and because inventors obtain fewer EPO patents per invention.[37]

Second, this concept of patent "value" does not directly correspond to the notion of discounted patent rents estimated by the studies above. Selling a patent to a rival means that the firm can no longer practice the invention (see Harhoff et al. 2003a). This means that the firm not only gives up patent rents, but it also gives up rents that it earned on the invention by lead time advantage, learning-by-doing, etc. That is, this notion of value might be described as "invention value" rather than the value of patent rents. A rough calculation comparing my estimate of the mean value of μ above (0.24) with the mean value of uP (.028), suggests that the value of all of the rents associated with an invention might be an order of magnitude larger than the rents from patents alone.

Finally, the survey responses might be inflated for those cases where there are multiple patents on an invention. Selling just one of these patents to a rival might prevent the firm from practicing the invention at all, so the reservation value might reflect the value of *all* of the patents covering an invention. Although some survey respondents might mentally prorate this value across all of the patents involved,[38] many respondents might not make such an adjustment, leading to an inflated average value.

Thus, accounting for differences in the nature of the patents and in the concept of value measured, the survey results do not necessarily conflict with my estimates and may well be consistent with them.

5. Conclusion

The model developed in this paper shows that clear but limited inferences can be drawn about the magnitude of patent rents from market value regressions. Coefficient estimates from carefully specified equations can be interpreted as upper bound estimates of the discounted value of future rents earned by patents. Moreover, I show that that these estimates are robust to a variety of considerations including the possibility that a "fat" upper tail in the distribution of patent values may make finite sample estimates unreliable.

Only limited inferences can be drawn using these coefficient estimates because they are known to

37 According to Dietmar Harhoff, one of the authors of the PATVAL study, there are 4 US patents corresponding to each invention that also has an EPO patent application and only 70% of these EPO applications result in a grant (email 11/1/2007). He estimates that EPO patents correspond to the most valuable one-third of US patents, suggesting that the mean should be several times larger.

38 Dietmar Harhoff also noted that at least one survey respondent who was interviewed in depth did just that.

be biased upwards. For example, comparisons across sub-samples might be difficult because the bias may change by an unknown amount when different sub-samples have different patent propensities.

Nevertheless, upper bound estimates are useful because they can be compared to other estimates of patent value as a check. Economists have inferred patent values from data on patentee behavior using a variety of methods. However, these studies suffer from a common weakness: since they rely on an extrapolation for the “upper tail” of the distribution of patent values, these estimates may be understated if that tail is “fatter” than predicted. In contrast, market value regressions directly reflect the value of the patents in the upper tail.

I compare market value estimates to estimates based on patentee behavior and I find a reasonably close correspondence except for large pharmaceutical firms. Indeed, except for the pharmaceutical industry, I find that a significant number of estimates obtained using a variety of different methods yield roughly equivalent estimates. Although these estimates are not very precise, this correspondence should provide some confidence in their accuracy. This is further supported by some benchmark calculations.

However, large pharmaceutical firms are an important exception. I find that market value regressions for these firms yield much higher estimates of patent value than do estimates based on renewal data. I also find that the estimates based on market value regressions account for the majority of pharmaceutical firm’s profits, supporting the accuracy of my estimates. It might be that the regulatory approval process for drugs affects the evolution of patent value over time in a way that conflicts with the renewal estimates. In any case, I conclude that the renewal estimates are understated for pharmaceuticals.

Of course, large pharmaceutical firms make up only a couple percent of the sample. Nevertheless, because these patents are so valuable, they account for a majority of aggregate patent value among public firms. This highlights the importance of performing separate economic analysis for patents in this industry.

Finally, this analysis only concerns the mean value of the private returns to patents. To the extent that market value regressions validate estimates of mean patent value obtained from studies based on patentee behavior, other statistics of the distribution of patent values derived from these latter studies are also supported.[39] In any case, as noted above, estimates of median patent value from these studies do not suffer from the “upper tail problem.” More important, the private returns from patents are only

39 It is, of course, possible that the means could correspond but that the distribution of value in the upper tail could be “lumpy” so that estimates of, say, the 99th percentile might not correspond to each other. This seems unlikely, however.

one element that goes into the policy performance of the patent system. A social welfare calculation needs to also consider the social returns, which might be larger or smaller than the private returns, and the costs of the patent system, both to innovators and to other parties (see Bessen and Meurer 2008 for some analysis of this comparison). Nevertheless, to the extent that the literature on private returns can provide some solid estimates, we are one step closer to a complete welfare analysis.

Appendix

Derivations of (5) and (7)

Substituting (2) into (1)

$$V_{jt} = q_t(K_{jt} + W_{jt}) = q_t(K_{jt} + u\cdot P_{jt} + \mu_j K_{jt}) = q_t(1+\mu_j)\left(K_{jt} + \frac{u\cdot P_{jt}}{(1+\mu_j)}\right).$$

Then, substituting in (3) and (4),

$$V_{jt} = q_t(1+\mu_j)\left(A_{jt} + \alpha R_{jt} + \beta P_{jt} + \frac{u\cdot P_{jt}}{(1+\mu_j)}\right).$$

Introducing γ, dividing by A, and taking logs yields (5):

(5) $$\ln\frac{V_{jt}}{A_{jt}} = \ln q_t + \ln(1+\mu_j) + \ln\left(1 + \alpha\frac{R_{jt}}{A_{jt}} + \gamma\frac{P_{jt}}{A_{jt}}\right), \quad \gamma \equiv \frac{u}{1+\mu_j} + \beta\cdot$$

Since $\gamma P << K$ (as found in the nonlinear estimation), then the last term can be expanded using a Taylor series approximation:

$$\ln\left(1 + \alpha\frac{R_{jt}}{A_{jt}} + \gamma\frac{P_{jt}}{A_{jt}}\right) = \ln\left(\frac{A_{jt}+\alpha R_{jt}}{A_{jt}}\right) + \ln\left(1 + \gamma\frac{P_{jt}}{A_{jt}+\alpha R_{jt}}\right)$$

$$\approx \ln\left(\frac{A_{jt}+\alpha R_{jt}}{A_{jt}}\right) + \gamma\frac{P_{jt}}{A_{jt}+\alpha R_{jt}} + \frac{1}{2}\left(\gamma\frac{P_{jt}}{A_{jt}+\alpha R_{jt}}\right)^2 + \ldots$$

Substituting this into (5) and assuming that $\alpha \approx 1$, so that $K^* \equiv A + R \approx A + \alpha R$,

$$\ln\frac{V_{jt}}{A_{jt}} \approx \ln q_t + \ln(1+\mu_j) + \ln\left(\frac{K^*_{jt}}{A_{jt}}\right) + \gamma\frac{P_{jt}}{K^*_{jt}} + \frac{1}{2}\left(\gamma\frac{P_{jt}}{K^*_{jt}}\right)^2 + \ldots$$

or

$$\ln\frac{V_{jt}}{K^*_{jt}} \approx \ln q_t + \ln(1+\mu_j) + \gamma\frac{P_{jt}}{K^*_{jt}} + \frac{1}{2}\left(\gamma\frac{P_{jt}}{K^*_{jt}}\right)^2 + \ldots$$

which is equivalent to (7).

Imputations used in Table 6

Cockburn and Griliches (1988) use a sample of large, publicly held manufacturing firms. Their regression equation is equivalent to a first-order Taylor series approximation of (5) except that the patent term they use is P/A rather than $P/(A+R)$. To obtain the equivalent to γ, I multiply their coefficient (.111) times 1.2 (the mean ratio of $(A+R)/A$ for my sample of large public firms in 1980), yielding .133, which, deflated to 1992 dollars, is .213.

Megna and Klock (1993) use a similar term, and I use the mean ratio of $(A+R)/A$ for my sample of semiconductor firms for 1979-91 (1.62).

Hall et al. (2005) use a rather different specification that is not so easily compared to (6) or (7). Instead, I calculate the implied increase in firm value from an additional patent, holding all else constant. This is .018* V/R at the sample mean and .022* V/R at the sample median using the regression coefficients in their column 1 Table 3. Using the reported mean and median values of V and R, and deflating, this yields equivalent estimates of γ of .093 and .252.

Putnam (1996) reports that the mean value of a family of international patents held in the United States is $245,000 in 1974 dollars. Only 36% of the patents filed in the US are also filed abroad. Putnam also reports that for Germany, the aggregate value of all patents (both those filed internationally and those filed only at home) is 5% greater than the aggregate value of internationally filed patents. This implies that the aggregate value of patents is 1.05 x $245,000 x no. of int'l patents. Therefore the mean value of all patents is aggregate value/total no. of patents = 1.05 x $245,000 x .36 = $92,600, which, deflated to 1992 dollars, is $230,000. This number is not too sensitive to the 5% figure; e.g., if it is calculated assuming that domestic-only patents add 10% to the aggregate value of patents, then the mean value is $241,000.

Putnam also reports that patents granted in the US and also filed abroad were worth $75,700 in 1974 dollars. Assuming the same relationship between domestic and international patent value as in Germany, the mean value for US patents should be ($75,700 + .05*$245,000)*no. of international patents/total no. of US patents = $31,700 in 1974 dollars, or $78,800 in 1992 dollars. The ratio of domestic patent value/worldwide patent value is then $78,800/$245,000 = 32%.

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Table 1. Sample Statistics

	Mean (million $)	Median (million $)
Firm market value, *V*	1568.0	80.5
Ln *Q*	0.85	0.61
Patent stock, *P*	86.8	4.5
R&D stock, *R*	227.9	15.5
Accounting assets, *A*	917.0	34.9
Percent observations with no patents	19%	

Notes: 25,861 observations for 3,451 firms from 1979 – 97.

Table 2. Basic Specifications

	1	2	3	4	5
Estimation technique	NLLS	OLS	FE	D4	D4
Dependent variable	Ln V/A	Ln $V/(A+R)$	Ln $V/(A+R)$	Ln $V/(A+R)$	Ln $V/(A+R)$
γ	0.370 (0.024)	0.445 (0.037)	0.205 (0.061)	0.351 (0.072)	0.290 (0.084)
α	0.992 (0.023)	--	--	--	--
Other variables					
$(P/(A+R))^2$		-0.070 (0.018)	-0.019 (0.020)	-0.031 (0.007)	-0.031 (0.007)
$(P/(A+R))^3$		0.003 (0.002)	0.001 (0.001)		
$(C/(A+R))$					0.003 (0.002)
No. observations	25,861	25,861	25,861	13,317	13,317
Adjusted R^2	0.625	0.066	0.550	0.157	0.157

Notes: Standard error in parentheses are heteroscedasticity-robust. All regressions include year dummies. NLLS regression uses equation (6); other regressions use equation (7). γ is measured in millions of 1992 dollars.

Table 3. Additional Regressions

	1	2	3	4	5
	year<1990	year>=1990			
Estimation technique	NLLS	NLLS	D4	D4	D4
Dependent variable	Ln *V*/*A*	Ln *V*/*A*	Ln *V*/(*A*+*R*)	Ln *V*/(*A*+*R*)	Ln *V*/(*A*+*R*)
γ	0.373 (0.034)	0.353 (0.033)	0.462 (0.091)	0.353 (0.072)	
α	1.542 (0.050)	0.737 (0.024)	--	--	--
Other variables					
$(P/(A+R))^2$			-0.059 (0.015)	-0.032 (0.007)	-0.019 (0.007)
$P/(A+R)$ * (yr>1989)			-0.161 (0.098)		
$(P/(A+R))^2$ * (yr>1989)			0.041 (0.018)		
Log rival's patents				-0.016 (0.007)	
$(P/(A+R))$ / domestic share of profits					0.070 (0.027)
$(P/(A+R))$ * (no foreign profits reported)					0.239 (0.063)
No. observations	13621	12240	13317	13317	13317
Adjusted R^2	0.323	0.350	0.157	0.157	0.156

Notes: Standard error in parentheses are heteroscedasticity-robust. All regressions include year dummies. NLLS regression uses equation (6); other regressions use equation (7). Rival's patents are the sum of distance-weighted patent stocks; the distance measure is described in the text. γ is measured in millions of 1992 dollars.

Table 4. Estimates from Separate Industry Regressions

Industry	SIC Code	γ		No. observations	Share of aggregate value	Patent rents / R&D (1997)
Chemicals, excluding large pharmaceuticals	28	**1,465**	(344)	1,543	25%	37%
Large pharmaceutical firms	2834	**7,177**	(2,801)	272	60%	79%
Other industries		**260**	(75)	11,502	15%	6%
Machinery Including computers	35	-60	(215)	2,285		
Electronics	36	407	(211)	2,304		
Instruments	38	**380**	(149)	2,225		
Other manufacturing		-13	(317)	3,276		
Business services, including software	73	76	(1,041)	645		
Other non-manufacturing		243	(182)	767		
WEIGHTED MEAN		798	(88)	13,317		18%

Note: Estimates are for separate industries using the specification in Table 2, Column 4 for 1979-97 using equation (7) with dependent variable Ln $V/(A+R)$. Robust standard errors in parentheses. Bold coefficients are significant at the 5% level or better. The large pharmaceutical category includes firms whose primary business is in SIC 2834, who have over 500 employees and are not identified as primarily manufacturers of generic drugs. The mean and share of aggregate value are weighted by the stock of patent applications in the observation year. The aggregate value calculation ignores the role of β and assumes that γ entirely represents the discounted value of patent rents. The patent rents/R&D ratios are aggregate patent rents (depreciated patent stock times γ times a flow rate of 15%) divided by deflated depreciated R&D stock. γ is measured in thousands of 1992 dollars.

Table 5. Regressions on Squared Residuals

	1	2
1 / *P*	0.355 (0.057)	0.181 (0.056)
No patents dummy	5.785 (0.469)	3.222 (0.477)
Ln employment		-2.963 (0.123)
Ln deflated R&D stock		1.527 (0.126)
Constant	4.396 (0.165)	0.417 (0.449)
No. observations	13317	13317
Adjusted R^2	0.013	0.064

Note: Heteroscedastic-robust standard errors in parentheses. Dependent variable is square of residuals from regression in Table 2, column 4.

Table 6. Comparison to Other Market Value Regressions

Study	Sample	γ or equivalent
Cockburn and Griliches (1988)	US public manufacturing firms, 1980	\$213
Megna and Klock (1993)	US public semiconductor firms, 1972-90	\$343
Hall et al. (2005), using means	US public manufacturing firms, 1979-88	\$119
Hall et al. (2005), using medians		\$322
<u>This paper</u>	US public firms, 1979-97	
Table 2, Column 1		\$370
Table 2, Column 4		\$351

Note: γ and equivalents are measured in thousands of 1992 dollars.

Table 7. Comparison to Estimates Based on Patentee Behavior (pooled data)

Study	Sample	Value of worldwide patents	Value of domestic patents
Putnam (1994)	All international patents applied for in 1974 by US patentees	$230	
Barney (2002)	US patents granted in 1986		$62
Serrano (2006)	US patents granted to small patentees, 1983-2002		$47
Bessen (2008)	All US patents granted to domestic parties, 1991		$78
	US patents granted to US public manufacturing firms, 1985-91		$113
<u>This paper</u>	US public firms, 1979-97		
Table 2, column 4		$351	
" prorated			$112
Table 3, column 5			$70

Note: see text and Appendix for details of comparisons. The prorated estimate of domestic patent value is calculated by multiplying $351 by 0.32, a ratio of domestic US patent value to worldwide patent value derived in the Appendix. Patent values are measured in thousands of 1992 dollars.

Table 8. Comparison to Estimates Based on Renewal Data by Technology/Industry

Study	Technology/Industry	Value of domestic patents
Bessen (2008)	Chemical patents	\$497
	Drug & medical patents	\$120
	Other technologies	\$39 - 86
<u>This paper, Table 4</u>		
	Chemical industry excluding large pharmaceutical firms	\$469
	Large pharmaceutical firms	\$2,297
	Firms in other industries	\$83

Note: Figures in the bottom panel are calculated by multiplying estimates of γ from Table 4 by 0.32, a ratio of domestic to worldwide patent value derived in the Appendix. Patent values are measured in thousands of 1992 dollars.