Long-Term Debt, the Term Structure of Interest and the Case for Accrual Taxation

Theodore S. Sims
Boston Univeristy School of Law

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I. Introduction

During the past 25 years, the Internal Revenue Code has become increasingly sophisticated in its treatment of long-term debt. That transformation occurred as part of a wider set of legislative changes, changes that have made the Code generally more sensitive to the consequences of compound interest and discounted (or present) values. Much of this was dictated by necessity. By ignoring the effects of compound interest, the Code often measured income in a way that was economically unsound, and thereby allowed taxpayers to take advantage of the statutory shortcomings, often with dramatic, unanticipated results.[1]

Congress responded by incorporating into the statute provisions that more realistically take present values into account.[2] Those features tend to be complex. By enhancing the accuracy with which the Code measures income, however, they reduce the extent to which manipulation can occur. A key aspect of this transformation is the set of provisions that governs the taxation of long-term debt.[3] Their centerpiece is a require-

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1 See generally Peter C. Canellos & Edward D. Kleinbard, The Miracle of Compound Interest: Interest Deferral and Discount After 1982, 38 Tax L. Rev. 565 (1983); Joint Comm. on Tax'n, 98th Cong., 2d Sess., Proposals Related to Tax Shelters and Other Tax-Motivated Transactions 60-95 (Comm. Print 1984).

2 Comprehensive discussions of these provisions can be found in David C. Garlock, A Practical Guide to the Original Issue Discount Regulations (1990); and Lawrence Lokken, The Time Value of Money Rules, 42 Tax L. Rev. 1 (1986). For analyses of important aspects of the rules, see Daniel I. Halperin, Interest in Disguise: Taxing the "Time Value of Money," 95 Yale L.J. 506 (1986); Canellos & Kleinbard, note 1.

3 See generally IRC §§ 1271-1286.

ment that interest accrue using what is often called "economic" accrual.[4] It measures income from debt in a fashion that seems to be essentially coherent and economically sound.

Nevertheless, a recent article by Professors Joseph Bankman and William Klein draws attention to the possibility that, because of a persistent economic phenomenon known as the "term structure of interest," the Code continues to mismeasure the accrual of interest in a pervasive, but unpredictable way.[5] Bankman and Klein suggest that the distortions they identify may be susceptible to manipulation. They assert that, under a system that taxes asset gains on realization, in principle the mismeasurement cannot be solved. They raise the possibility that the dimensions of the problem may be substantial.[6] More broadly, their analysis suggests that the influence of the term structure on the taxation of long-term debt may lend weight to the case for adopting a periodic "accrual" tax, in lieu of our current realization based tax.[7]

The possibility that the existing taxation of long-term debt may be seriously flawed deserves to be explored in greater detail. That is the objective of this article. Section II.A begins by describing the problem more concretely. Then, to furnish a perspective on subsequent observations, Section II.B briefly reviews the developments (which will be familiar to many) that have led us to tax long-term debt as we currently do. The next three sections explore the matter in depth. Section III reviews the literature on the term structure of interest (or "yield curve").[8] Prevailing economic understanding of that phenomenon suggests that the mis-

[4] IRC § 1272(a). The development and operation of this provision is described in some detail in Section II.B. The concept of "economic" accrual is described at notes 11-15, 33-34 and accompanying text.

[5] Joseph Bankman & William A. Klein, Accurate Taxation of Long-Term Debt: Taking Into Account the Term Structure of Interest, 44 Tax L. Rev. 335 (1989). Bankman and Klein draw on earlier work by Bruce Kayle. See Bruce Kayle, Where Has All The Income Gone? The Mysterious Relocation of Interest and Principal in Coupon Stripping and Related Transactions, 7 Va. Tax Rev. 303 (1987).

The nature of the term structure of interest and the impact it may have on the taxation of long-term debt is described in Section II.A. Its economic causes are explored in Section III.

[6] Bankman & Klein, note 5, at 346 & n.24. According to the Federal Reserve, there was, at the end of 1989, a little less than $10 trillion of net credit market debt outstanding in the United States. Board of Governors of the Federal Reserve, Flow of Funds, 77 Fed. Reserve Bull. A44-A45 (Jan. 1991) [hereinafter 1989 Flow of Funds Data].

[7] Bankman & Klein, note 5, at 348. Under an accrual tax, assets would be valued periodically, typically annually, and changes in value occurring between the current valuation and the preceding valuation would be taken into account for tax purposes as gain or loss, regardless of whether the asset was sold or otherwise disposed of during the year. See Jeff Strnad, Periodicity and Accretion Taxation: Norms and Implementation, 99 Yale L.J. 1817 (1990) [hereinafter Periodicity]. Compare IRC §§ 61(a)(3), 1001(a). See generally David J. Shakow, Taxation Without Realization: A Proposal for Accrual Taxation, 134 U. Pa. L. Rev. 1111 (1986).

[8] The expressions "term structure of interest," "term structure," and "yield curve" all are used to denote the phenomenon in question. Throughout this article, those terms are used interchangeably, although, for consistency, yield curve generally is preferred.

measurement of income it induces might be rectified within the framework of the existing system, contrary to the conclusion arrived at by Professors Bankman and Klein. Doing so would add, however, to the complexity of an already formidable body of rules. It is therefore appropriate to ask how much might be gained by the endeavor. That question is investigated in Section IV. An examination of post-war term structure data discloses that, in comparison with what Bankman and Klein's examples suggest, the actual yield curve typically is not pronounced. Hence, its impact on the measurement of income characteristically will be modest, especially when compared to other inaccuracies in our taxation of long-term debt. So the gains to be achieved might very well prove to be small.

Sections III and IV establish that, if we thought the effort justified the cost, we could correct for the effects of the yield curve on the taxation of long-term debt. Considered in isolation, then, this particular phenomenon adds little to the argument for an accrual tax. But the yield curve is only one of several phenomena that can affect the measurement of income from debt. So, in a somewhat more tentative way, Section V explores the broader question of the extent to which inaccuracies generally in the taxation of debt—including, but not limited to those induced by the yield curve—contribute to the case for a periodic accrual tax. A somewhat unexpected conclusion suggested by this exploration is that, short of accruing gain to long-term debt on an essentially continuous basis—a procedure that would be administratively impracticable even if it were feasible—the existing rules, although in some sense an approximation, may actually furnish the best approximation that we can hope to achieve.

II. The Problem and its Background

A. Original Issue Discount and the Yield Curve

Professors Bankman and Klein illustrate the impact of the yield curve on the taxation of long-term debt using a debt instrument issued at a discount.[9] An obligation of that sort typically is referred to as an original issue discount (OID) obligation. In pure form, it requires the borrower to make a single repayment at maturity to the lender, or holder. Compensation to the lender takes the form of the "discount"—the difference between the price at which the instrument was issued and the single payment to be received when it is redeemed.[10]

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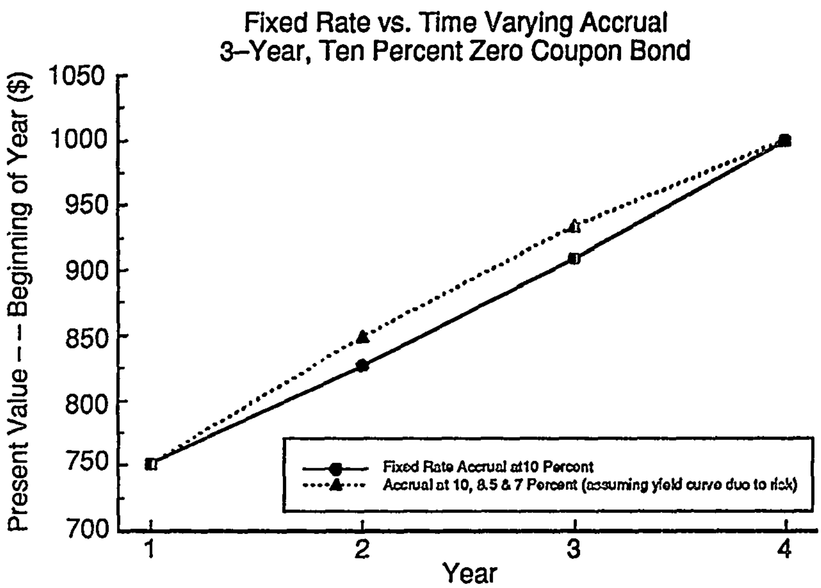
[9] Bankman & Klein, note 5, at 337.

[10] The most familiar example of an original issue discount obligation is probably a U.S. savings bond. For purposes of this article, that example, although illustrative, is slightly misleading. U.S. savings bonds are exempt from the original issue discount rules. IRC § 1272(a)(2)(B). For debt instruments subject to those rules, original issue discount is, with

Section 1272(a) prescribes the tax treatment of original issue discount obligations. In its existing form, that provision generally requires that the discount be treated as accruing over the life of a debt instrument using a constant interest rate,[11] and assumes that interest compounds at least once every year.[12] The periodic compound interest methodology prescribed by the statute, although often referred to as "economic accrual," is actually a discrete version of what in other settings is denoted "exponential growth."[13] The pattern of accrual it produces is illustrated, for a $1,000 face amount, three-year, 10% zero-coupon bond, by the line denoted with circles in Figure I.[14] As so calculated, the discount that

FIGURE I



minor exceptions, defined as the difference between the instrument's "stated redemption price at maturity" and its "issue price." IRC § 1273(a)(1)-(2), (b).

[11] IRC § 1272(a)(3), (4).

[12] The Code itself prescribes the use of semi-annual compounding, except as otherwise provided by regulation. IRC § 1272(a)(5). Prop. Reg. § 1.1272-1(d) has modified this provision to allow, in effect, compounding at a frequency of the parties' choosing, provided that compounding occurs at least annually.

[13] See note 35; cf. Richard Courant, Differential and Integral Calculus 178-80 (1937). Exponential growth at a *constant rate* produces a pattern of growth that *increases in amount* over time. See notes 14, 34.

[14] Figure I plots the value of the instrument on the date of its issue ("Time 0," or the beginning of "Year 1") and at the beginning of each succeeding year. Figure II plots the interest that accrues to the instrument's holder during each of the three years it is outstanding. The statutory accruals of interest to the holder of this obligation would be $75.13, $82.64 and $90.91 during Years 1, 2 and 3, as calculated at note 34.

accrues each year is includable in the income of the holder of the instrument and deductible by its issuer.[15]

Succinctly stated, Bankman and Klein's insight is that the use of a constant interest rate—which they refer to as the "single rate convention"—to calculate the accrual of interest over the life of a long-term debt instrument is inconsistent with economic reality.[16] At any time, the interest rate charged by the market may vary with the term of a debt instrument, all other factors that influence interest rates held constant.[17] The contours of this "term structure of interest," or "yield curve,"[18] vary (sometimes widely) over time.[19] Nevertheless, the phenomenon itself persists. Hence, accruals of interest calculated using the single rate convention characteristically are inaccurate, certainly when compared to periodic market valuations of outstanding debt, which implicitly would take the yield curve (as well as changes in other economic variables) into account.[20]

The nature of the discrepancy the yield curve can induce is illustrated in Figures I and II. The line denoted with triangles in Figure I depicts successive market valuations, under a hypothetical yield curve, of the same three-year, zero-coupon bond.[21] Figure II depicts the difference

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[15] IRC §§ 1272(a)(1), 163(e).

[16] Bankman & Klein, note 5, at 335-36; see also Kayle, note 5, at 313-24 & tbl. 2.

[17] See generally Richard A. Brealey & Stewart C. Myers, Principles of Corporate Finance 547-58 (3d ed. 1988); Robert J. Shiller, The Term Structure of Interest Rates, in Handbook of Monetary Economics 627 (Benjamin M. Friedman & Frank H. Hahn, eds. 1990).

[18] See note 8.

[19] An informative graphical depiction of the post-war contours of the U.S. term structure can be found in Shiller, note 17, at 630 (Figure 13.1).

[20] The nature of the inaccuracies is illustrated in Bankman & Klein, note 5, at 338-46. Periodic market valuations would form the basis for accruing gains and losses from long-term debt under an accrual tax. See note 7; notes 178-84 and accompanying text; Shakow, note 7, at 1113-18.

[21] The example assumes that, throughout the three-year period, the term structure of interest was such that lenders insisted on yields of approximately 10%, 8.5% and 7%, respectively, for instruments with three years, two years and one year remaining to maturity. On that assumption, the instrument's present (market) value at the time of issue and at the beginning of years two and three would be:

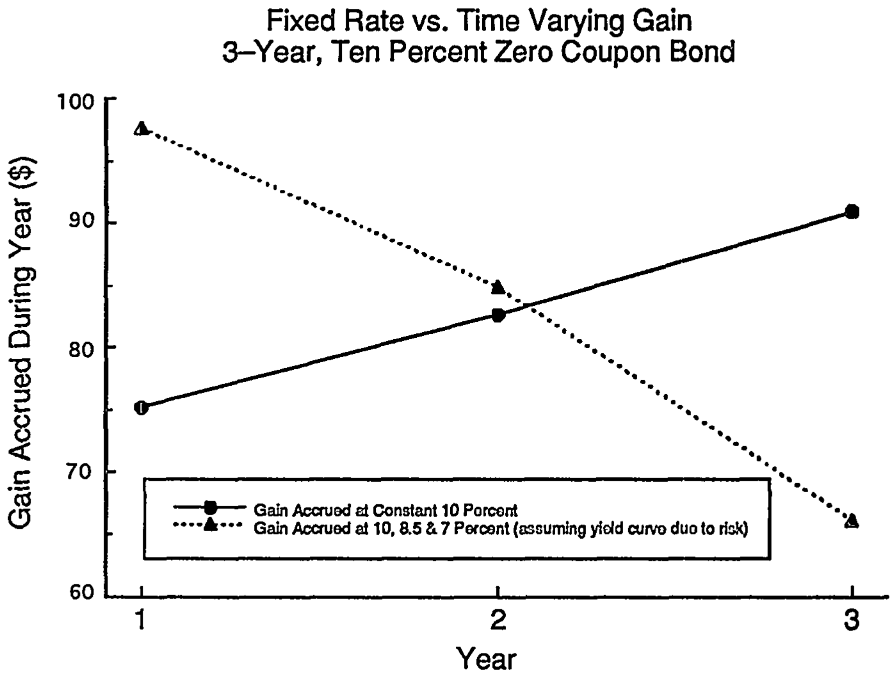
Begin Year	*Computation of Present Value*		*Present Value*
1	$\$1{,}000/(1.1)^3$	=	\$751.31
2	$1{,}000/(1.085)^2$	=	849.46
3	1,000/1.07	=	934.58
4	1,000		

The gain accrued each year is the increase in the instrument's value between the beginning of that year and the beginning of the succeeding year, or \$98.15, \$85.12 and \$65.42, respectively.

In comparison with actual experience, the slope of the yield curve in this example has been substantially exaggerated for purposes of illustration. See notes 114-28 and accompanying text. The example assumes that the yield curve is attributable to risk. See notes 57-66, 74-85 and accompanying text.

between the interest accruing annually to the holder of the obligation under the single-rate convention used in the original issue discount rules and the gain implied by successive market valuations. As that figure illustrates, the yield curve (in theory) can alter significantly the rate at which income accrues.[22]

FIGURE II



Some events that influence the market's valuation of debt *after* it is issued—principal among them subsequent changes in interest rates generally, or in the creditworthiness of the issuer—cannot be anticipated at the time of issue. Bankman and Klein capture such changes with the observation that OID calculations under the existing scheme almost invariably are inaccurate ex post.[23] Their point, however, is that statutory OID calculations, which accrue interest at a constant rate and thereby ignore the term structure to begin with, are even inaccurate ex ante. The insight seems undeniably correct.

[22] The gains depicted in Figure II are the year-to-year changes in the values depicted in Figure I. See notes 14, 21. On the assumptions about the yield curve used in constructing this example, the yield curve accelerates the rate at which income accrues. Id.

It should be emphasized that the influence of the yield curve is not confined to original issue discount debt. See notes 97-98, 139-141, and accompanying text.

[23] Bankman & Klein, note 5, at 336.

B. *Origins of the Rules Governing Taxation of Long-Term Debt*

Before exploring the causes and magnitude of this phenomenon, it is worth reviewing how far the taxation of debt instruments has come. Sections 1272 and 1273 are embedded in a wider framework of statutory provisions that, although unquestionably imperfect, take a largely coherent approach to the taxation of long-term debt. The unifying theme is that, as an economic matter, interest accrues to *all* indebtedness on a compound basis—in essentially the same way as interest credited to savings deposited in a bank.[24] The provisions themselves so account for interest and currently include it in income on that basis. But it took nearly 30 years of legislative effort, beginning more than 40 years after the advent of the income tax, to reach that point.

Historically, much of the difficulty has involved instruments issued at a discount. It now seems obvious that original issue discount is part of the compensation paid by a borrower to a lender for the extension of credit.[25] It is interest, in addition to whatever interest is explicitly stated and paid. That, however, has not always been self-evident. Before 1954, the statute sometimes was interpreted as classifying gain accruing to the holder of a debt instrument as long-term capital gain,[26] to be taxed at favorable rates at the time of surrender or sale. When it was, the holder was taxed on a deferred and otherwise preferential basis.

Congress began chipping away at this characterization beginning with the 1954 Code. It enacted what originally was § 1232, which expressly provided that gain from the sale of a debt instrument originally issued at a discount would be characterized as ordinary income.[27] Even with that provision, however, the periodic increase in the value of a debt instrument, although characterized as interest, was not taxed until the instrument was surrendered or sold. That was so even though issuers of discount obligations, typically corporations using the accrual method of

[24] See notes 33-34 and accompanying text. This analogy is made explicit by a number of authorities. See, e.g., Garlock, note 2, at 73-74; Lokken, note 2, at 10-18; see also Halperin, note 2, at 509-10.

The essential similarity between economic accrual on discount debt and on bonds issued with coupons attached is described at note 46.

[25] See, e.g., Commissioner v. National Alfalfa Dehydrating & Milling Co., 417 U.S. 134, 142-45 (1974); United States v. Midland-Ross Corp., 381 U.S. 54, 57-58 (1965).

[26] See, e.g., Commissioner v. Caulkins, 144 F.2d 482 (6th Cir. 1944).

[27] Former IRC § 1232(a)(2)(B), (b), Pub. L. No. 83-591, 68A Stat. 326, 326-27 (1954). When the original issue discount provisions were amended in 1984, they were also recodified. The provisions of former IRC § 1232(a)(2)(B) are now found in IRC § 1271(c)(2)(A), Pub. L. No. 98-369, 98 Stat. 533, 533 (1984).

After promulgation of the 1954 Code, the courts also fell into line, repudiating holdings like that in *Caulkins*, 144 F.2d 482. This line of cases culminated with *Midland-Ross Corp.*, 381 U.S. 54.

accounting, could deduct the discount currently as it accrued.[28] Consequently, the deferral of income experienced by the holder of a discount obligation was not offset by any corresponding deferral of deductions (and acceleration of income) to the issuer.

That state of affairs prevailed until 1969, when Congress first provided for the current inclusion of original issue discount.[29] It enacted what was then § 1232(a)(3),[30] which provided for periodic accrual of income to the holder of a debt instrument issued with OID. That section prescribed a ratable (or straight-line) methodology that had the effect of spreading the discount evenly over the life of the instrument.[31] This treatment was identical to the method, which continued to be prescribed by regulation, of calculating the interest deductions allowed to issuers of discount obligations.[32] Hence, for the first time, the 1969 legislation brought the inclusion of discount to the holder into line with the allowance of the deduction to the issuer. That was an important advance.

Since that time, however, it has come to be understood that ratable accrual of original issue discount is not economically sound. In the simple instance of a lender extending credit under an agreement that does not provide for periodic payments of interest, the issue price of the debt instrument is strictly analogous to the deposit of money in an account with a bank. Over time, interest is credited on a compound basis, not only to the original deposit, but also to any accumulated interest. So, too, with a debt obligation issued without stated interest: The holder implicitly agrees to leave with the borrower not only the amount originally lent at issue, but also the interest accruing over time.[33] If interest is assumed to accrue at a constant *rate* over the life of the obligation, there exists a unique rate that, when multiplied initially by the issue price, and thereafter by the sum of the issue price and all prior accruals of interest, produces a sequence of accruals that, when added to the issue price, equals the stated redemption price on the maturity date. That rate corre-

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[28] The regulatory provision that sanctioned this treatment is now found in Reg. § 1.163-3(a)(1), which provides that original issue discount on pre-1969 bonds is to be "prorated or amortized over the life of the bonds." Versions of this regulation predated the enactment of the 1954 Code, and generally were interpreted as permitting ratable deduction of the discount. The history of this aspect of the tax system is set out in *National Alfalfa*, 417 U.S. at 142-45.

[29] Tax Reform Act of 1969, Pub. L. No. 91-172, 83 Stat. 487.

[30] The provisions of former IRC § 1232(a)(3) now are found in IRC § 1272(b) (recodified in 1984).

[31] Former § 1232(a)(3)—and existing § 1272(b), which by its terms, continues to govern the taxation of any outstanding corporate debt issued before the effective date of the Tax Equity and Fiscal Responsibility Act of 1982, Pub. L. No. 97-248, § 231, 96 Stat. 324, 496-99—divide the aggregate original issue discount into equal monthly installments, which were includable in the holder's income for as many months as the instrument was held during the taxable year.

[32] Reg. § 1.163-4.

[33] See note 24 and accompanying text.

sponds to the rate at which interest is credited to the account with the bank. It is usually denoted as the instrument's "yield to maturity."[34]

The immediate significance is this. The *rate* at which interest is credited remains constant. Nevertheless, the *amount* of interest accruing in each period increases exponentially with time.[35] Since, however, the *aggregate* amount of discount to be accrued under either ratable or economic accrual is ultimately the same, the ratable accrual prescribed in 1969, which spread the total discount evenly over the life of the obligation, was, at least by comparison with fixed-rate exponential accrual, "too fast." Consequently, the ratable inclusion rules operated to acceler-

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[34] See, e.g., Brealey & Myers, note 17, at 548; Prop. Reg. § 1.1272-1(f).

To take a concrete illustration, from which the constant-rate accrual depicted in Figures I and II was derived, a deposit of $751.31 to an account with a bank that agrees to pay interest at a 10% annual rate and to credit and compound interest annually will grow to about $1,000 in three years. Assuming annual compounding, the yield to maturity of a three-year debt instrument issued for $751.31 that will be redeemed for $1,000 is approximately 10%. The addition of interest at 10%, first to the $751.31 issue price and then to the sum of the issue price and all prior accruals of interest (as with the addition of interest to the account with the bank) increases the instrument's value to about $1,000 by the end of three years, as illustrated in the following table:

Begin Year	*Value at Beginning of Year*	*Computation of Discount Accrued During Year*		*Interest*
1	$751.31	$751.13 x .10	=	$75.13
2	826.44	826.44 x .10	=	82.64
3	909.08	909.08 x .10	=	90.91
4	999.99			

The resulting pattern of accrual also is depicted by the lines marked with asterisks in Figures I and II.

With investments other than bonds—for example, investments in physical capital—the same concept is usually referred to as the investment's "internal rate of return." See Brealey & Myers, note 17, at 77-85, 548.

[35] It is worth emphasizing here that the choice of compounding period is arbitrary. It can be shortened (or lengthened) and the periodic interest rate appropriately adjusted at will. Financial institutions now typically compound interest (on both deposits and consumer loans) on a daily basis, and the parties to a debt instrument subject to the original issue discount rules are free to select whatever compounding period they like, as long as compounding occurs at least annually. See note 12. (The formula used to convert a rate of interest based on one compounding frequency into the equivalent rate for a different compounding frequency is exemplified in Prop. Reg. § 1.1274-3(e)(4)(iii).)

As the subdivision of each compounding period becomes arbitrarily small, the algebraic formula used to compute future (and, by inversion, present) values using discrete compounding—$FV = PV(1 + {}^{r}/_{k})^{nk}$, where r denotes the rate of interest, n the number of compounding periods and k the number of times each compounding period is subdivided—takes on the continuous form e^{rn}, which more generally is the expression that describes continuous *exponential* growth at rate r (when n is reinterpreted as denoting the continuous passage of time). See Courant, note 13, at 178-80. In other words, economic accrual of interest of the sort now used in the original issue discount rules is simply a discrete version of what in the limit is continuous exponential growth.

ate the income of holders and the interest deductions of issuers of obligations subject to those rules.[36]

Inaccuracies in tax accounting have a rich history of being vulnerable to manipulation. Accounting inconsistencies tend to fare even worse. The ratable inclusion rules enacted in 1969 were inaccurate, but not generally inconsistent, as they required ratable accrual of original issue discount by holders and issuers alike. Hence, the advantage to issuers of interest deductions that were economically accelerated was offset by premature and therefore disadvantageous taxation of the holders, at least when both issuers and holders were taxed.[37] In crude terms, the ratable inclusion rules created a zero-sum game between holders and issuers as groups. Consequently, opportunities to exploit those rules may not have been immediately apparent.

Over time, however, such possibilities emerged. Exempt pension trusts and other tax-exempt organizations with portfolio wealth were (and still are) sizable holders of debt.[38] To them, as a group, the accelerated rate at which interest technically accrued to the holder of a discount obligation was inconsequential.[39] Thus, the late 1970's witnessed the emergence of corporate obligations issued at a discount for placement with pension trusts and other exempt organizations.[40] While the holders were indifferent to the rate at which interest accrued, the accrual method issuers, under the then prevailing rules, were free to deduct the original issue discount using the ratable methodology prescribed by the regulations.[41] By comparison with the rate at which interest accrued and was deductible on nondiscount debt, the resulting deductions were accelerated and therefore advantageous.[42] Such developments, together with other, equally serious shortcomings in the taxation of arrangements involving

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[36] See, e.g., Joint Comm. on Tax'n, 97th Cong., 2d Sess., General Explanation of the Tax Equity and Fiscal Responsibility Act of 1982, at 159-61 (Comm. Print 1982) [hereinafter 1982 Bluebook].

[37] Id. at 161.

[38] As of the end of 1989, private domestic holdings of credit market debt amounted to about $8.9 trillion, of which approximately $2.1 trillion, or 23%, was held by insurance and pension funds. 1989 Flow of Funds Data, note 6, at A45. This figure does not include debt held by the other species of exempt organizations enumerated in § 501(c), whose holdings are not separately accounted for in the Flow of Funds accounts.

[39] Cf. 1982 Bluebook, note 36, at 161.

[40] E.g., N.Y. Times, Apr. 24, 1981, at D8, col. 3; N.Y. Times, June 22, 1981, at D3, col. 1.

[41] See notes 28-32 and accompanying text.

[42] See 1982 Bluebook, note 36, at 159-61. The illustration used in the 1982 Bluebook seems strained. A more spectacular illustration could have been furnished by so-called "coupon-stripping" transactions, also dealt with in the 1982 legislation, which highlight the essential identity between economic accrual of interest on a discount obligation and the payment of stated (or coupon) interest at a fixed rate on nondiscount debt. See note 46.

present values, culminated in major revisions to the applicable rules in 1982 and, then again, in 1984.[43]

For purposes of this discussion, the most salient of these changes were the 1982 amendments to § 1232(a)(3) (revised in 1984 and recodified as § 1272). Those revisions shifted the taxation of interest on long-term discount debt from ratable to periodic exponential accrual, for purposes of determining both the amount to be included in the income of holders and to be deducted as accrued interest by issuers.[44] The statute now requires the accrual of discount to be calculated using the yield to maturity that sets the issue price of an instrument equal to its stated redemption price at maturity by the maturity date, on the assumption (except where altered by the terms of the debt) that interest compounds semi-annually.[45]

It seems safe to say that the revisions that have been enacted since 1969, although at times intricate in detail, have achieved progressively greater accuracy in the measurement for tax purposes of accruals to the wealth of holders of debt. In fact, they achieve a high measure of consistency between the treatment of original issue discount and the taxation of interest on other forms of debt.[46] As they stand, however, these rules

[43] Tax Equity and Fiscal Responsibility Act of 1982, Pub. L. No. 97-248, §§ 231-232, 96 Stat. 324; Tax Reform Act of 1984, enacted as the Revenue Provisions of the Deficit Reduction Act of 1984, Pub. L. No. 98-369, §§ 41-44, 98 Stat. 494. The statutory remedies to some of these problems are enumerated briefly in note 45.

[44] IRC §§ 1272(a), 163(e).

[45] IRC § 1272(a)(3)-(5); Prop. Reg. § 1.1272-1(c)-(f); see also note 12.

Among the more conspicuous of the other features of this overhaul were provisions that (1) require interest on debt arising in connection with many transactions involving the sale of property to accrue on an economic basis (§ 1274); (2) corrected a severe misallocation of basis in connection with coupon-carrying debt (§ 1286) (see also note 46); and (3) characterize gain on the sale of debt instruments purchased in the market at a discount as accrued interest rather than as gain from the sale of a capital asset (§ 1276).

Descriptions and analyses of these provisions can be found in the authorities cited at notes 1-2.

[46] The essential similarity between the accrual of interest on discount and interest-bearing debt will be of importance later in this article. See notes 95-99, 139-45, and accompanying text. The degree of consistency achieved by the 1982 and 1984 legislation is highlighted most dramatically by the coupon-stripping rules, enacted in 1982 as § 1232B, and recodified in 1984 as § 1286. See Canellos & Kleinbard, note 1, at 572-76; see generally Kayle, note 5.

Before then, conventional treatment of debt that provided for periodic payments at the prevailing market rate, and hence was issued at a price in the vicinity of par, was that the coupons, as "interest," were includable in income; the amount to be received at maturity was a recovery of the holder's basis and, therefore, was not. This implied that the purchaser's basis was properly allocable in its entirety to the underlying bond. When detached from the coupons, however, the bond was worth just the present value of the payment to be made at maturity, identical to that of an otherwise comparable zero-coupon bond, and *less than* the cost of acquiring both the coupons and the bond. Given the conventional basis allocation, the bond without the coupons could therefore be disposed of immediately after issue at a loss.

The key to this problem lay in the fact that, in essence, each constituent element of a coupon-carrying bond was itself a separate obligation. Canellos & Kleinbard, note 1, at 572-76. In effect, a coupon bond is a portfolio consisting of its constituent payment obligations. Hence, the solution, when a coupon was detached (except to surrender it at the time it fell due), was to constitute an advance, not perfection. As with pretty much everything else in the Code, they contain their share of concessions to practicality and political expedience.[47] Even with their imperfections and complexity, the system as it stands is more accurate and less vulnerable to manipulation than it was before the advent of these rules.

There is one unrectified shortcoming that bears on matters to be taken up below. That is the treatment of debt acquired in the market at a discount. The 1984 legislation characterizes as ordinary income any gain on disposition of a debt instrument attributable to market discount that has accrued between the date the instrument was acquired and the date of disposition.[48] In contrast with original issue discount, however, market discount is not taxed currently as it accrues.[49] The discrepancy be-

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treat the bond and each coupon as a separate "zero-coupon" obligation, each subject to the original issue discount rules, to which basis was required to be allocated in accordance with its respective present value. IRC §§ 1286, 1281(b)(1)(F).

The illuminating phenomenon is this: If a bond were to be disassembled into its constituent obligations when it was trading at par, the sum of the amounts accrued annually as original issue discount on the collection of separate obligations would *equal* the amount *stated* as interest on the undisassembled bond. Hence, treating each coupon as interest in its entirety (and taxable as "income") when it falls due is a good proxy for separately accruing the discount on each constituent element of the bond. Canellos & Kleinbard, note 1, at 572-73; cf. Garlock, note 2, at 216.105.

In practice, this year-by-year identity may break down, because the yield curve values obligations of different maturities using different discount rates. E.g., id. at 216.105-07; Kayle, note 5, at 315-22 & tbl. 2. Even in the absence of a yield curve, the identity may be obscured—but in this case it *does not* break down—if the instrument is disassembled at a time when its market value differs from par. In that event, the sum of the amounts accrued each year pursuant to §§ 1286 and 1272(a) on all constituent elements add up to the amount explicitly stated as interest *plus* that portion of the market discount or premium produced by the sale that properly (that is, exponentially) accrues. See Canellos & Kleinbard, note 1, at 574-76.

[47] Among their concessions to practicality, the rules do not, for example, generally insist on accrual of discount on obligations having a term of one year or less; and, when they do, they do not insist on using economic (as contrasted with ratable) accrual of interest over periods of less than one year, in as much as ratable interpolation of the accrual of discount is prescribed both for short-term instruments, and, within accrual periods (which may be as long as one year) for longer-term debt. IRC §§ 1283(b)(1), 1272(a)(3); Prop. Reg. § 1.1272-1(d). Retention of the first of these approximations apparently rested on the judgment that "the tax benefit from a one-year deferral is not large enough to warrant subjecting taxpayers to the additional complexity of accrual accounting." Joint Comm. on Tax'n, 98th Cong., 2d Sess., General Explanation of the Revenue Provisions of the Deficit Reduction Act of 1984, at 100 (Comm. Print 1984) [hereinafter 1984 Bluebook]. Ratable accrual for periods of less than one year presumably was retained because it conforms to existing financial practice, which allocates between purchaser and seller interest accrued on debt that changes hands between interest payment dates using ratable approximations.

As a matter of political expedience, the provisions subjecting debt instruments that arise out of transactions involving the sale of property to the original issue discount rules contain a variety of special exceptions. See IRC §§ 1274(c)(3), 1274A.

[48] IRC § 1276(a). Market discount generally is treated as accruing ratably, but the holder may elect exponential accrual. IRC § 1276(b)(1)-(2).

[49] In this respect, the treatment of market discount is very much like the treatment of original issue discount under the 1954 Code, before the enactment of the ratable accrual rules in 1969. See notes 27-28 and accompanying text. The holder of an obligation acquired in the

tween the treatment of market discount and original issue discount may be the product of expedience.[50] Congress explicitly acknowledged in enacting these provisions that, from the standpoint of the holder, market and original issue discount function in identical ways.[51]

Nevertheless, there are distinctions between the two that bear on the ease with which they may be taxed. The amount of original issue discount, and the rate at which it is to accrue, are fixed when the instrument is issued. Once fixed, the rate of accrual is not affected, even when the instrument changes hands.[52] As a result, the basic OID calculations need be made only once (and can be made by the issuer).[53] In contrast, market discount might arise *whenever* an instrument changes hands. The amount of discount (and the implied yield-to-maturity) would have to be calculated separately on every such occasion.[54] Consequently, from a computational standpoint, mandatory current accrual of market discount would be more burdensome than the accrual of OID. It is evidently for such reasons that Congress elected to treat market discount in the fashion that it did.[55]

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market at a discount may, however, elect to include the discount in income currently as it accrues. IRC § 1278(b).

50 It is also somewhat surprising in view of the fact that when a bond is acquired in the market at a premium, the holder is permitted to deduct the premium over the life of the bond. IRC § 171. That provision, like § 1276, is, in fact, elective, but, since § 171 applies *only* to debt instruments acquired at a premium, a taxpayer may elect to deduct acquisition premium currently while simultaneously deferring the accrual of interest on instruments acquired at a discount.

51 1984 Bluebook, note 47, at 93.

52 In general, the original issue discount rules preserve the rate of accrual in the hands not only of the original purchaser, but also of subsequent holders. The amount of original issue discount is reduced, however, when the instrument is acquired at premium in relation to (approximately) its adjusted issue price at the time of acquisition, and is reduced to zero if the instrument is acquired for a price in excess of its stated redemption price at maturity. IRC § 1272(a)(7), (d)(1). The effect of these provisions is to amortize any premium against the accruing original issue discount when the instrument has been acquired in the market at a premium. By the same token, when an original issue discount obligation trades after issue at a discount to its revised issue price, the original issue discount continues to accrue at the rate originally fixed at issue. IRC § 1278(a)(2).

53 Under IRC § 6049(a), (d)(6), original issue discount is among the categories of interest for which issuers are required to furnish to holders information returns showing the amount includable in income each year. The OID calculations thus are made for the holder by the issuer. In the case of an OID obligation that has changed hands after issue, OID is reported as it would have been to the original holder, so only the adjustments required by § 1272(a)(7) need be made by the holder. See note 51.

54 Market discount arises whenever an instrument not subject to the original issue discount rules changes hands for less than its "stated redemption price at maturity." IRC § 1278(a)(2)(A). For an obligation with original issue discount, it arises if the instrument changes hands for less than the sum of the issue price plus all accruals of OID up to the date of transfer, referred to as its revised issue price. IRC § 1278(a)(2)(B), (4).

55 See 1984 Bluebook, note 47, at 93. It is worth observing that the complications that characterize the taxation of original issue discount debt always may be avoided by refraining from issuing debt obligations at a discount. Indeed, it is likely that the legislative enactments

The net effect, in any event, is that the taxation of fluctuations in the market value of debt is a good deal less refined than the taxation of OID. Until realization, market gains and losses, to both issuers and holders, generally are ignored. When debt changes hands, however, the purchaser may deduct acquisition premium, but also may defer the inclusion of market discount; for issuers, both continue to be ignored.[56] Some implications of these provisions are taken up in Section IV.

More important than the continuing imperfections, however, are the improvements that the system has achieved. Income from most debt is now characterized as ordinary income, and is taxed currently as it accrues. In particular, the system provides for mandatory accrual on debt that does not explicitly state and pay interest at a market rate. In so doing, it prescribes the use of an exponential methodology that, with some allowance for imprecision, is widely regarded as accurate in an economic sense. Bankman and Klein's analysis suggests, however, that, owing to the influence of the yield curve, that belief may be unsound.

III. Explanations of the Term Structure

There are two principal explanations for the existence of a yield curve.[57] According to the first, the "expectations hypothesis," the yield curve in some fashion incorporates the beliefs of market participants about the direction in which interest rates will move.[58] The second ex-

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since 1969 have curtailed the ownership of discount obligations by taxable, individual investors. See notes 136-38 and accompanying text; see also N.Y. Times, Apr. 24, 1981, at D8, col. 3; N.Y. Times, June 22, 1981, at D3, col. 1 (both observing that new issues of deep discount obligations were not especially desirable investments for taxable individuals).

Market discount, on the other hand, is generated by market forces and cannot be avoided. Providing for mandatory accrual of market discount therefore would "bite" in a way that the mandatory accrual of original issue discount does not.

There also are uncertainties about the way in which a regime of accruing gain to holders of market discount obligations should be framed. For one thing, when debt sells at an extraordinary discount because the issuer is financially in distress, it is not clear that exponential accrual of the discount would be sound. It is believed to be for such reasons that, when the Ways and Means Committee proposed current taxation of market discount in 1987, they imposed a limit on the amount of discount that would be subject to annual accrual. See H.R. Rep. No. 391, 100th Cong., 1st Sess. 1056-57 (1987).

It is likewise unclear whether mandatory accrual of market discount should be accompanied by an adjustment to the interest deductions allowable to the issuer of the debt or whether it should be regarded simply as a delayed offset for the loss presumably realized on transfer by some prior holder of the debt. The 1987 House proposal did not provide for an adjustment to the interest deductions of the issuer. Id.

[56] As noted above, the relevance of the failure to adjust the interest deductions of issuers is unclear. The accrual of market discount may be regarded as properly offsetting the loss deduction realized by some prior holder. See note 55.

[57] For a brief summary, see Brealey & Myers, note 17, at 547-61. A more detailed current summary of the economic literature may be found in Shiller, note 17.

[58] See generally, e.g., Brealey & Myers, note 17, at 553-58; Shiller, note 17, at 644-45; notes 67-72 and accompanying text. A somewhat different form of the expectations hypothesis is

planation attributes it to differences in the risks of holding debt of different maturities.[59]

Central to Bankman and Klein's analysis is their illustration of the implications of each of those explanations for the taxation of long-term debt.[60] In connection with those illustrations, they make two important observations. First, the slope of the yield curve has diametrically opposed implications for future market valuation of long-term debt, depending on whether it is attributable to investors' expectations about future interest rates or to risk.[61] (Using the examples from Figures I and II, these implications are illustrated in Figure III.[62]) Their second observation is that it is not possible to specify in what measure the yield curve is attributable at any given time to expectations or risk.[63] In conjunction, these two propositions imply that, in principle, the tax system cannot incorporate the yield curve into the treatment of long-term debt.[64] If the cause of the yield curve at any time is not (because it cannot be) known, and if different causes imply differing future valuations of long-term debt, it is impossible to say what adjustment *should* be made to the mechanism by which discount is accrued on long-term debt to take the yield curve into account.

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that the yield curve incorporates expectations about inflation. See Robert J. Shiller, Market Volatility 218, 237-50 (1990).

[59] See generally Brealey & Myers, note 17, at 558-60; notes 73-85 and accompanying text. This view attributes the term structure simply to the *risk* that interest rates will move, inducing the value of long-term debt to change, *not*, as with the expectations hypothesis, to beliefs about the *direction* in which they will move.

A term structure is exhibited by yields on securities that are essentially free of any risk of default (that is, U.S. Treasury securities) as well as by yields on securities that are not. Hence, in this setting, "risk" refers primarily to risks associated with uncertainty induced by the passage of time, principally fluctuations in the rate of expected inflation (which generally induces changes in interest rates), other fluctuations in interest rates or fluctuations in returns on substitute investments. See, e.g., Strnad, Periodicity, note 7, at 1865 n.137; John Y. Campbell & Robert J. Shiller, A Simple Account of the Behavior of Long-Term Interest Rates, 74 J. Am. Econ. Ass'n 44, 47 & n.2 (1984).

[60] Bankman & Klein, note 5, at 337-38, 344-45.

[61] Id. at 345.

[62] The yield curve normally (but not invariably) exhibits a positive slope, in the sense that yield to maturity increases with maturity. See note 106 and accompanying text.

If a positive slope were attributable to the fact that longer-term debt entails greater risk of market fluctuation, as time passed, the instrument's remaining maturity and riskiness would decline. It therefore would be valued using successively lower discount rates, producing more rapid accrual of gain than would have been the case had the discount rate remained unchanged. Bankman & Klein, note 5, at 337-38. The lines denoted by triangles in Figures I-III depict this possibility.

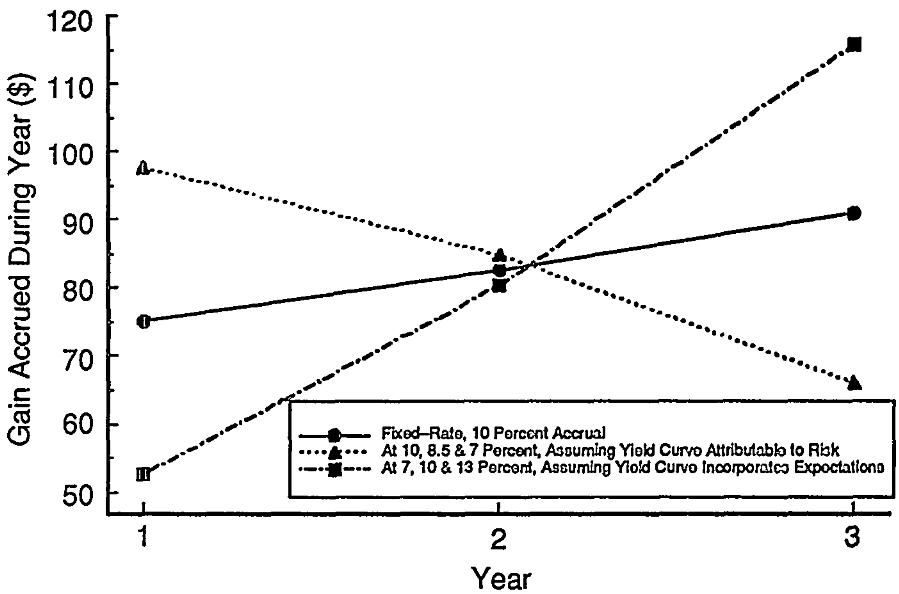
The positive slope, on the other hand, might reflect investors' expectations that interest rates would rise. If it did, and to the extent those expectations were fulfilled, subsequent market valuations of the debt, using the realized higher interest rates, would increase more slowly than if the interest rate had remained unchanged. Id. at 344-45. This explanation is depicted by the line denoted with squares in Figure III.

[63] Bankman & Klein, note 5, at 348.

[64] Id.

FIGURE III

Risk, Expectations & Market Gain
3–Year, Ten Percent Zero Coupon Bond



That proposition forms the basis for Bankman and Klein's conclusion that accurate taxation can be accomplished *only* by ex post market valuation of long-term debt.[65] That, moreover, is their pivotal analytic conclusion. Consequently, some further examination of each of these hypotheses is appropriate. What will be found is that, despite intense economic study, the determinants of the yield curve are not very well understood.[66] One thing, however, does seem to be accepted. There is little empirical support for the expectations hypothesis. So, at least in principle, it turns out that we *might* feasibly incorporate the yield curve into the existing taxation of long-term debt. That we can do so, and how we might go about doing so, are explored in the remainder of this section.

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[65] Id.

[66] What follows draws heavily on the extensive, recent survey of both the theoretical and empirical literature to be found in Shiller, note 17. Parts of this survey are technical in nature. Somewhat more accessible treatments (in roughly ascending order of technical difficulty) are to be found in Brealey & Myers, note 17, at 547-60; N. Gregory Mankiw, The Term Structure of Interest Rates Revisited, 1 Brookings Papers on Economic Activity 61 (1986); Shiller, note 58, at 219-36; Jonathan E. Ingersoll, Jr., Theory of Financial Decision Making 387-409 (1987). See also John C. Cox, Jonathan E. Ingersoll & Stephen A. Ross, A Re-examination of Traditional Hypotheses About the Term Structure of Interest Rates, 36 J. Fin. 769 (1981), much of which, however, is mathematically advanced.

A. The Expectations Hypothesis

The expectations hypothesis has been the focus of much economic study.[67] The hypothesis is actually a collection of somewhat differing propositions, all of which in some way relate investors' expectations about the interest rates that will prevail in the future to the actual future interest rates—usually denoted "forward" rates—that are given implicitly by differences in current yields to maturity on debt of differing maturities.[68] In part, the inquiry has been theoretical,[69] but the bulk of the

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[67] See, e.g., Shiller, note 17, at 644-45; Cox et al., note 66, at 774. Although development of the expectations hypothesis reflects the contributions of many, its initial formulations are usually traced to some combination of the work of Irving Fisher, John R. Hicks and F.A. Lutz. Irving Fisher, The Theory of Interest 206-10, 381-82 (1930); John R. Hicks, Value and Capital 138-46 (1946); F.A. Lutz, The Structure of Interest Rates, 55 Q.J. Econ. 36, 49 (1940).

[68] There are a variety of ways of formulating this relationship, the differences among which sometimes can be subtle. See, e.g., Ingersoll, note 66, at 389-92; Cox et al., note 66, at 774-77; Robert J. Shiller, John Y. Campbell & Kermit L. Schoenholtz, Forward Rates and Future Policy: Interpreting the Term Structure of Interest Rates, Brookings Papers on Economic Activity 173, 176 (1983).

One common formulation is cast in terms of the equivalence of expected future short-term rates and the forward rates implied by the prevailing term structure. A forward rate is just the marginal increase (or decrease) in yield to maturity associated with a marginal increase in maturity. See, e.g., Brealey & Myers, note 17, at 554-58; Shiller et al., supra at 176-85; Shiller, note 17, at 639-44. In this formulation, prevailing forward rates are investors' optimal estimates of future rates. E.g., Shiller, note 58, at 217-22; see also Shiller, note 17, at 644-45. Bankman and Klein use this formulation in illustrating the effect of the expectations hypothesis. See Bankman & Klein, note 5, at 344-45.

In a slightly different formulation, it is taken as a starting point that the certain one-period (for example, annual) return from holding one-period (one year) debt must equal the expected one-year return from purchasing (for example) a two-year bond and selling it at the end of one year. If, however, investors expect that the one-year rate will be higher in year 2 than it was in year 1, they necessarily also expect that the rate that will prevail will have a depressing effect on the price of the two-year bond as of the end of year 1. Hence, at the beginning of year 1, the two-year bond will have to offer investors a yield to maturity that exceeds the certain return from the one-year investment, to offset the expected year-end decline in the value of the two-year bond. The yield to maturity on the two-year bond at the beginning of year 1 that satisfies this requirement turns out to be a geometric average of the certain year 1 and expected year 2 rates. See, e.g., Brealey & Myers, note 17, at 556-58. This formulation is often described as requiring the equivalence of "holding returns" on debt of differing maturities.

In settings not involving uncertainty, the equivalence of current forward rates and expected future spot rates, and the equivalence of yields on short-term instruments and expected holding returns on long-term instruments, are themselves equivalent formulations. See Cox et al., note 66, at 778-79; Shiller, note 17, at 648-49; see also Brealey & Myers, note 17, at 558.

[69] At this level, much of the effort has been devoted to formulating models of investor behavior, particularly models that accommodate attitudes other than neutrality towards risk, that might furnish a theoretical foundation for the existence of a term structure. See, e.g., Shiller, note 17, at 644-53; Lutz, note 67, at 49; see generally Burton Malkiel, The Term Structure of Interest Rates: Expectations and Behaviour Patterns (1966); J.E. Stiglitz, A Consumption-Oriented Theory of the Demand for Financial Assets and the Term Structure of Interest Rates, 37 Rev. Econ. Stud. 321 (1970). There also has been controversy about the extent to which different versions of the expectations hypothesis are mutually compatible and consistent with economic equilibrium in a general equilibrium setting in which economic agents act on rational expectations in the face of uncertainty. Compare Cox et al., note 66, at 774-78, with

research has been empirical. The variety of ways in which the hypothesized link between the term structure and investors' expectations can be formulated has led to a proliferation of tests of the validity of the expectations hypothesis.[70] As a consequence, the literature is voluminous.

For purposes of this discussion, the common and conspicuous feature of empirical studies of the expectations hypothesis is how poorly the hypothesis has fared.[71] It has been consistently rejected, so consistently that a leading student of the subject recently concluded an extensive survey of the literature with the observation that empirical "work on the term structure has produced consensus on little more than that the rational expectations model, while perhaps containing an element of truth, can be rejected."[72] However appealing the hypothesis that the slope of

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John Y. Campbell, A Defense of Traditional Hypotheses about the Term Structure of Interest Rates, 41 J. Fin. 183 (1986). See also Shiller et al., note 68, at 176.

[70] See Shiller, note 17, at 654-58. Among the more commonly tested and intuitively accessible propositions are (1) that the slope of the yield curve should predict future changes in the rate of interest on long-term debt, e.g., Mankiw, note 66, at 76; Shiller, note 17, at 654; (2) that the slope of the yield curve should predict future changes in the short-term (or "spot") rate of interest, e.g., Mankiw, note 66, at 81 n.11; Shiller, note 58, at 228-29; and (3) that "excess returns" from holding long-term bonds during some future period should *not* be predictable from the current slope of the term structure, e.g., Mankiw, note 66, at 75-82; Shiller, note 17, at 656; Shiller, note 58, at 227-29.

[71] Of the three formulations described in note 70, only the second—that the yield curve should predict subsequent changes in the short rate—seems to command any empirical support whatsoever. E.g., Mankiw, note 66, at 63, 81 & n.11; Shiller, note 58, at 228-29. Even there, moreover, the findings have been that the relationship between the slope of the yield curve, although significant in a statistical sense to subsequent changes in the short rate, has very little explanatory power. There is, however, some evidence that the predictive power of the yield curve was greater before the founding of the Federal Reserve System in 1915. N. Gregory Mankiw & Jeffrey A. Miron, The Changing Behaviour of the Term Structure of Interest Rates, 101 Q.J. Econ. 211 (1986).

The consistent rejection of the expectations hypothesis seems to have stimulated the investigation of ever more sophisticated formulations of the hypothesis, which (for example) explore the limits on the variability of long-term interest rates implied by the expectations hypothesis; but, even in these studies, the hypothesis has not done well. E.g., Shiller, note 17, at 658-60; Shiller, note 58, at 256, 276-77.

[72] Shiller, note 17, at 670. One group of investigators was moved to describe it as "useless for interpreting the data provided by recent history," Shiller et al., note 68, at 215, adding the picturesque observation that:

> The simple expectations theory, in combination with the hypothesis of rational expectations, has been rejected many times in careful econometric studies. But the theory seems to reappear perennially in policy discussions as if nothing had happened to it. It is uncanny how resistant superficially appealing theories in economics are to contrary evidence. We are reminded of the Tom and Jerry cartoons that precede feature films at movie theatres. The villain, Tom the Cat, may be buried under a ton of boulders, blasted through a brick wall (leaving a cat-shaped hole), or flattened by a steamroller. Yet seconds later he is up again plotting his evil deeds.

Id. at 174-75 (footnotes omitted); see also, e.g., Mankiw, note 66, at 63, 81-82. But see note 71.

Shiller, Campbell & Schoenholtz, note 68, made the point in still a different way. If the term structure *were* attributable to expectations that were formed optimally given all currently available information, a steep, positively sloped yield curve, suggesting that interest rates were expected to rise, would make it optimal to engage in long-term rather than short-term borrow-the term structure incorporates investors' expectations about future interest rates, it seems to be lacking in factual support.

B. Risk, Term Premia and the Term Structure

The view that the normally positive slope to the yield curve is attributable to the risks associated with holding long-term debt is usually credited to John R. Hicks.[73] In its simplest form, this view holds that investors prefer the safety (and liquidity) of investing short term. They must, therefore, be induced by a premium return to invest in more volatile (and, hence, "riskier") long-term debt.[74] This is frequently expressed by attributing the slope of the yield curve to the presence of "liquidity premia" or "risk premia."[75] (For reasons to be described below, however, the label "term premium" is probably to be preferred.)

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ing to avoid the still higher borrowing costs expected to prevail just around the corner. By the same token, it would imply that *investors* should *avoid* long-term lending, since the expected rise in interest rates would lead to depreciation in the market value of long-term debt, thereby reducing holding returns to investments in such debt. The authors found, however, that their results suggested "contrary to the [expectations] theory, that the six-month returns to holding [long-term] bonds are higher than on [short-term] bills when the bond rate is relatively high"; and, by the same reasoning, that

> companies should delay long financing until long-term rates fall relative to short-term rates, and householders should not switch from floating to fixed rate mortgages until this occurs. It is perhaps surprising only to students of the expectations theory that this is what a naive person might have done without the guidance of a sophisticated model.

Id. at 196-97 (footnotes omitted).

[73] See, e.g., Brealey & Myers, note 17, at 558 & n.18; Cox et al., note 66, at 784; Shiller, note 17, at 649.

[74] John R. Hicks, Value and Capital (2d ed. 1946). In the passage most frequently quoted, Hicks observed that:

> the forward market for loans . . . may be expected to have a constitutional weakness on one side, a weakness which offers an opportunity for speculation. If no extra return is offered for long lending, most people (and institutions) would prefer to lend short, at least in the sense that they would prefer to hold their money on deposit in some way or other. But this situation would leave a large excess of demands to borrow long which would not be met. Borrowers would thus tend to offer better terms in order to persuade lenders to switch over into the long market . . . .

Id. at 146; see also Lutz, note 67, at 62.

[75] The classic formulation of a risk premium stems from the observation that, confronted with a gamble having a known expected value, there is some smaller amount of money, to be received with certainty, that a risk-averse individual will prefer. The smallest certain amount that the individual would prefer to the gamble is often referred to as the "certainty equivalent" of the gamble. The difference between the expected value of a gamble and its certainty equivalent is the individual's risk premium for the gamble. E.g., John W. Pratt, Risk Aversion in the Small and in the Large, 32 Econometrica 122, 124 (1964).

If short-term debt is perceived as offering a certain return, the difference between the yield available on (volatile) long-term debt and the (typically) lower return on short-term debt can be regarded as investors' required risk premium for investing long.

Frequently, the notion of a premium associated with the longer maturity of a bond is given a more intricate formulation than that it is simply the difference between yields to maturity on

A key assumption in most empirical investigations of the expectations hypothesis has been that, although long-term debt may have to offer premium returns, the dimensions of the risk premia do not vary (or vary only slowly) over time. Consequently, unanticipated revisions in investors' expectations about future interest rates are hypothesized to be what induce the term structure to change.[76] The principal alternative, to which empirical investigators have been forced to turn by the absence of evidence for the expectations hypothesis, is that changes in the yield curve must in some way be attributable to changes in the risk premia themselves.[77]

There is, however, a logical difficulty with this view, at least if long-term debt is regarded as invariably more risky than short-term debt in the sense originally conceived by Hicks. It implies that long-term debt is always less attractive than short-term debt, and hence that the slope of the yield curve should always be positive. In fact, the yield curve sometimes exhibits a negative (or "inverted") slope.[78] Subsequent work has suggested, however, that, in theory, the nature of the premia associated with returns on debt of different maturities rests on more intricate foundations than the simple observation that long-term debt is more variable and hence, less attractive to risk-averse lenders.

One alternative has it that different investors may have preferred (and differing) lending horizons (often referred to in the literature as "habitats") that are dictated by their personal circumstances. For particular investors, the risk-minimizing choice of maturity may correspond with their preferred horizon.[79] Investing in debt of any longer *or shorter* maturity exposes them to greater risk.[80] The habitat theory therefore implies that short-term investments may not naturally be preferred.

instruments of different maturities. E.g., Shiller, note 17, at 647-49. For these purposes, however, these differences are not important.

[76] E.g., Mankiw, note 66, at 75; Shiller, note 17, at 647; Shiller et al., note 68, at 174-76.

[77] E.g., Mankiw, note 66, at 82-85; Shiller, note 17, at 667-68; Shiller et al., note 68, at 197-200.

[78] See note 106.

[79] This view is usually credited to Franco Modigliani & Richard Sutch, Innovations in Interest Rate Policy, 56 Am. Econ. Rev. 178, 183-84 (1966). See Shiller, note 17, at 650. The typical example is that of a family saving for their children's education: They may find it least risky to "lock in" a return for a duration that coincides with the time when the educational expenses are expected to fall due, rather than for the immediate future. Id.

Cox et al., note 66, at 784, have observed that the view that investors will want to minimize risk in the immediate future is just a special instance of the "preferred habitat" theory, with a preferred horizon equal to the shortest term for which an investment can be made.

[80] Investing in longer-term debt entails the normal risk that the long-term debt might have declined in *value* at the time the investor expects to consume. Engaging in a series of short-term investments, on the other hand, exposes the investor to the risk that short-term *yields* may decline between the time the investor selects the investment strategy and the time the investor expects to consume.

Furthermore, although it is generally (but not invariably) true that the value of long-term debt is more responsive to changes in interest rates than short-term debt,[81] it does *not* follow that long-term debt is on that account *necessarily* less attractive to risk-averse investors, even those with short-term lending horizons. To the contrary, uncertainty can quite generally render more variable (and, at least superficially, "riskier")[82] long-term debt a *more* attractive investment, both to risk neutral *and* to moderately risk-averse investors, than the certain short-run returns from holding short-term debt.[83]

For these reasons, it now seems to be accepted that risk (or term) premia on long-term debt in theory can be either positive or negative. It follows that the risks associated with interest rate fluctuations[84] can, in principle, produce a yield curve with a positive or negative slope.[85]

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[81] This proposition and its qualifications are developed in Appendix B.

[82] One common measure of the riskiness of an asset, widely used in asset pricing and portfolio selection models, is the "variance" of the asset's price over time. See, e.g., Ingersoll, note 66, at 82-101. A somewhat more general measure of whether one probability distribution is "more risky" than another is whether the first was obtained from the second through a series of alterations known as "mean-preserving spreads." E.g., id. at 114-21; Michael Rothschild & Joseph E. Stiglitz, Increasing Risk I: A Definition, 2 J. Econ. Theory 225 (1970).

[83] This is a mathematically subtle point. The observation in the text holds as long as the "expectations" of market participants, when faced with uncertainty, are taken to be "expected values" of a random variable (or, more precisely, of a function of a random variable) in a mathematical sense. The assumption that they are is similar to the widely employed assumption that optimizing consumers, faced with uncertainty, will maximize "expected utility."

On that assumption, it turns out, by a proposition known as "Jensen's inequality," that, because the computation of present values involves a mathematical transformation that is "convex," to a risk-neutral investor, the mathematical expectation of the present value of a future sum, discounted in the presence of uncertainty about future interest rates, will *exceed* the present value of the same sum discounted at the expected (or mean) value of the uncertain interest rate. In a sentence, the (expected) present value of the random return exceeds the present value computed using the expected interest rate. See Appendix B.

The relevance of this proposition to the present discussion is that, if given a choice, say, between (1) a one-year note, discounted at some *known* interest rate, and (2) a two-year note, valued by computing the note's expected present value after one year using a probability distribution for future interest rates for which the expected future interest rate was *equal to* the known interest rate, a risk-neutral investor would find the uncertain investment to be more attractive. Consequently, she would insist on a *higher* (known) return on the one-year note than on the two-year note. See Appendix B.

This sort of relationship also appears to hold for *risk-averse* investors, at least when they exhibit relative risk aversion with a coefficient less than or equal to one. The proposition that it does evidently originated in print with Stiglitz, note 69, at 322-23, 326-31. More recently, much the same conclusion was arrived at by Cox et al., note 66, at 784-86, on the more general assumption that the horizon over which investors seek to minimize risk differs from the immediate future. See text accompanying notes 79-80.

It is to be emphasized that this point is different from the observation by Bankman and Klein (and others) that when the yield curve is inverted, even risk-averse investors seeking to "lock in" high, long-term yields may gravitate to long-term bonds. See Bankman & Klein, note 5, at 340 & n.16; see also Shiller, note 17, at 645.

[84] See note 59.

[85] E.g., Shiller, note 17, at 650.

C. *Incorporating the Term Structure into the Taxation of Long-Term Debt*

The lack of empirical support for the expectations hypothesis is at odds with Bankman and Klein's assertion that, in principle, the tax system cannot incorporate the yield curve into the treatment of long-term debt.[86] That assertion turns on the claimed inability to determine the extent to which, at any time, the term structure is attributable to expectations or to risk.[87] The evidence, however, seems to warrant the conclusion that changes in the yield curve are attributable principally to changes in investors' perceptions of risk.[88] On the assumption that they are, it is possible to incorporate the yield curve into the mechanism currently used to accrue interest on long-term debt. This section concludes by describing briefly the nature of the adjustment that would be required, and by exploring some drawbacks to that endeavor.

In practice, it is feasible to extract from observable economic data the interest rates that prevail at the margin at different locations on the yield curve.[89] It is likewise feasible to incorporate the term structure prevailing when a debt instrument is issued directly into the accrual of original

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[86] Bankman & Klein, note 5, at 348; text accompanying notes 62-64.

[87] As described by Bankman & Klein, note 5, at 345, and as noted above, the slope of the term structure has different implications for the future valuation of debt depending on whether it is attributable to expectations or risk. See note 61 and accompanying text. Consequently, the adjustment to any formula used to accrue interest on long-term debt depends on what gives rise to the yield curve.

If, for example, the term structure exhibited a positive slope that was assumed to be attributable to risk, incorporating it into the original issue discount rules would imply that interest should be accrued by successively valuing long-term debt over a series of years using discount rates that sequentially *declined*. If, on the other hand, the same slope was taken to reflect investors' expectations that interest rates would rise, the appropriate adjustment to the original issue discount rules would be to accrue interest during the first year using the one-year interest rate prevailing at the time of issue, and during subsequent years using discount rates that sequentially *rose*.

The kinds of difficulties that would arise if both explanations were valid can be illustrated in the following way: Suppose that the term structure had a positive slope, and suppose further that, on the assumption that the slope was attributable to risk, the term structure were to be incorporated into the rate at which original issue discount accrued. As time passed and maturity approached, the instrument would be valued using progressively *lower* discount rates.

But if, in reality, the positive slope to the yield curve reflected expectations that proved to be fulfilled, *higher* (rather than lower) discount rates subsequently would be used *by the market* in valuing the debt. On assumptions like these, incorporating the term structure into the original discount rules would aggravate rather than ameliorate the inaccuracy in the measurement of income from debt.

[88] See, e.g., Shiller, note 17, at 667-70; Shiller et al., note 68, at 174-76, 197-200; Mankiw, note 66, at 82.

[89] These are just the "forward rates" referred to in the text accompanying note 68. They correspond to what Bankman and Klein denote by "annual interest rates" in their examples. Bankman & Klein, note 5. Forward rates for Treasury securities of maturities up to 25 years for the period 1947-1986 are tabulated in Shiller, note 17, app. B, at 688-702, tbl. 13.A.2.

issue discount (or other interest)[90] on the instrument over time. For instruments issued in pure discount form, and subject to the original issue discount rules in any event, such an adjustment is not difficult to specify. It consists simply of accruing interest annually over the life of the instrument using the set of forward rates implied by the yield curve at the time of issue, beginning with the forward rate corresponding to the obligation's initial term and working backwards from there.[91]

Even for pure discount obligations, however, implementation, although not infeasible in principle, would not be especially appealing to anyone preoccupied with simplicity of administration of the income tax. The problems would be most modest for debt, principally U.S. Treasury securities, for which the actual yield curve can readily be observed, and from which the necessary forward rates can be computed.[92] Even there, however, and even assuming that ordinary individuals can be expected to do the OID calculations currently required,[93] it would be patently unreasonable to expect anyone other than issuers of discount obligations to be able to extract forward rates from observable data. For debt that is less regularly traded and for which the yield curve itself might have to be inferred, the computational problems would be significantly more severe.[94]

Coupon-carrying debt originally issued at par, on the other hand, as well as self-amortizing obligations, such as home mortgages, would pose even more substantial difficulties. Such obligations are not now subject

90 As noted immediately below, such adjustments would be required for interest-paying and discount obligations alike. See text accompanying notes 95-99.

91 The forward rates implied by the simple example depicted in Figures I and II may be determined by dividing the gain accrued each year by the instrument's market value at the beginning of that year. The resulting sequence of forward rates in that example is approximately 13%, 10% and 7%. See text accompanying notes 21-22. Accruing interest by applying this sequence of rates to the sequence of beginning-of-the-year values in that example reproduces the sequence of accruals given in note 21. The proposition that this procedure generally produces the correct sequence of annual valuations is developed in Appendix B.

92 See, e.g., Shiller, note 17, app. B, at 672-715.

93 A casual glance at § 1272(a) and the proposed regulations thereunder does not inspire confidence in the soundness of that assumption.

94 In the aggregate, Treasury debt amounts to significantly less than the bulk of outstanding U.S. credit market debt. According to the Federal Reserve Flow of Funds accounts, as of the end of 1989, Treasury and U.S. agency securities (including mortgage-backed securities and debt of federally sponsored credit agencies itself backed by credit market debt) amounted to about 30% of credit market debt in private hands. See 1989 Flow of Funds Data, note 6, at A44-A45. Of this total, moreover, it is unlikely that much of it consisted of original issue discount debt in the hands of taxable private holders. See text accompanying notes 136-38.

For other discount obligations, the yield curve, which would have to be adjusted for differences in risk, probably would have to be inferred. In particular, for debt that is not publicly traded, some adjustment would have to be made on the basis of the yield curve exhibited by traded debt of "comparable risk," an adjustment that of necessity would be judgmental. See Shiller, note 17, at 637.

to the original issue discount rules.[95] The taxation of their holders generally is based on interest that is explicitly stated and paid.[96] Nevertheless, from a conceptual standpoint, each constituent payment on an interest-bearing obligation can be regarded as an individual, pure discount bond. The obligation as a whole amounts to a portfolio of discount bonds.[97] Since the yield curve influences the accrual of interest on each constituent discount bond, it influences the interest accruing to the holder of the entire portfolio. It follows, as Bankman and Klein point out, that the yield curve affects the accrual of gain to the holders of interest-bearing debt.[98] In contrast with a single discount obligation, however, there is no simple adjustment to the accrual of gain on an interest-bearing obligation that will take the term structure into account.[99] As a result, incorporating the yield curve into the accrual of income on interest-paying obligations would necessitate a wholesale revision in their taxation. The necessary changes almost surely would produce a regime of taxation that is dramatically more complex than existing law.[100]

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[95] In general, debt issued at a price approximately equal to its "stated redemption price at maturity" has no original issue discount and is not subject to § 1272(a). IRC § 1273(a)-(b). There is, however, a de minimis exception, the amount of which varies with the maturity of a debt obligation, the general effect of which is to prevent normal fluctuations in the prices of coupon carrying bonds during the underwriting process from creating original issue discount. IRC § 1273(a)(3).

[96] As noted above, in the absence of distortions such as those induced by the yield curve, this treatment is conceptually sound: The stated interest is a reasonable proxy for the interest that economically accrues. See note 46. In instances where a debt obligation is structured so that it is not, the statute tends to require that interest be restated and accrued in a fashion that *is* economically sound. IRC §§ 1274, 1274A.

[97] See note 46; see also Shiller et al., note 68, at 177.

[98] Bankman & Klein, note 5, at 341-44. The influence of the yield curve on such obligations is, however, attenuated by comparison with its influence on zero-coupon debt. See text accompanying notes 139-43.

[99] The difficulty is that interest accruing to a single discount obligation in any year is a function of the single forward rate appropriate to the instrument's remaining maturity during that year. In contrast, an interest-bearing obligation consists of a collection of separate payment obligations, each having a *different* maturity date. Consequently, the interest accruing to the holder of the obligation in any year is a function of the entire *set* of forward rates appropriate for all such maturity dates.

[100] For purposes of taxation, such instruments would have to be disassembled into their constituent obligations, each of which would be taxed under the original issue discount rules. The rate of accrual then could be adjusted for the yield curve in the manner described above. See text accompanying note 91.

The net effect would be a scheme of taxation much like the "serial bond" approach prescribed by the existing original issue discount regulations for debt that provides for partial repayment of principal before final maturity, but using compound interest concepts and adjusted for the influence of the yield curve. Reg. § 1.1232-3(b)(2)(iv)(c); see also Garlock, note 2, at 111-12.

The amount of debt subject to such a change almost certainly would be substantial. The sum of corporate bonds, mortgages and consumer credit, outstanding as of the end of 1989, alone amounted to approximately $5.25 trillion, or about 55% of aggregate credit market debt. 1989 Flow of Funds Data, note 6, at A44.

There are grounds, moreover, to be skeptical about the degree of improvement these adjustments would produce in the measurement of income from either form of long-term debt. In theory, considerations of risk can produce a yield curve with a positive or negative slope.[101] In practice, the contours of the yield curve vary over time.[102] Consequently, the yield curve that exists when a debt instrument is issued may differ from the yield curves that prevail over the instrument's life. Such considerations suggest that incorporating the yield curve at the time of issue might produce only fortuitous improvements in the treatment of long-term debt.

Still, it is true that where future interest rates are uncertain, the volatility of a debt instrument's present (and therefore its market) value will tend to increase with its term.[103] To the extent that investors have short lending horizons and are more than weakly averse to risk, that characteristic may render long-term debt generally less attractive than short-term debt.[104] What is more, empirical evidence suggests that the slope of the yield curve manifests a "mean-reverting" tendency, so that it tends to exhibit a relatively persistent value.[105] Such findings are consistent both with the more casual observation that the yield curve typically has a positive slope, and with the view that long-term debt must offer a premium return.[106] On balance, then, it might be appropriate to incorporate the

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101 See text accompanying notes 84-85.

102 These variations typically do not, however, take the form of simple parallel movements of the sort envisioned by Bankman and Klein, which would at least preserve the *slope* of the yield when the level of interest rates changed. Bankman & Klein, note 5, at 337 & n.11. Compare Shiller, note 17, at 639 (noting that the yield curve rarely makes a parallel shift).

It does, however, seem to be a robust empirical finding, albeit one without an agreed-on explanation, that the long interest rate tends to be some sort of weighted average of current and past short-term rates. E.g., Mankiw, note 66, at 69; Campbell & Shiller, note 59, at 44. Such findings suggest that when the short rate moves, the long rate eventually also will move. E.g., Mankiw, note 66, at 69. They also suggest that the volatility of the yield curve increases with interest rate volatility.

Interest rates became significantly more variable after the Federal Reserve Board altered its operating procedures in 1979. See, e.g., Campbell & Shiller, note 59, at 44.

103 See text accompanying note 81; Appendix B.

104 See text accompanying note 83.

105 E.g., Mankiw, note 66, at 69-70.

106 During about 70% of the post-war era, the U.S. term structure has exhibited a monotonic (or approximately monotonic) positive slope. About 17% of the time, it sloped monotonically (or approximately monotonically) down.

The foregoing observations are based on tabulations by the author from data compiled by J. Huston McCulloch and published as McCulloch, U.S. Term Structure Data, 1946-1987, in Shiller, note 17, app. B, at 672-715 [hereinafter McCulloch Term Structure Data]. The data are described in more detail at note 110 and Appendix A. See also J. Huston McCulloch, The Tax-Adjusted Yield Curve, 30 J. Fin. 811 (1975); J. Huston McCulloch, Measuring the Term Structure of Interest Rates, 44 J. Bus. 19 (1971).

As noted above, an instructive, three dimensional plot of the movements of the post-war U.S. term structure, based on the McCulloch data, may be found in Shiller, note 17, at 630, fig. 3.1.

term structure that prevailed when a debt instrument was issued into the accrual of interest, on the assumption that, at least on average, it would not be too bad an approximation of the yield curves that actually prevailed throughout the instrument's life.

D. Conclusion

To sum all of this up, the evidence suggests that it is appropriate to attribute the yield curve not to expectations, but to risk. On that assumption the system could, both in principle and in practice, incorporate the yield curve into the taxation of long-term debt. Doing so would add more than trivially to the already complex taxation of discount debt (of which there may, in any event, be little in the hands of taxable holders).[107] It would make very much more onerous the still relatively straightforward treatment of coupon-carrying bonds and other interest-bearing debt. Overall, the administrative costs could be expected to be great. At the same time, for reasons to be developed in the section that follows, there is reason to believe that the gains in accuracy to be achieved by incorporating the yield curve into the taxation of long-term debt may not be as substantial as one might otherwise suspect. So, the net effect might be to achieve little improvement in the measurement of income, and, even then, only at substantial administrative cost.

The theoretical feasibility of these adjustments turns, moreover, on the absence of evidence for the expectations hypothesis. The yield curve may yet be found to incorporate information about expectations as well as risk. In the absence of such findings, however, adjustments of the sort outlined above, *even if* they were costly and *even if* they did not achieve much of a gain, would go a long way towards eliminating *in principle* the distortions in the measurement of income identified by Bankman and Klein.

IV. The Impact of the Yield Curve on the Valuation of Debt

Any assessment of whether the impact of the yield curve on the taxation of debt warrants remedial attention—either in the form of adjustments to the existing regime, or by adopting an accrual tax based on market valuation—ought to take into account the degree of improvement in measuring income the remedy might achieve. That depends in part on how pronounced a phenomenon the yield curve is. Bankman and Kleir observe that its impact increases with both the term of a debt obligatioı and with the slope of the curve.[108] As a means of developing a feel fo

[107] See text at notes 136-38.

[108] Bankman & Klein, note 5, at 339-40. This observation, which Bankman and Klc develop by example, is only accurate with qualifications. As shown in Appendix B, for re

the dimensions of its influence, they illustrate the impact of hypothetical curves with differing slopes on debt of maturities ranging from one to twenty years.[109]

It is unnecessary, however, to confine the assessment of this question to conjecture. Information is available with which to form a more realistic picture of the term structure, and to examine its magnitude and the resulting impact on the valuation of debt. It is also possible to compare its influence to that of other economic variables, particularly changes in the level of interest rates generally, that also affect the valuation of debt. This section develops such comparisons, using post-war data on yields on Treasury securities of differing maturities, developed by Professor J. Huston McCulloch for the express purpose of investigating economic hypotheses about the yield curve.[110]

Anticipating the conclusions,[111] the data suggest that, on average during the post-war period, the yield curve has been substantially flatter than Bankman and Klein's illustrative range of values suggests.[112] In addition, much of the action has been at the short end of the yield curve, where its influence on the value of debt in general is comparatively less.[113] Overall, the data suggest that variations in the valuation of debt that might be induced by the yield curve are smaller by at least an order of magnitude than the variations induced by fluctuations in the level of interest rates generally. This indicates that, on average, the inaccuracy identified by Bankman and Klein will be smaller by that amount than other, market-induced fluctuations in value. What is more, the relative magnitude of these distortions renders it unlikely that they could be exploited by private investors in any significant way.

A. The Magnitude of the Term Structure

The McCulloch term structure data include yields to maturity on both zero-coupon and coupon-carrying U.S. Treasury securities of maturities

tively low combinations of interest rates and maturities—specifically, when their product is smaller than one—the interest rate volatility of a discount obligation does increase with its time remaining to maturity. It also is shown in Appendix B that, other factors held constant, interest rate volatility always decreases with the *level* of interest rates.

[109] Bankman & Klein, note 5, at 339-40 & tbl. 3.

[110] McCulloch Term Structure Data, note 106. The McCulloch data, which give refined yields to maturity calculated for both coupon-carrying and discount obligations, for all available maturities of 25 years or less and for each month between the beginning of 1947 and the end of 1986, are described in greater detail in Appendix A.

[111] The conclusions set out below are based on the author's calculations using the McCulloch data, and are summarized in Appendix A.

[112] It appears that during the post-war era, the U.S. term structure has been less pronounced than that of other industrialized countries, in particular, the United Kingdom and West Germany. See Mankiw, note 66, at 66-68 tbl. 3, 79.

[113] See text accompanying notes 81 & 108.

ranging from three months to 25 years,[114] calculated monthly for the period 1947-1986. They permit exploration, in a variety of ways, of the dimensions of the term structure.

One way of doing so is simply to calculate what might be denoted its "width": the difference between the lowest and highest yield to maturity among all maturities outstanding at a given time, irrespective of whether the yield curve has a positive or negative slope.[115] Using a quarterly subset of the data, and beginning with calculated yields to maturity on zero-coupon securities with maturities ranging from one to 25 years, the width of the term structure rarely has been more than two percentage points. It has averaged about 1.16%.[116] For coupon-carrying debt, for which (as described below)[117] the term structure should tend to be less pronounced, the width averaged only about 1.05% over the same period.[118]

At times when the yield curve exhibited a monotonic slope—that is, when yield to maturity either steadily increased or steadily decreased

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114 The yields to maturity on zero-coupon obligations were derived from observed yields on coupon-carrying debt because, until recently, the Treasury's practice was to issue debt with maturities longer than one year only in coupon-carrying form. McCulloch Term Structure Data, note 106, at 672.

115 This is similar to what is often taken to be the "slope" of the yield curve, the difference between the yield to maturity on long-term and short-term debt (also referred to as "the spread"). See, e.g., Shiller, note 58, at 227; Mankiw, note 66, at 76.

Since, however, I am interested only in how sizeable a phenomenon the term structure is, I need not be concerned with the *sign* of its slope. Hence width, as defined here, is the absolute value of the spread, and the spread is taken to be the difference between yields on those maturities that exhibit the highest and lowest yields, rather than between yields on debt of the longest and shortest maturity. It will always be at least as great in magnitude as (and on average will be greater than) the slope. Since the width is an absolute value, years in which the slope had positive or negative values will not tend to cancel one another out in the computation of its mean.

The width of the term structure was calculated using yields to maturity on instruments with maturities of between one and 25 years. Maturities of less than a year are not relevant to the taxation of long-term debt, the interest on which is taken into income not more frequently than annually, so yields for maturities of less than one year were ignored, as were (for reasons of computational convenience) maturities of two, three or four years.

116 Specifically, as set out in Appendix A, the width of the term structure for zero-coupon debt was greater than four percentage points in only a single quarter (out of 160), greater than three percentage points during only three quarters, and greater than two points in only 18 quarters. The standard deviation of the width was about 0.74% (or about 74 basis points).

117 See text accompanying notes 142-43.

118 The figures (corresponding to those in note 116) are that for coupon-carrying debt, the width of the term structure was greater than three percentage points during only one quarter, greater than two percentage points in only 11 quarters, and was never as much as 4%. The standard deviation was about 65 basis points. See Appendix A.

It is to be noted that these calculations, based on the McCulloch data, are grossly consistent with the calculations of others that the difference between average yields on long-term and short-term Treasury debt calculated for the period 1926-1985, was about 1.2%. See, e.g., Brealey & Myers, note 17, at 560. The width as calculated here may be somewhat smaller because it is based on maturities ranging from one to 25 years, rather than the entire set of outstanding maturities, which typically has ranged from three months to 30 years.

with maturity—what is here denoted the width is also the difference between yields to maturity on Treasury debt of the shortest and longest of the outstanding maturities surveyed. At such times, the difference between yields on instruments that were any more proximate in maturity would have to be less than (or equal to) the width of the entire term structure.[119] This measure suggests, then, that, at least *on average* during the post-war era, yields on Treasury securities changed by not more than roughly five hundredths of a percentage point for each one-year increase in maturity.

This picture could be misleading for two reasons. At times, for reasons that varied, instruments of all maturities were not outstanding.[120] For those years, the width of the yield curve might have been curtailed. Averaging such years with those for which yields on the full range of maturities were available conceivably could depress the apparent width of the curve. More importantly, at times when the slope of the yield curve was not strictly monotonic—there were more than 60 quarters in which it was not[121]—the difference between yields on instruments whose maturity differed by as little as five years could, in theory, be as great as the width of the entire term structure at that time.[122]

As a check against these possibilities, it is feasible to calculate directly the difference in yields for instruments having adjacent maturities.[123] The picture that emerges remains essentially unchanged. On average over the entire period, the difference in yields on instruments separated in maturity by five years was less than three-tenths of a percentage point. That translates (again on average) into a difference in yield of less than six hundredths of a percentage point for each year change in maturity.[124]

To be sure, there were instances, although not many, in which a specific interval of the yield curve had a particularly steep slope. The extreme was a difference of nearly three percentage points between the

[119] If, for example, the extreme values consisted of an 8% yield to maturity on 25-year debt and a 6% yield on one-year debt, the difference between yields on 10-year and 15-year debt necessarily would be less than 2%.

[120] McCulloch Term Structure Data, note 106, at 672-73.

[121] See Shiller, note 17, at 630, fig. 13.1.

[122] For example, the pattern of yields on instruments of 1, 5, 10, 15, 20 and 25 years might have been 6%, 8%, 7.9%, 7.8%, 7.7% and 7.6%, respectively. In that event, the "width" of 2 points, if averaged over the entire range of maturities, would imply an average change in yield of about 0.08% of a point for each one-year change in maturity. The actual average change in yield between years one and five is 0.5%.

[123] Since the subset of the data that was employed consists of debt with maturities of 1, 5, 10, 15, 20 and 25 years, adjacent maturity in this setting means maturity separated by not more than five years.

[124] For zero coupon bonds, the average difference in yield between instruments whose maturity differed by five years was 0.2996%; for coupon-carrying debt, the corresponding figure was 0.2717%. See Appendix A.

yields on instruments of one year and five years' maturity.[125] But, on average, even the steepest five-year segment of the yield curve in each of the 160 quarters exhibited a difference of only about seven-tenths of a point, a change in yield to maturity of about 0.14% for each year change in maturity, and the balance of the yield curve would have tended to be disproportionately flat.[126] Of greater interest for present purposes is that the steepest five-year interval on the yield curve tended disproportionately—roughly two-thirds of the time—to fall at the short end of the yield curve.[127]

The picture that emerges, as set out in Appendix A, is that, in isolated instances, the yield curve was characterized by some five-year interval over which it was steep. But by less isolated measures—for example, the average width of all five-year intervals over the 40-year period, or even the average width of the steepest five-year interval in each quarter—the change in yield to maturity as maturity changed by a year was markedly less pronounced. On average over the entire yield curve for all years covered by the data, yield to maturity changed by less than six-hundredths of a percentage point for each one-year difference in maturity. The intervals of greatest change in each period occurred nearly two-thirds of the time between instruments having maturities of one and five years.[128]

B. Implications of the Magnitude of the Term Structure For the Taxation of Long-Term Debt

The object of this exercise is to put Bankman and Klein's examples into perspective. On average, actual rates of change in yield to maturity with respect to duration are at the extreme low end of the range of possibilities they survey.[129] Consequently, the degree of income mis-

[125] Id. Interestingly, in *each* of the 10 quarters that exhibited the greatest difference in yield between instruments whose maturities differed by five years, the difference was exhibited by the short end of the yield curve, between instruments of one-year and five-year duration.

[126] For zero-coupon bonds, the average, aggregate width of all five-year segments of the yield curve was only 1.27%, while the width of the steepest five-year interval averaged 0.70%, so that the remaining 20-year span could have had an aggregate average width of not more than 0.57%, or an average year-to-year change in yield to maturity of about 0.0285%.

For coupon-carrying instruments, the corresponding figures have an average, aggregate width of 1.135% and an average maximum five-year width of 0.687%, so that the balance of the yield curve on average was no wider than 0.448%, which implies an average year-to-year change in yield to maturity over the balance of the yield curve of about 0.0224%.

[127] The steepest segment of the yield curve for zero-coupon securities occurred between one and five years' maturity in 107 quarters out of 160 (67% of the time); for coupon-carrying debt the figure is 102 quarters out of 160 (or 64% of the time). See Appendix A.

[128] The average width between maturities of one year and five years was a little over 0.6%, consistent with the fact that this interval was rather consistently the steepest segment of the yield curve. Id.

[129] See Bankman & Klein, note 5, at 340, tbl. 3.

measurement induced by the discrepancy between the term structure and the single-rate convention employed by the original issue discount rules is typically at the very low end of what their examples suggest. The resulting inaccuracy in measurement is further reduced by the tendency of the yield curve to be disproportionately steep at the short end of the term structure. It was disproportionately flat at the long end, where its effect on the valuation of debt would generally be most pronounced.[130]

These conclusions can be illustrated using an example similar to that in Bankman and Klein's Table 2,[131] with the term structure altered in a simple fashion that is plausibly consistent with the data. The results are set out in Table 1 and depicted graphically in Figure IV.[132] In this example, the interest accrued under the original issue discount rules exhibits an extreme deviation from the market valuations implied by the term structure of about 13% of the income accrued during the final year.[133] Over the life of the instrument, the unweighted average of the absolute values of the deviations is about 5.5%.[134] It illustrates that the inaccuracies in income measurement induced by the yield curve *on average* are not materially different from the smallest of the distortions implied by Bankman and Klein's Table 3.[135]

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130 See text and notes at 81 & 108; Appendix B.

131 Bankman & Klein, note 5, at 339, tbl. 2.

132 Under the yield curve in this example, yield to maturity increases by one-tenth of a percent (0.001%) for each of the first four one-year increases in maturity, and by four one-hundredths of a percent (0.0004%) for each additional one-year increase thereafter. This reflects the fact that the yield curve is typically steepest between maturities of one and five years. It is slightly flatter than the actual average slope over that portion of the curve. See Appendix A. In the aggregate, this curve has a width of 1% over the illustrated 20-year range of maturities, consistent with an aggregate width of 1.25% over a 25-year range. This is slightly steeper than the actual average width of the yield curve for zero-coupon securities, which was about 1.16%. See text accompanying note 116; Appendix A.

133 The extreme deviation occurs during the final year despite the fact that the responsiveness of an instrument's value to changes in the interest rate generally declines as maturity approaches. See text accompanying note 108. This is because the disparity between the yield to maturity prescribed by the original issue discount rules (8% in this example) and the sequence of forward rates implied by the term structure is most pronounced in the final year.

134 Since the figure given in the text is an average of the absolute values, it ignores the fact that, over the life of an instrument, these deviations offset one another. See note 147.

135 Bankman & Klein, note 5, at 340.

TABLE 1

PATTERN OF INTEREST ACCRUAL IMPLIED BY THE AVERAGE TERM STRUCTURE FOR A 20-YEAR ZERO-COUPON BOND

(1) Years to Maturity	(2) Yield to Maturity[a]	(3) Value Beginning of Year[b]	(4) Increase in Market Value[c]	(5) Implied Forward Rate[d]	(6) OID Rate	(7) Basis Beginning of Year	(8) Accrued Gain	(9) Percent Deviation[e]
20	8.00%	$215	$19	8.76%	8.00%	$215	$17	−8.71%
19	7.96%	233	20	8.68%	8.00%	232	19	−8.51%
18	7.92%	254	22	8.60%	8.00%	250	20	−8.23%
17	7.88%	275	23	8.52%	8.00%	270	22	−7.88%
16	7.84%	299	25	8.44%	8.00%	292	23	−7.45%
15	7.80%	324	27	8.36%	8.00%	315	25	−6.95%
14	7.76%	351	29	8.28%	8.00%	340	27	−6.36%
13	7.72%	380	31	8.20%	8.00%	368	29	−5.69%
12	7.68%	412	33	8.12%	8.00%	397	32	−4.94%
11	7.64%	445	36	8.04%	8.00%	429	34	−4.10%
10	7.60%	481	38	7.96%	8.00%	463	37	−3.17%
9	7.56%	519	41	7.88%	8.00%	500	40	−2.15%
8	7.52%	560	44	7.80%	8.00%	540	43	−1.03%
7	7.48%	604	47	7.72%	8.00%	583	47	0.18%
6	7.44%	650	50	7.64%[f]	8.00%	630	50	1.49%
5	7.40%	700	55	7.80%	8.00%	681	54	−0.27%
4	7.30%	754	57	7.60%	8.00%	735	59	2.55%
3	7.20%	812	60	7.40%	8.00%	794	64	5.72%
2	7.10%	872	63	7.20%	8.00%	857	69	9.27%
1	7.00%	935	65	7.00%	8.00%	926	74	13.23%
Maturity:		$1,000				$1,000		
Average Deviation:								5.39%[g]
Future Value of Taxes:			$382.19[h]				$375.39[h]	

[a] As described in note 132, yield to maturity increases by .10% for each additional year to maturity between years one and five, and by .04% for each additional year thereafter. This column corresponds to the "Blended Interest Rate" column in Bankman & Klein, note 5, at 339 tbl. 2.

[b] The value at the beginning of the year is the $1,000 payment to be received at maturity discounted for the number of years remaining until maturity, using the yield to maturity from Column 2.

[b c] The increase in market value is the change in the value of the bond, between the beginning of the year and the beginning of the succeeding year.

[d] The implied forward rate is the value in Column 4 (Increase in Market Value) divided by the value in Column 3 (Value Beginning of Year). This column corresponds to the "Annual Interest Rate" column in Bankman & Klein, note 5, at 339 tbl 2.

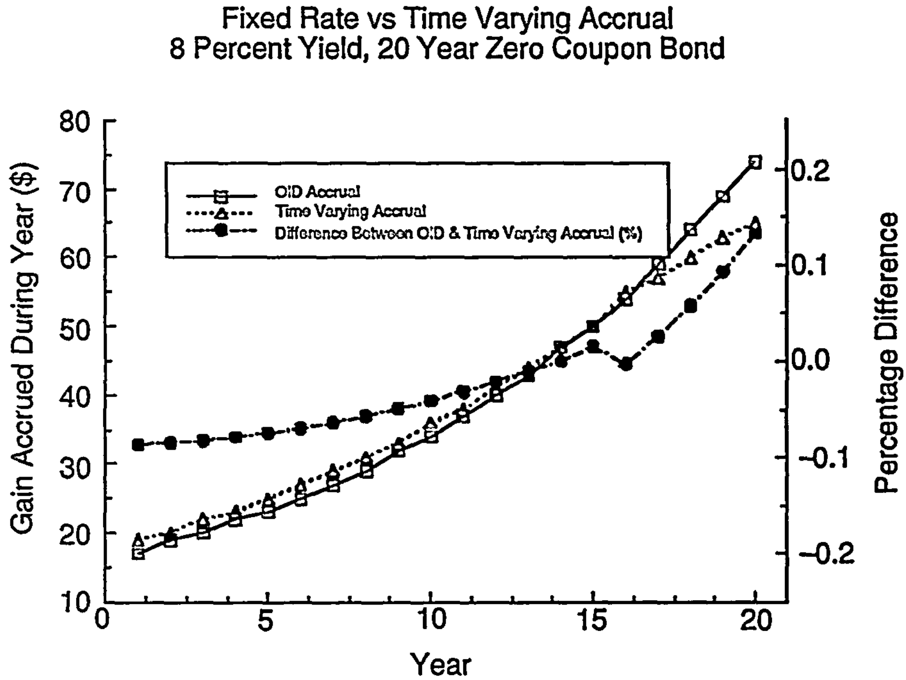
[e] The percentage deviation is the difference between the value in Column 8 and the value in Column 4, divided by the value in Column 4.

[f] The discontinuity observed at Year 6 in Columns 5 and 9 reflects the change in the slope of the yield curve at that point. See notes 132 and 133.

[g] The average deviation is the average of the *absolute values* of the entries in Column 9.

[h] The entries in Columns 4 and 8 are the sum of the taxes imposed during each year at a 30% marginal rate on the change in market value (Column 4) or the accrued income (Column 8), together with interest, compounded annually from the close of the year in which the tax accrued until maturity, using in each compounding period the interest rate for that period (Column 5), and assuming that the interest was taxed at 30%. The difference between these two, expressed as a decimal fraction of the tax on the accrued gain, is $6.80/$382.19 = .017792.

FIGURE IV



To put Table 1 into context, it should be noted that the amount of outstanding discount debt in the hands of taxable holders and subject to the original issue discount rules is probably small. Tax advantages aside, historical practice has been to issue debt instruments with maturities longer than a year with coupons attached rather than in discount form.[136] Furthermore, while favorable taxation may have elicited some proliferation of discount obligations issued to taxable holders before 1969, between 1969 and 1982 the treatment of discount obligations was explicitly more onerous to taxable holders than the taxation of coupon-carrying debt.[137] Since 1982, it has been at least as onerous as that of coupon-carrying debt.[138] It is therefore likely (at least since 1969) that the original issue discount rules, which in form *prescribe the taxation* of obligations issued at a discount, operate in fact to *eliminate* discount debt from the portfolios of taxable investors. Hence, at least as far as the taxation of zero-coupon obligations is concerned, the impact of the yield curve may only be a matter of theoretical interest.

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136 E.g., Shiller et al., note 68, at 177. Note that strips are now issued by Treasury, but primarily for sale to exempt institutions. See generally Frank J. Fabozzi & Irving M. Pollack, The Handbook of Fixed Income Securities 94 (1987).

137 See text accompanying notes 35-36.

138 See text accompanying notes 31-46.

It is also the case, however, as Bankman and Klein observe,[139] that the impact of the term structure on the valuation (and the accuracy of taxation) of long-term debt is not confined to discount obligations. It affects coupon-carrying bonds—which may be regarded as a portfolio of distinct instruments having differing maturities[140]—and other forms of debt that explicitly state and pay interest as well.[141] In this respect, the influence of the term structure is unquestionably widespread.

On the other hand, the impact of the yield curve on instruments like coupon-carrying debt of a given nominal maturity is substantially more modest than its impact on a discount obligation having the same nominal maturity. The true "duration" of coupon debt, with coupon payments as well as "principal" taken into account and all payments weighted in accordance with their present values, is materially shorter than the instrument's nominal maturity.[142] Consequently, given zero-coupon and coupon-carrying debt of identical nominal maturities, the influence of the term structure on the latter should be noticeably less pronounced.[143]

A feel for the influence of the term structure on the valuation of coupon-carrying debt that is more consistent with the data can be developed either by valuing the constituent elements of a coupon bond using term structure data for zero-coupon obligations, or, alternatively (and perhaps more simply), by examining the influence of the term structure exhibited by coupon-carrying instruments on a coupon-carrying bond. To facili-

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139 See text accompanying notes 97-98; Bankman & Klein, note 5, at 341-45.

140 See text accompanying notes 46, 98-99; Shiller et al., note 68, at 177.

141 Bankman & Klein, note 5, at 341-44 and tbls. 5 and 6.

142 This is because the coupons redeemed before maturity are in effect separate obligations having distinct (earlier) maturity dates. See, e.g., notes 46, 99; Fabozzi & Pollack, note 136, at 81-98; Shiller et al., note 68, at 177, 201 (noting that average present-value weighted "duration" of 30-year Treasury bonds in one sample was about 13.5 years).

This observation also applies to other forms of interest-paying obligations, such as self-amortizing home mortgages and consumer installment obligations. The point is recognized explicitly in the proposed regulations implementing the original issue discount rules. There exists a de minimis exception to the operation of those rules, the amount of which varies with the "maturity" of a debt obligation. IRC § 1273(a)(3). For debt obligations that make repayments of principal before maturity, the proposed regulations compute the "maturity" of the obligation as a whole by weighting each constituent payment by its maturity. See Prop. Reg. § 1.1273-1(a)(3)(ii).

143 This phenomenon is implicit in Bankman and Klein's examples. As developed in their Table 3, Bankman & Klein, note 5, at 340, a half-point change in yield as maturity changes by a year introduces a 25% distortion into the measurement of income from a discount obligation with five years remaining to maturity. In contrast, as illustrated in their Table 5, id. at 342, a steeper yield curve produces only a 16% distortion in the income of the holder of a coupon-carrying bond with five years remaining until nominal maturity.

As might be expected, the McCulloch data calculations of yield to maturity on zero-coupon instruments discloses a pattern essentially identical to that for coupon-carrying obligations—the latter were derived from the former—except that it is more pronounced, reflecting the fact that coupon bonds (and their observed yields) were treated as a portfolio of instruments the maturities of most of which were shorter than the nominal maturity of the bonds. McCulloch, Term Structure Data, note 106, at 672.

tate comparison with the type of example developed by Bankman and Klein,[144] however, the former approach is used. The resulting example is set out in Table 2 and depicted in Figure V.

It illustrates the accrual of interest implied by the yield curve to the holder of a 20-year, 8% coupon bond.[145] Once again, when data typical of the actual term structure are used, the deviations exhibited are substantially smaller than suggested by Bankman and Klein's Table 5. Taxing the holder on the coupon each year produces a maximum deviation from the accrual implied by the term structure of about 10%, and deviations with an average of less than 2.5%.[146]

Although the examples in Tables 1 and 2 are in some sense faithful to the values typically exhibited by the yield curve, they remain no more than examples. They fail, in particular, to capture the effects of random (or "stochastic") variations in both the level of interest rates and the term structure. They do, on the other hand, exhibit one important feature that would not be altered by the introduction of fluctuations in the economic variables of interest. In both instances, taxing the holder using a constant yield in lieu of the successive valuations implied by the yield curve will overstate income during some part of the instrument's term and understate it during the balance. Over the life of the instrument, these variations cancel out.[147] That does not, of course, imply that they are without any overall effect. By maturity, they usually will have had some net influence on the value of the taxes paid by, and the after-tax value that has accrued to, the holder. But simple comparisons of the amount by which the single-rate convention deviates from the pattern of accrual implied by the yield curve, either year-by-year or on average, are not especially meaningful as an index of the overall effects.

A more useful way to assess the collective impact of these annual deviations is to value them as of some single moment in time, for example by discounting each of them to present value as of the date the instrument was issued, or by extending them to future value as of the date it is to be surrendered. For each example, a computation of this sort has been

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144 Bankman & Klein, note 5, at 341-44, tbl. 5.

145 The 20-year term and 8% coupon were used in Table 2 in lieu of the five-year term and 9% yield used in Bankman and Klein's Table 5 to facilitate comparison with Table 1. Consistent with the approach in this example of valuing each constituent element of the bond as a separate discount obligation, the yield curve employed was again based on the term structure exhibited by discount obligations. It therefore has the same slope as the curve that was used in developing the example in Table 1. Id. The initial yield to maturity was, however, adjusted upwards from 8% so that, on the hypothetical date of its issue, the instrument was valued at par.

146 This is an unweighted average of the absolute values of the deviations. See note 134.

147 The aggregate gain to be accrued either with or without adjustment for the effects of the yield curve is ultimately the same. Therefore, the sum of any year-by-year differences produced by the two methods must add up to zero. See text accompanying notes 35-36.

TABLE 2

PATTERN OF INTEREST ACCRUAL IMPLIED BY THE AVERAGE TERM STRUCTURE FOR A 20-YEAR 8% COUPON BOND

(1) Years to Maturity	(2) Coupon Payment	(3) Yield to Maturity[a]	(4) Value Beginning of Year[b]	(5) Income Accrued[c]	(6) Implied Forward Rate[d]	(7) Percent Deviation[e]
20	$80	8.21%	$1,000.00	$82.00	8.200%	−2.44%
19	80	8.17%	1,002.00	82.00	8.184%	−2.44%
18	80	8.13%	1,004.00	81.98	8.166%	−2.42%
17	80	8.09%	1,005.99	81.94	8.146%	−2.37%
16	80	8.05%	1,007.93	81.88	8.124%	−2.30%
15	80	8.01%	1,009.81	81.79	8.100%	−2.19%
14	80	7.97%	1,011.61	81.67	8.073%	−2.05%
13	80	7.93%	1,013.28	81.52	8.045%	−1.86%
12	80	7.89%	1,014.79	81.32	8.014%	−1.62%
11	80	7.85%	1,016.11	81.08	7.980%	−1.33%
10	80	7.81%	1,017.20	80.79	7.943%	−0.98%
9	80	7.77%	1,017.99	80.45	7.903%	−0.56%
8	80	7.73%	1,018.44	80.04	7.859%	−0.06%
7	80	7.69%	1,018.48	79.57	7.813%	0.54%
6	80	7.65%	1,018.05	79.02[f]	7.762%	1.24%
5	80	7.61%	1,017.08	80.06	7.871%	−0.07%
4	80	7.51%	1,017.14	78.56	7.724%	1.83%
3	80	7.41%	1,015.70	76.84	7.565%	4.12%
2	80	7.31%	1,012.53	74.86	7.393%	6.87%
1	80	7.21%	1,007.39	72.61	7.208%	10.18%
Maturity:			$1,000.00			
Average Deviation:						2.37%[g]
Future Value of Taxes:	$873.39		$879.98[h]			

[a] As described in note 145, yield to maturity increases by .10% for each additional year to maturity between years one and five, and by .04% for each additional year thereafter. The slope of the yield curve is identical to that used in constructing Table 1. The starting point — the 20-year yield of 8.21% — differs. Because of the nonlinearity of the present value function, a 20-year yield to maturity of 8% (corresponding to the coupon rate) would have produced an issue price different from $1,000. This column corresponds to the "Blended Interest Rate" column in Bankman & Klein, note 5, at 342 tbl. 5.

[b] For simplicity, a separate calculation of the present value of each constituent element of the bond has not been set out in Table 2. The figures in this column consist of the sum of (1) the $1,000 payment to be received at maturity, discounted for the number of years remaining until maturity, using the yield to maturity from Column 3, plus (2) the number of coupons still outstanding as of the beginning of the year, each discounted for the number of years remaining until that coupon is redeemed, using the yield to maturity from Column 3 for the number of years that remain until it is redeemed. This figure *excludes* the $80 value of the coupon surrendered at the close of the preceding year.

[c] Accrued income consists of the change in the value of all remaining coupons and the bond, between the beginning of the year and the beginning of the following year, plus the value of the coupon redeemed at the close of the year (which is excluded from the computation of value as of the beginning of the following year). See note b.

[d] The implied forward rate is the value in Column 5 (income accrued during the year) divided by the value in Column 4 (value at the beginning of the year). This column corresponds to the "Annual Interest Rate" column in Bankman & Klein, note 5, at 342 tbl. 5.

[e] The percentage deviation is the difference beween $80 and the value in Column 5, divided by the value in Column 5.

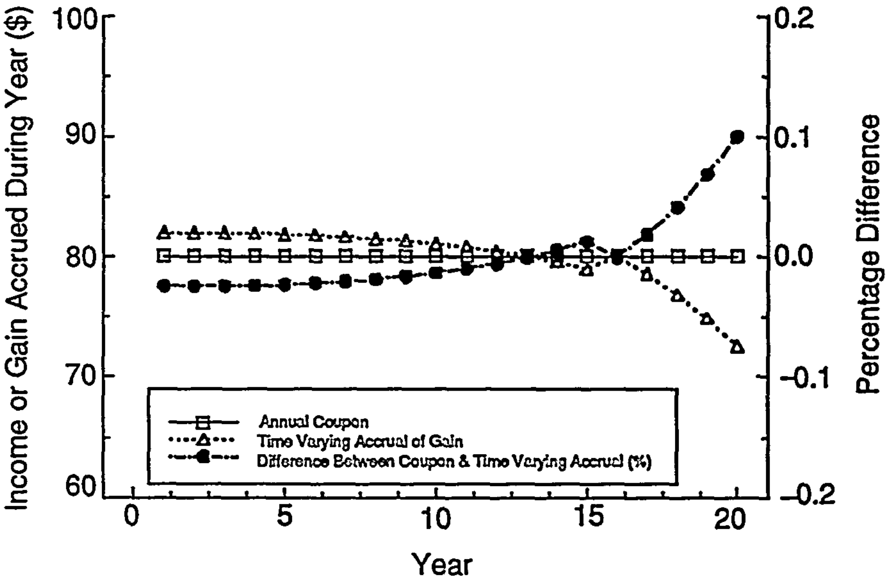
[f] The discontinuity observed at Year 6 in Columns 5-7 reflects the change in the slope of the yield curve at that point. See note a.

[g] The average deviation is the average of the absolute values of the entries in Column 7.

[h] The entries in Columns 2 and 5 are the taxes imposed during each year at a 30% marginal rate on the Coupon Payment (Column 2) or the Accrued Income (Column 5), together with interest, compounded annually from the end of the year in which the tax accrued until maturity, using in each compounding period the interest rate for that period given in Column 6, and assuming that the interest itself was taxed at 30%. The difference between these two, expressed as a decimal fraction of the tax on the accrued gain, is $6.59/$879.98 = .007489.

FIGURE V

Fixed Rate vs Time Varying Income
8 Percent, 20 Year Coupon Bond



made. Specifically, a calculation has been made of the value, as of the date of surrender, of all taxes paid by the holder over the life of the obligation, taking into account the times at which they were paid.[148] By these calculations, the sum of the annual deviations in measurement over the life of the instrument produces an aggregate difference of less than 2% in the value of the taxes paid by the holder of the discount obligation. For the coupon bond, the corresponding figure is a deviation of less than 0.75%.

The examples set out in Tables 1 and 2 are both based on actual term structure data. Table 1 suggests that the distortions induced by the yield curve in measuring income from debt are modest by comparison with the range of values illustrated by Bankman and Klein. Table 2 suggests that

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148 The methodology of this comparison is that suggested in Strnad, Periodicity, note 7, at 1828-30. In each instance, the taxes imposed on the holder (who was assumed to be taxed at a marginal rate of 30%) were computed, first on the basis of the gains implied by the yield curve and then using the single-rate approach of the original discount rules. Those taxes were then extended to their value as of the date of surrender. The future values were calculated by accruing interest on the tax liability from the time it fell due until the date of surrender at the time-varying rates implied by the yield curve, assuming that this interest also was taxed at a 30% rate.

A comparison of the after-tax values of the *investments* as of the date of surrender (assuming reinvestment until maturity of all periodic returns after payment of taxes) would lead to the same conclusion (in percentage terms) as comparing the value of the taxes paid.

the attenuated influence of the yield curve on coupon-carrying debt, given the probably low incidence of discount debt in the hands of taxable holders, renders even more modest the impact this phenomenon will have.[149]

One comparison can assist in putting the dimensions of the problem into perspective. The value of debt is influenced by phenomena other than the yield curve.[150] Rather than canvassing the full range of possibilities, one illustration will do. On average, over the past 40 years, the year-to-year change in the rate of interest on one-year Treasury debt has been about 1.25%. That is more than 25 times the typical year-to-year change in yield to maturity from movement along the yield curve. Furthermore, changes in the level of interest rates enter into the market's valuation of debt in the same way as changes in yield to maturity as time passes and an instrument moves along the yield.[151] Consequently, the impact of changes in interest rates and variations in yield with maturity may be directly compared. The comparison implies that the effects of the yield curve typically will be swamped by variations in interest rates alone.

C. *The Feasibility of Manipulation*

Even though the distortions induced by ignoring the yield curve may be slight, Bankman and Klein suggest that investors may capitalize on them by adjusting their "asset holdings to take advantage of the misstatement of interest under present law."[152] They caution, however, that other distortionary features of the tax law might "make this distortion difficult or impossible to detect."[153] While the precise nature of these adjustments is left unspecified, there are two ways to interpret this suggestion.

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[149] For the sake of completeness, a further calculation, not reproduced in a table, was carried out using a hypothetical yield curve that was more pronounced by about one standard deviation than the average. Under this curve, yield to maturity changed by 0.0025% per year between maturities of one and five years, and by five hundredths of a percent (0.0005%) for each additional year of maturity up to 20 years. Compare note 132.

This produces a yield curve with a width of 1.75% based on a 20-year span, consistent with a width of just under 2.2% for the 25-year range of observed maturities in the McCulloch Term Structure Data, note 106. There were only 15 quarters in the quarterly sample of the entire period 1947-1986 during which the actual yield curve was more pronounced.

That curve was used as the basis for a coupon bond example like that set out in Table 2. The average of the absolute values of the discrepancies between the accruals using the single rate convention and the valuations implied by this yield curve was under 3.75%. The change these discrepancies induced in the after-tax, date-of-surrender value of the instrument was still less than 1%. Compare text accompanying notes 134, 146, 148.

[150] See Bankman & Klein, note 5, at 336; text accompanying note 23.

[151] See Appendix B.

[152] Bankman & Klein, note 5, at 346-47.

[153] Id. at n.26.

One possible interpretation is that the tax advantage conferred on long-term debt by ignoring the term structure renders it relatively more attractive than other financial investments. Interest, however, has consistently been the most heavily taxed investment return, and the magnitude of the advantage from ignoring the term structure suggested by the data is relatively slight.[154] It is unlikely that tax induced advantages of this general magnitude would be sufficient to overcome the significant tax induced disadvantages otherwise historically borne by debt, thereby inducing shifts in portfolio composition away from more lightly taxed assets towards debt.

A more plausible interpretation is that the distortions induced by the yield curve may influence the choice between short- and long-term debt. Specifically, when the yield curve has a "normal" (that is, positive) slope, ignoring the yield curve for purposes of taxation—thereby deferring recognition of income from and increasing the after-tax return to long-term debt[155]—should enhance the relative attractiveness of long-term debt, at least when, as the economic evidence suggests, the slope is attributable not to expectations but to risk.[156]

Regardless of interpretation, however, it seems unlikely, both for conceptual and practical reasons, that investors could make effective adjustments to any appreciable extent. Identifying the advantages would be a difficult task. They would have to be detected against a background of stochastic variations in both the level of interest rates and the contours of the yield curve. Such fluctuations typically would be far greater in magnitude than the advantages conferred upon long-term debt by ignoring the yield curve for purposes of taxation.[157] The very size of the advantage would change with those fluctuations.[158] Even if detected, these tax-induced advantages would amount quantitatively to only a fraction of whatever premium return was conferred on long-term debt by the yield curve itself.[159] Even more significantly, whatever the tax-induced distortions might be, they never would alter the relative attractiveness of long-

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154 It usually will equal the product of the applicable tax rate and the percentage (typically less than five) by which the asset's return is mismeasured. See Tables 1 and 2.

155 Bankman & Klein, note 5, at 338-39.

156 See notes 21-22 and accompanying text.

157 See Tables 1 and 2 and accompanying text.

158 If the slope of the yield curve were initially negative, ignoring the yield curve would be disadvantageous to the taxpayer.

159 The advantage induced by ignoring the term structure for purposes of taxation is never more than the product of the applicable tax rate (currently at a maximum of about .3) and the additional yield for longer maturity typically provided by the term structure. Assuming a yield to maturity on a 20-year bond of 10%, a one-year rate of 9%, and a distortion from ignoring the yield curve of about 5% of yield, the tax advantage would be .09 x .05 x .3 = .00135, an additional advantage of less than one-seventh of the premium return to long-term debt provided by the yield curve itself.

term debt qualitatively.[160] Therefore, it does not appear as though distortions induced by ignoring the yield curve would be independently amenable to exploitation.

It is, then, one thing to observe that ignoring the yield curve for purposes of taxation may produce an advantage. Because that advantage typically will be smaller than the advantage conferred by the yield curve itself, and given the nature and extent of the variations in both interest rates and the slope of the yield curve, in practice it would be quite difficult to detect and exploit.

D. *The Aggregate Significance of the Yield Curve*

All things considered, it does not appear that the influence of the yield curve on the accuracy of the measurement of income accruing to long-term debt is great. The data suggest that the phenomenon is not particularly pronounced. Given both its stochastic variation and characteristic magnitude in comparison with that of other economic variables, it does not seem that the effect of the yield curve is especially vulnerable to manipulation. Consequently, although on occasion the yield curve does take on extreme values, it seems unlikely that its impact on individual incomes typically will be great.

This does not establish, however, that in the aggregate (or even in particular instances) its impact will be trivial. There is a good deal of outstanding credit market debt.[161] Even a small percentage of a sufficiently large number can be a matter of concern. To arrive at a precise fix on the true magnitude of the issue would require an analysis of the holdings of credit market debt by maturity, and by type and taxability of both the borrower and lender. An analysis at that level of detail is not undertaken here; nevertheless, it is possible to acquire a feel for the general dimensions of the problem using the Federal Reserve Flow of Funds Accounts.

At the end of 1989, approximately $9 trillion in taxable credit market debt was outstanding, of which about $8.1 trillion was in private hands.[162] About $2.1 trillion of the debt in private hands, however, was

[160] If the single rate convention operated to defer the recognition of income, as it would when the term structure had a positive slope, that effect would fractionally enhance the additional return to long-term debt. If, on the other hand, the single rate convention accelerated the recognition of income (and thereby reduced the after-tax return to long-term debt), as it would when the term structure exhibited a negative slope, the yield disadvantage on long-term debt would be magnified.

[161] See note 162.

[162] See 1989 Flow of Funds Data, note 6, at A44-A45. Aggregate credit market debt outstanding at that time, net of credit market debt issued by public and private financial intermediaries (principally credit market debt of private financial institutions that themselves held credit market debt, and publicly-sponsored mortgage securities backed by mortgage loans) was about $9.8 trillion, of which approximately $800 billion consisted of tax-exempt

held by pension and insurance trusts, leaving about $6 trillion in taxable private hands.[163] If all the credit market debt were assumed to be held directly by individuals, plausible assumptions about yield and the contours of the term structure would imply a gross annual deviation in the measurement of individual income from ignoring the yield curve of about $18 billion.[164]

Quite apart from the uncertainties in the derivation of this number,[165] it suffers from a serious conceptual flaw. As described above, *annual* deviations tend to offset one another over time.[166] For that reason, they are not a sound guide to the impact of the yield curve on the measurement of income. A better index would be a consistently valued *sum* of those deviations.[167] That sort of calculation, also based on the $6 trillion of taxable credit market debt in taxable private hands and translated into annual terms, suggests that the mis-measurement induced by the yield curve would be more on the order of $5 billion.[168]

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state and local debt. Of this amount, about $8.9 trillion, including over $800 billion of state and local debt, was in private hands.

163 Id. at A45. The $2.1 trillion of debt in the hands of exempt holders apparently excludes holdings of debt by other categories of exempt organizations.

164 At the end of 1989, the composite yield on Treasury securities having a maturity of 10 years or more was 8.64%. The difference between yields on one-year and 30-year Treasury securities (approximately the width of the term structure) was approximately zero. 1989 Flow of Funds Data, note 6, at A24. Hence, the calculation in the text assumes that the overall yield on taxable credit market debt in taxable hands (only a fraction of which was U.S. government debt) was 10%, and that the deviation in income induced by ignoring the yield curve was 3% of yield. The latter figure is consistent with the average data used in constructing Tables 1 and 2, and with the further assumption that most debt in private taxable hands is interest-paying (rather than zero-coupon) debt. See notes 136-38 and accompanying text. The term structure that actually prevailed during 1989 would have produced essentially no distortion whatsoever.

On those assumptions, the figure given in the text is the product $6 trillion × .1 × .03 = $18 billion.

165 The computation conveys, in particular, a false sense of precision. The examples from which it is derived assume that the extent to which accrual under the original issue discount rules deviates from market valuations can be feasibly measured, an assumption that frequently will be false. See notes 198-202 and accompanying text.

166 See notes 134 & 147 and accompanying text.

167 See note 148 and accompanying text.

168 This estimate begins with a calculation that is procedurally identical to that carried out in connection with Tables 1 and 2, see note 148, but values the pretax *differences in annual income*, rather than the taxes themselves. When carried out for either the zero-coupon or coupon-carrying bond, this calculation produces values with a date-of-surrender difference of approximately $40.

That figure, however, is the future value of differences that accumulated over 20 years. To convert the $40 into a yearly equivalent, it was simply annualized over 20 years using (for simplicity) an 8% nominal discount rate. This conversion places an annualized equivalent value on the aggregate distortions of $0.874 per $1,000 of outstanding long-term debt.

If differences with this annual value were taken to be characteristic of every $1,000 of long-term debt, and the entire $6 trillion of taxable credit market debt in taxable private hands was assumed to be long-term, the implied aggregate annual value of the deviations would be ($6 trillion/$1,000) × 0.874 = $5.24 billion.

Even this estimate must be interpreted with caution. It has shortcomings, as a guide both to the aggregate, system wide effect of ignoring the yield curve, and as a guide to the aggregate impact of doing so on individuals. As to the former, the estimate assumes, first, that outstanding credit market debt consists entirely of long-term debt. That assumption obviously is false. Eliminating short-term debt would lead to a reduction in the estimate.[169] More importantly, it ignores the impact of the yield curve on taxable issuers of debt, whose interest deductions (and taxable income) would be distorted in a complementary fashion that would offset the distortions in the income of taxable holders.[170] That phenomenon alone could reduce the aggregate estimate by as much as 70%.[171]

As an index of the overall effect on individuals, the estimate also is flawed. It tacitly assumes that all credit market debt in taxable private hands is held directly by individuals. More than two-thirds is actually held by financial intermediaries alone, offset by deposits with financial intermediaries by nonfinancial investors.[172] More importantly, at the individual level there are other offsetting effects to consider. Individuals

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169 The maturity structure of the U.S. public debt suggests that a very substantial fraction of credit market debt is short term debt. As of 1989, approximately 34% of the marketable Treasury debt in private hands had a maturity of one year or less; about 48% had a maturity of two years or less; and only 17% had a maturity of more than 10 years. Treasury Bull., Mar. 1990, at 35.

170 This point is recognized by Bankman & Klein, note 5, at 346. It also has been recognized by other students of accrual taxation, who, like Bankman and Klein, observe that the point is devoid of force if the debt is either issued or held by a tax-exempt party. Id; see also Shakow, note 7, at 1131.

The 1989 Flow of Funds Data, note 6, suggest, however, that substantially more than two-thirds of the taxable credit market debt in the hands of (nominally) taxable lenders is issued by (nominally) taxable borrowers. Of approximately $2.3 trillion of U.S. Treasury debt (other than debt issued or sponsored by official financial institutions and in turn backed by credit market debt) some $690 billion was itself in official or foreign hands, leaving about $1.6 trillion in private domestic hands. If this *all* were assumed to be in taxable hands, that would *still* leave about $4.4 trillion of the $6 trillion of taxable credit market debt held by private taxable lenders as having been issued by private taxable borrowers.

In attempting to assess the dimensions of the problem, it seems more appropriate to take account of *aggregate* taxable holdings of taxable debt issued by taxable borrowers, rather than, as in Shakow, note 7, at 1131, restricting attention to debt held directly by individuals.

171 See note 170. A common objection to the relevance of this sort of observation is that, even though there may exist offsetting effects as between issuers and holders, private participants in a transaction may structure it to take advantage of differences in the *rates* at which they are taxed. See, e.g., Halperin, note 2, at 509-12.

In the present setting, however, concerns of this nature do not seem well founded, because of the difficulties investors will have detecting the advantages, and, because both the *direction* of the effects and the intensity of any advantages will vary unpredictably over time. See notes 157-60 and accompanying text.

172 While there is no a priori reason to assume that the yield curve would not affect the measurement of the income of financial intermediaries holding credit market debt, these effects almost surely would be complex, involving the influence of the yield curve on the value of debt held by financial institutions, on the value of debt issued by financial institutions and (in theory at least) on the market valuation of time deposits with financial intermediaries having maturi-

are not just holders of credit market debt; they are also substantial borrowers. About a third of the total outstanding credit market debt at the end of 1989 consisted of residential mortgages and consumer credit.[173] The yield curve presumably affects the accrual of interest obligations of individuals on such debt[174] in a way that tends to offset its impact on them as credit market lenders.[175]

It is reasonable to expect that, after adjustments for these considerations, the actual net effects of the yield curve, either in the aggregate or on total individual income, would be materially less than the $5 billion estimate developed above. At the aggregate level, an adjustment for the offsetting effects on borrowers and lenders alone would reduce the annual estimate to about $1.4 billion. At the individual level, it is doubtful that a phenomenon generously estimated to be of this general magnitude would produce dramatic variations in income as reported on many individual returns.[176]

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ties longer than a year. As of the close of 1989, time deposits with financial institutions amounted to more than $2.5 trillion. 1989 Flow of Funds Data, note 6, at A45.

[173] As of the end of 1989, these two items amounted to nearly $3.2 trillion. See 1989 Flow of Funds Data, note 6, at A44.

Calculations by Shakow, note 7, at 1125-26, indicate that, as of the end of 1984, individual holdings of taxable credit market debt amounted to about $857 billion; and individual holdings of savings and time deposits with financial intermediaries (excluding money market shares), which indirectly would have been backed by credit market debt, amounted to $1.876 trillion, so that the total of such holdings was approximately $2.7 trillion. At the same time, the individual obligations on home mortgages and consumer credit amounted to about $1.9 trillion. Id. at 1126.

Not surprisingly, Professor Shakow's calculations, based on holdings by individuals, suggest a higher ratio (70%) of mortgage and consumer credit liabilities to direct and indirect holdings of credit market instruments than the 1989 ratio of home mortgage and consumer credit liabilities to *total* private taxable holdings of taxable credit market debt of about 53%.

[174] See Bankman & Klein, note 5, at 342-44 & tbl. 6.

[175] Professor Shakow's calculation that the ratio of individual obligations on home mortgages and consumer credit to direct and indirect holdings of credit market assets was about 70% (see note 173) suggests that these offsetting effects at the individual level would be significant.

[176] Using the $5 billion estimate in the text, *unreduced* by any of the adjustments suggested in the text, and even assuming that the effects were concentrated in the upper 12% (those reporting adjusted gross incomes in excess of $50,000) of all individual returns—approximately 14 million in number in 1988—the average distortion in the income reported on each of those returns would be about $357, producing a variation in tax liability (at the maximum current individual rate) of perhaps $110 per return. In fact, the top 12% of all returns reported only about $78 billion, or 47%, of the roughly $168 billion of taxable interest reported on all individual returns for 1988. See Dep't of Treasury, Internal Revenue Service, Statistics of Income Division, Individual Income Tax Returns, Preliminary Data, 1988, Stat. of Income Bull., Spring 1990, at 15, tbl. 1.

Even if we confine our attention to the 65,184 returns reporting adjusted gross incomes of more than $1 million, which accounted for 6.7% of all taxable interest, id., their share of the unadjusted $5 billion estimate would be about $335 million, or an average distortion of about $5,140 per return, producing an average variation in tax liability of about $1,600 per return.

V. Accrual Taxation and Long-Term Debt

The preceding sections suggest that the distortions in measuring income induced by the term structure are remediable both in principle and (at some administrative cost) in practice. They also seem small in magnitude, especially when compared to fluctuations in the value of long-term debt induced by other economic variables, and they are not, in any event, especially vulnerable to exploitation. The principal objective of this article has been to suggest, in light of those conclusions, that, although the influence of the yield curve on the taxation of long-term debt is of analytical interest, as a practical matter it may not be cause for serious concern. In particular, the probably modest distortions it induces appear to add little to the case for an accrual tax.

Still, such observations reasonably might be regarded as begging the more important question. The conclusion that the impact of the term structure is modest flows largely from a comparison of its influence with that of economic variables, whose effects unquestionably are more pronounced. So it seems appropriate to conclude by considering more generally how important the treatment of long-term debt may be to the case for accrual taxation. The issue has multiple dimensions. Apart from the larger question of the possible gains from adopting a system of accrual taxation, there are some more immediate policy implications, particularly for the treatment of market discount debt.[177]

The development here is somewhat more tentative than that of the preceding sections. It suggests, however, three conclusions. First, given

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177 On the principal issue—the wisdom of an overall shift to an accrual tax—the belief seems widely held that accrual taxation of asset gains and losses—often referred to as a "mark-to-market" system—is part and parcel of an ideal tax based on accretion. E.g., Shakow, note 7, at 1119; Daniel N. Shaviro, Risk and Accrual: The Tax Treatment of Nonrecourse Debt, 44 Tax L. Rev. 401, 403-04 & n.1 (1989). For purposes of what follows, I take that proposition as given.

Renewed interest in accrual taxation appears to have been stimulated at least partly by the adoption, in 1981, of accrual taxation for commodities futures contracts. IRC § 1256. In contrast with other financial investments, such contracts, *independently of tax considerations*, are actually valued and settled in cash on a daily basis, a process denoted in that world as "marking" the contracts "to market" (and, evidently, the source of the alternative way of describing an accrual tax). Consequently, when § 1256 was originally enacted, there already existed a mechanism by which gains and losses on commodities contracts were valued on a daily basis and therefore, could readily be taxed. Pub. L. No. 97-34, § 503, 95 Stat. 327, 327-30 (1981). In some sense, the gains and losses could be regarded as having been "realized" daily.

The accuracy with which commodities contracts are valued daily does not, of course render them unique. There are other assets, including traded corporate equity securities and traded debt as well as derivative securities of various sorts, for which markets, characterized by a high volume of trading, exist. But, for many assets, this is not so. In addition, commodities futures contracts *are* unique in being settled in cash in accordance with their daily market valuations. That is, when such contracts are marked to market daily, any net gain (or loss) on that day is actually received by (or paid to) the holder *in cash*.

differences between debt and assets that may be regarded broadly as "equity"—differences in both their inherent characteristics and the way in which they currently are taxed—the gains from accrual taxation would be far more conspicuous for equity than for debt. At the same time, the absence of regular trading in a good deal of outstanding debt implies that the "market valuations" essential to accrual taxation would consist of approximations rather than true market valuations. When required in quantity, they would not be unburdensome to obtain. The second point, then, is that in practice accrual taxation of debt might prove to be both costly and inescapably imprecise. Finally, and most intriguingly, recent research suggests that the way in which debt instruments change value in response to stochastic variations in interest rates, the principal determinant of the value of debt, may be approximated best by a pattern of exponential growth of the sort already incorporated in the law. Establishing that final proposition, however, would require mathematical modelling of a sophistication far beyond the original objectives of this article.

A. The Advantages of Accrual Taxation of Debt

Under an accrual tax, in contrast with our existing realization system in which most asset gains (and losses) are not taxed until sale or other disposition,[178] taxation would be based on periodic valuation.[179] At each valuation date, the gains and losses that had accrued since the immediately preceding valuation date would be taken into account for purposes of taxation.[180] For some assets—notably, corporate equity securities, real estate and other physical assets—an accrual system would mark a shift to periodic taxation of gains the taxation of which under existing law is frequently long delayed.[181] For assets held by individuals, moreover, taxation delayed often translates into taxation forgone by reason of the step up in basis at death.[182]

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[178] IRC § 1001. The principal exception involves commodities futures contracts marked to market. IRC § 1256; see note 177.

[179] See, e.g., Shakow, note 7, at 1111-20. Consistent with existing practice of reporting income on an annual basis, most observers assume that valuation would occur annually. Id. Strnad, Periodicity, note 7, at 1825-30, examines in detail selection of the appropriate period to use in implementing an accrual tax.

[180] See generally Shakow, note 7, at 1111-18, and authorities there cited.

[181] The issue for real estate (and, to some extent, for other physical assets) is actually somewhat more complex. For structures, the importance of an accrual tax depends upon the accuracy with which depreciation is allowed as a deduction for purposes of measuring taxable income. If depreciation were accurate, which in practice it never is, an accrual tax would be unnecessary for structures or other depreciable assets. On the other hand, for undeveloped land, the choice between accrual and realization taxation would have about the same significance as it does for corporate stock. Since, however, structures invariably rest on land, the accuracy with which a realization-based tax treats developed real property depends on *both* the accuracy of depreciation *and* the extent to which the value of the underlying land is changing.

[182] IRC § 1014.

For other assets, however, accrual taxation would produce only a refinement in the inter-period *allocation* of amounts that already *are* periodically included in income. Much outstanding debt falls into this category. Income from most debt already is subject to some form of periodic inclusion,[183] and, in most instances, inclusion is now based on exponential accrual. So, instituting an accrual tax would simply change the method of calculating the amount of income from debt to be taxed each year.[184]

In ordinary circumstances, the principal implication of an accrual tax for debt would be to adjust its value as interest rates change. Given the variability of interest rates noted above,[185] it would seem at first glance that accrual taxation might significantly improve the measurement of income from debt. Nevertheless, several considerations suggest that in practice there may only be modest gains to be achieved. These considerations turn on the fact that, at least insofar as accrual taxation is concerned, debt and equity differ in a quite fundamental way.

Equity—whether in the form of corporate stock or through direct ownership of physical assets—constitutes a residual claim to the assets themselves, and can fluctuate freely in value. In particular, there exists no inherent limit on the extent to which equity claims may appreciate in value. In contrast, debt typically consists of a claim to a finite stream of payments that is prespecified, usually in nominal terms. That characteristic limits the extent to which debt instruments may fluctuate in value. Like equity, debt can, at one extreme, decline in value to zero. It can also increase in value. A debt obligation, however, in contrast with an equity claim, generally cannot appreciate to more than the undiscounted (finite) sum of its nominal payments.[186] Consequently, the value of debt is limited in a way that the value of equity typically is not.[187] Since, moreover, virtually all the improvements to be achieved through accrual taxation involve the taxation not of losses, but of gains, that is an important constraint.[188]

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[183] See Section I.B & Appendix A.

[184] The principal exception is market discount debt. For such debt, a market valuation based accrual tax would produce a change from taxation only on realization to periodic inclusion. Market discount debt could, of course, be taxed periodically using formulary accrual in somewhat the same fashion as original issue discount (and market premium). See notes 48-56 and accompanying text.

[185] See notes 150-51 and accompanying text.

[186] This would occur if nominal interest rates were to decline to zero. Excluded from consideration here is a debt instrument that consists *solely* of a stream of interest payments in perpetuity (a "consol"), whose value would increase without limit as the nominal interest rate approached zero.

[187] This typically would not be the case for debt that contained so-called "hybrid" features, such as convertibility into equity securities.

[188] Under a realization system, the absence of accrual is not an obstacle to the enjoyment of the tax benefits associated with losses, that may be voluntarily realized, in the absence of spe-

This difference in character between equity and debt has several implications for the extent to which the value of debt will fluctuate as interest rates change. As time passes and a debt instrument's maturity draws near, the fixed amount to be received at maturity increasingly limits the extent to which its value will fluctuate, *regardless* of how much interest rates may change.[189] Hence, in contrast with corporate stock, the variability of whose value is generally regarded as increasing with time,[190] the variance in the value of a debt instrument typically will decline.

In addition, over the life of a debt instrument, the *sum* of the fluctuations in its value is constrained in a somewhat different way. Once *any* formulary method of accruing interest—whether ratable accrual of the sort that existed before 1969, or exponential accrual of the sort that is currently used, or indeed some other method—has been specified, in the absence of default or extension, that method will accrue *all* gain inherent in the instrument at the time it was issued (or, if subsequently acquired in the market, at the time of acquisition) by the specified maturity date.[191] It follows that, even though the instrument's "market" valuation may deviate over time from the path at which interest is being accrued, the *sum* of the deviations over the life of the instrument in all instances is zero.[192]

The force of these observations is that the case for accrual taxation of equity is more pressing than it is for debt. The gain accruing to debt is already taxed periodically, and the extent to which the value of debt may diverge from some prespecified path of accrual is limited in ways that the value of equity is not.

One further consideration, alluded to above,[193] is of some relevance in assessing the overall improvements to be achieved. Much of the gain attributable to holders of debt by accrual taxation would be offset by losses experienced symmetrically by issuers (or vice versa). Roughly 70% of all taxable credit market debt in the hands of taxable holders

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cific limitations, at the discretion of the taxpayer. See generally Strnad, Periodicity, note 7, at 1868-79, 1884-91.

[189] As developed in Appendix B, at short maturities the interest-rate volatility of a debt instrument unambiguously declines as maturity declines, even though the same may not be true at long maturities, especially at high interest rates. See Appendix B, expression (4).

[190] E.g., Strnad, Periodicity, note 7, at nn.143 & 153 and accompanying text.

[191] See notes 48-56 and accompanying text.

[192] See text accompanying notes 147-48. The fact that the undiscounted sum of these deviations is zero does not, of course, imply that their net effect on the tax burdens from holding the instrument is zero. Id.; see also Strnad, Periodicity, note 7, at 1828-30.

Even for debt on which interest is *not* periodically accrued, such as debt acquired at a market discount, fluctuations in value are constrained in a somewhat similar way. Using market discount debt as an example, the sum of *all* fluctuations between the time of acquisition and the time of surrender, regardless of how extreme *individual* fluctuations may be, must in all cases add up to the aggregate market discount to be accrued.

[193] See notes 170-71 and accompanying text.

appears to be issued by taxable borrowers.[194] From a systemic standpoint, gains in the accuracy with which the income of holders and issuers was measured would, to that extent, offset one another: No aggregate improvement in the measurement of system-wide income would be achieved. What is more, even though the exact revenue consequences of this fact would depend on the relative rates at which holders and issuers were taxed, it does not seem likely, given the uncertainties that characterize the movement of interest rates, that tax rate differentials could be exploited systematically.[195]

This leavening effect on the system-wide improvement in the measurement of income will not, of course, carry over to the measurement of the income of particular taxpayers. To many, accuracy in the measurement of individual incomes is the only ideal worth pursuing.[196] Even there, however, the existence of offsetting effects on individuals as both borrowers and lenders on credit market debt,[197] when considered in connection with the imprecision (to be discussed momentarily) that in practice would afflict accrual taxation, casts doubt on how much might realistically be achieved.

B. Shortcomings in the Case for Market-Based Accrual Taxation of Debt

In considering the desirability of an accrual tax based on market valuations, especially given the objective of achieving greater accuracy in the measurement of income, it should be kept in mind that such a tax would have shortcomings of its own. In practice, it would be beset by imprecision. There are, moreover, respects in which even its advantages are open to question on conceptual grounds.

At a practical level, markets for debt with sufficient trading to produce meaningful valuations on a regular basis are far from universal. Such markets are available for U.S. Treasury and some agency securities, and for some (but not all) corporate debt.[198] They are not generally available for a good deal of corporate debt, or for consumer credit and mortgage loans.[199] Thus, for half or more of all outstanding long-term debt, the

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194 See note 170.

195 See notes 150-60 and accompanying text. Since the gains and losses from fluctuations in the value of debt are generally unpredictable, it is unlikely that planners could arrange transactions to take advantage of them.

196 Shakow, note 7, at 1131 & n.75; cf. Halperin, note 2, at 509-12.

197 See notes 173-76 and accompanying text.

198 According to the 1989 Flow of Funds Data, note 6, at A44, a total of about $11.6 trillion of taxable credit market debt—including debt of officially sponsored credit agencies, guaranteed mortgage pools and credit market debt of financial institutions—was outstanding. Of this total, about $3.6 trillion was either issued or backed by the federal government, and about $1.4 trillion consisted of corporate bonds of financial and nonfinancial issuers.

199 About $3.5 trillion of outstanding credit market debt as of the end of 1989 consisted of mortgage debt, about $790 billion consisted of consumer credit and another $765 billion consisted of otherwise unclassified bank loans. 1989 Flow of Funds Data, note 6, at A44.

recurring "market valuations" essential to accrual taxation will rarely be more than approximations. Often such approximations would consist simply of computing the present value of the sum of an instrument's payment obligations, applying a discount rate selected using judgments about what constitutes a "market" rate of interest for debt of comparable maturity and risk.[200]

To that extent, the sense of precision conveyed by the phrase mark-to-market accrual, at least when applied to nonpublicly traded long-term debt, may be something of an illusion. At times when interest rates fluctuate substantially, even periodic *approximations* of market value might be expected to provide a better measure of the gain accruing to holders of long-term debt than formulary accrual of interest. More typically, however, it will not be clear that such approximations produce a pattern of accrual that is more unambiguously "correct" than a system of accrual based on a mechanical, but conceptually sensible, formula.

To be sure, difficulty of valuation is one of the most commonly voiced objections to an accrual tax.[201] In the present setting, however, the gravamen of the objection differs from what is normally the case. When imprecision of valuation is advanced as an objection to accrual taxation, the alternative typically is taxation only at the time of realization. So the advocate of an accrual tax need only reply that even imprecise periodic taxation is preferable to no periodic taxation at all.[202]

In the setting of long-term debt, however, failing to adopt a market-based system of accrual taxation would still leave us with a formulary system of periodic taxation based on exponential accrual. To the extent that market valuations are imprecise, moreover, market-based accrual taxation itself will be an approximation. Here, then, the objection is that, at least as far as a significant amount of long-term debt is concerned, an accrual tax might accomplish little more than substituting one approximation for another. There is, moreover, no generally accepted, objective benchmark by reference to which one approximation can be said to be superior to the other. Market valuation is the standard of comparison to which one normally appeals. In this instance, the standard of comparison is nothing more than the competing approximation.

[200] This is essentially the procedure proposed for nontraded debt by Shakow, note 7, at 1130.

[201] Both Robert M. Haig and Henry C. Simons, the spiritual parents of accretion as a basis for taxation, acknowledged that the absence of markets and resulting difficulties of valuation would impose practical limitations on the basis for taxation they proposed. See Robert M. Haig, The Concept of Income—Economic and Legal Aspects, in The Federal Income Tax 16-20 (Robert M. Haig ed., 1921), reprinted in Am. Econ. Ass'n, Readings in the Economics of Taxation 54, 66-70 (Richard A. Musgrave & Carl S. Shoup, eds., 1959); Henry C. Simons, Personal Income Taxation 56 (1938). Valuation and liquidity are the two most commonly advanced objections to an accrual tax. See generally Shakow, note 7, at 1113-14.

[202] See, e.g., Shakow, note 7, at 1118.

On conceptual grounds, there are two different observations to be made. The first—one that has entered into the debate over the choice between an accretion tax and a consumption tax—is that (for example) the gain to the holder from the change in value of a long-term debt instrument is simultaneously attributable to and offset by a reduction in the instrument's yield.[203] Even though a debt instrument may appreciate in value, the holder, by realizing the gain and reinvesting in the market at the prevailing rate of interest, can still do no better than acquire an instrument whose value at maturity would be identical to that of the instrument sold. The inference drawn from this fact by some students of taxation has been that gains from fluctuations in the value of long-term debt are qualitatively different from (and inferior to) gains from changes in the value of assets (like corporate stock) that are attributable to changes in the earning power of the assets themselves.[204] Although there are differences of opinion on this matter,[205] on conceptual grounds it renders the case for accrual taxation of debt somewhat less clear than for other assets.

A second qualification relates to the form—involving annual valuation—in which implementation of an accrual tax is usually proposed.[206] The interest rate that happens to prevail at the close of a year, and that determines (or may be used to approximate) the year-end value of long-term debt, typically will differ from those that prevailed during the course of the entire year. The importance of this fact is highlighted by the suggestion, recently advanced by Professor Jeff Strnad, that much of the justification for the choice of income as a basis for taxation rests on propositions that imply that continuous accrual of gain should be the norm.[207] Against that standard, the accuracy to be achieved through accrual taxation based on annual valuations (*however* precise) may be both fortuitous and sporadic. Even *precise* annual valuations are only approximations. The year end value of debt, whether observed in a market or approximated in some indirect way, will not furnish an accurate measure of, only a better or worse proxy for, the continuous rate at

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203 In effect, this is a slightly different way of characterizing the fact that, over the life of a debt instrument, the deviations from economic accrual induced by variations in interest rates always will add to zero. See note 35, 147-48 and accompanying text.

204 This point was emphasized by Nicholas Kaldor in his work on consumption taxation. See Nicholas Kaldor, An Expenditure Tax 44-46, 69-70 (1955).

205 The force of Kaldor's observation was subsequently disputed in Alvin Warren, Would a Consumption Tax be Fairer than an Income Tax?, 89 Yale L.J. 1081, 1109-12 (1980), arguing, in effect, that when a bond increased in value because interest rates had changed, the accrual of gain to the holder of the bond accelerated, thereby making the holder better off, even though the aggregate amount of gain ultimately did not change.

206 What follows draws heavily on Strnad, Periodicity, note 7.

207 Id. at 1832-53.

which value actually accrued to the instrument during the course of the year.[208]

The extent to which this problem is soluble in principle rests on several considerations. As with the matter of valuation, it is more readily soluble for debt that is publicly traded than for debt that is not. For the former, an essentially complete solution would be to keep track of daily (perhaps monthly) changes in the instrument's value, and actually calculate the tax associated with the (approximately) continuous accrual of gain. The informational and administrative costs of that solution would be great.[209]

For both publicly traded and nontraded debt, Professor Strnad's analysis implies the existence of a secondary solution. Working with stock (rather than debt), he has shown that, in an environment characterized by uncertainty, it is possible to construct what in some sense is an "average" rate at which an investment's value may be expected to accrue.[210] Short of using continuous data on actual values, a procedure of this sort would, in a precise sense, furnish a "best" approximation of the rate at which the asset's value continuously changed.[211] It may be possible to modify that analysis to produce a model that is appropriate for debt.[212] If so, it would permit continuous accrual of gain from long-term

[208] Strnad, Periodicity, note 7, at 1825-30. One of the points developed by Professor Strnad is that, even if taxes are collected only once a year, the tax due in connection with gain calculated only once a year will be less than the tax due in connection with gain treated as accruing continuously throughout the year. Id.

Although the notion of a tax assessed more frequently than annually initially might seem strange, the existing income tax is—because of wage withholding and the requirement of periodic payments of estimated tax—effectively assessed and collected more frequently than annually. See IRC §§ 3402(a), 6315, 6654.

[209] The informational requirements would include more or less continuous data on both asset values and interest rates. The demands of computing essentially continuous accrual of taxes, even though machine-intensive once the necessary programming had been done, might reasonably be expected to be great.

[210] See Strnad, Periodicity, note 7, at 1868-74. Working with stock, the technique he develops consists basically of (1) positing a stochastic process for the value of the stock; (2) given the assumed stochastic process and an assumed initial value for the stock, calculating the expected asset value at the termination of a specified period of time (calculating the outcome of what he describes as an "unconstrained bridge process"); and (3) given the initial value, the stochastic process and the expected terminal value, calculating the probability-weighted average of all paths the asset might traverse between the initial and terminal values during the specified period of time (calculating the outcome of a "constrained bridge process").

The mathematical details of these calculations, which are quite involved, are set out in Jeff Strnad, Periodicity and Accretion: Norms and Implementation (Cal. Inst. of Tech. Social Science Working Paper, No. 721, app. B, 1990) [hereinafter cited as Working Paper].

[211] The estimate of accrual given by the procedure developed by Professor Strnad is known as a "minimum variance" estimate: It has the attractive statistical property that its expected value will vary from the true path of accrual by less than that of any other approximation. See Strnad, Periodicity, note 7, at 1868-74, 1893 & n.231.

[212] The technique developed by Professor Strnad is suggestive for the problem of measuring the accrual of value to debt. Because the terms of a debt obligation fix both the initial *and*

debt to be approximated from periodic data that can be observed and collected more readily.

Such a procedure, if feasible, would be theoretically appealing. Nevertheless, it would have practical shortcomings of its own. The information needed would differ from (and possibly be less demanding to obtain than) the information needed for continuous accrual.[213] In comparison with continuous accrual, however, the computational demands of this procedure would probably be great.[214] Those demands, while evidently tractable in constructing an isolated economic model or simulation, would almost surely be formidable if applied on any more substantial scale. Beyond that, for debt that was not publicly traded and whose market value had to be estimated, the accuracy of the resulting estimate of continuous accrual would remain hostage to the accuracy of the annual estimates of value. To the extent that the estimates of value were themselves clouded by uncertainty, it would remain impossible to say, even ex post, that the measured accrual was superior to accrual of interest as calculated by a mechanism like that prescribed by existing law.

For present purposes, what perhaps is most intriguing about Professor Strnad's study of continuous accrual is the nature of the approximations he develops of the path along which gain can be expected to accrue. These approximations are sensitive to the assumptions he employs. Nevertheless, under one of two sets of assumptions, the natural implication of his analysis is that the best estimate of the rate at which the value of debt will continuously accrue may be given by approximately exponential growth.[215] Exponential accrual, however, is precisely the pattern of approximation now prescribed by law.

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terminal value, the procedure for debt corresponding to the procedure described at note 210 would be reduced to steps (1) and (3). That is, the outcome of the "bridge process" described as step (2) (in note 210) could, where debt is concerned, be calculated ex ante.

There is, however, one other major difference. Since debt fluctuates primarily *in response to* interest rates, a stochastic process would be posited for the time derivative of asset value, rather than for the asset value itself. For this reason, it apparently would require substantial mathematical modifications to adapt Professor Strnad's model to an analysis of the accrual of gain from debt.

[213] The analysis developed by Professor Strnad could be implemented using annual (rather than continuous) data on interest rates and asset valuations. In this sense, its requirements would be similar to those for an annual accrual tax. It would, however, also require historical data on the mean and variance of the stochastic process followed by interest rates, to be used in valuing different categories of debt. See Strnad, Periodicity, note 7, at 1868-79, 1893.

[214] The procedure outlined in Strnad, Working Paper, note 210, involves the evaluation of complicated stochastic integrals in computing the average trajectory of asset values. Similar calculations presumably would be required for all outstanding debt if such a procedure were to be prescribed in practice.

[215] Strnad, Periodicity, note 7, at 1873-74. There are distinctions between the accrual of gains to equity, studied by Professor Strnad, and the accrual of gains to debt. See note 212. The principal difference is that the value of debt responds in a derivative manner, rather than directly, to movements in interest rates. Id. There is no a priori reason to believe, however,

The assumptions needed to arrive at this conclusion are important. They are, basically, that holders of debt not be able to engage in what Professor Strnad denotes "strategic trading": They may not, without cost or limitation, freely sell debt whenever its market value has declined below its adjusted basis, thereby realizing losses, while continuing to hold the debt (thereby deferring taxation) as long as it is trading at a gain.[216] Under existing conditions, it is not clear to what extent these assumptions will prevail. For most investors, turning over a portfolio of debt instruments to realize losses will not be costless.[217] On the other hand, existing law imposes little in the way of effective limitations on a holder's ability to realize accrued losses on debt.[218] Indeed, the absence of any effective limitation, in combination with the favorable treatment of market discount debt,[219] probably operates to encourage activities of precisely this sort.[220]

Plainly, this is a subject on which there is work remaining to be done. It may well be possible to modify Professor Strnad's analysis, taking account of both the plausible costs of strategic trading and differences in the modelling of equity and debt, in a fashion that illuminates the pattern in which gains continuously accrue to debt. That alone would be a substantial endeavor.[221] Nor is there necessarily agreement on the underlying proposition that continuous accrual should be the norm. Beyond that, if Professor Strnad's analysis does produce conclusions for debt

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that this difference alone would produce a qualitative change in the characteristics of the average path.

216 Strnad, Periodicity, note 7, at 1874-79. When strategic trading is possible, assets will tend to be sold when trading at a loss to realize and take advantage of the loss. Consequently, assets that continue to be held will tend to be those trading at a gain. The probability-weighted average of their paths will tend to be higher—i.e., they will have tended to accrue more gain at earlier times—than if losses could not be realized through sale.

217 Apart from the "bid-asked" spread that typically prevails, retail investors will incur commissions on the sale and simultaneous repurchase of comparable bonds.

218 The principal limitations are statutory provisions limiting the deductibility of capital losses and so-called "wash sales." IRC §§ 1211-1212, 1091. The effectiveness of these provisions in this setting are discussed in Strnad, Periodicity, note 7, at 1887-91. The wash sale limitation is probably even less effective for debt than it is for equity. That provision is triggered by the repurchase of stock or securities that are "substantially identical" to the securities that were sold at a loss. Debt typically provides simply for nominally pre-specified repayments, so debt as a class is inherently less heterogenous than equity. It should, therefore, be fairly simple to replace debt of one issuer sold at a loss with debt of a different issuer having essentially identical maturity and risk.

219 See notes 48-56 and accompanying text.

220 Because of the timing advantage from realizing market losses on debt, combined with immediate reinvestment, strategic trading would (depending on the magnitude of the legal obstacles and other costs) occur even if market discount were currently accrued for purposes of taxation. The additional tax advantage of deferral conferred on the income from reinvestment, assuming (as would be optimal) that reinvestment was in a market discount rather than a newly issued obligation, probably amounts to an additional inducement to strategic trading.

221 See notes 210, 212 and 214.

comparable to those for equity, it would make more pressing the case for revising the existing treatment of market discount debt. For it would imply that the favorable treatment currently enjoyed by market discount may not only be inaccurate in and of itself, but, by encouraging the strategic turnover of debt instruments trading at a loss, it may also affect the accuracy of the existing taxation of *all* outstanding debt.[222] This would necessitate more careful consideration of the difficulties, not encountered in the taxation of original issue discount, that periodic accrual of market discount would pose.[223]

Even at this stage, however, Professor Strnad's analysis is provocative. It suggests that, if continuous accrual is the appropriate ideal, and to the extent that strategic trading is costly (or might be curtailed), the best approximation *short of* continuous accrual of gain based on actual market valuations may be exponential accrual, essentially what is now embodied in the law.

VI. Conclusion

The observation that the term structure of interest affects the accrual of gain to long-term debt has proven to be illuminating, pointing as it does to the possible importance of market-based accrual for accurate taxation of long-term debt. Considered alone, however, the influence of the yield curve ultimately adds little to the case for an accrual tax. Even from a wider perspective, the case for accrual taxation seems distinctly more compelling for equity than it does for long-term debt. It may be, moreover, that with revisions to the existing system short of adopting outright market-based accrual—repealing the deferral now available to holders of market discount debt, and developing some means by which to curtail the advantages of discretionary realization with respect to debt trading at a loss—we could, at least as far as long-term debt is concerned, achieve about the same accuracy using a formula that prescribes exponential accrual as we could by adopting an accrual tax based on periodic market valuations. So, at the end of the day, it may turn out that the approximation incorporated into the law by the wave of legislation a decade ago is, really, not so bad an approximation after all.

222 To the extent that investors are free to engage in strategic trading, it becomes less likely, at least where equity is concerned, that the continuous accrual of gains will be approximately exponential. See Strnad, Periodicity, note 7, at 1873-79.

223 See notes 52-54 and accompanying text. In particular, the extension of a system of mandatory accrual to market discount would, in contrast with the treatment of original issue discount, almost surely have a genuine impact: With market discount, mandatory accrual could not as readily be avoided as it probably is with original issue discount. See note 55.

APPENDIX A

The data that form the basis for the observations in Section III were compiled by J. Huston McCulloch.[224] These data consist of three tables that "summarize the term structure of interest rates on U.S. Treasury Securities from December 1946 to February 1987."[225] Each table consists of monthly observations during that period. For each observation, the yield to maturity is given (when available) for securities having maturities of 3 months, 6 months, 9 months, 1 year and 2, 3, 4, 5, 10, 15, 20 and 25 years. (In some instances, the longest maturity outstanding was less than 25 years.) Table 1 contains yields on zero-coupon securities. Table 3 contains yields on coupon-carrying bonds. Table 2 consists of the "forward rates" corresponding to the yields in Table 1. The data given in Tables 1 and 2 were derived from observed data on coupon-carrying bonds.[226]

The analysis developed in Section III was based on a quarterly subset of the McCulloch data consisting of the March, June, September and December entries from Tables 1 and 3 for the entire 40-year period. For each entry, the yield to maturity was used for Treasury securities having maturities of 1, 5, 10, 15, 20 and 25 years. This subset was analyzed using a standard micro-computer spreadsheet. The following characteristics of the term structure were derived from the analysis of the data:

> "Width": The absolute value of the difference between the highest and lowest yield to maturity during each quarter surveyed. When used with respect to some interval of the curve during any quarter, the absolute value of the difference between yields to maturity at the endpoints of that interval.
>
> "Width (1-5)": The absolute value of the difference between the yields to maturity exhibited by maturities of one and five years.
>
> "Summed Five-Year Width": The sum of the absolute values of the differences between yields to maturity at each observed interval of the curve, during each quarter surveyed. (Thus, if the yields observed during some quarter at maturities of 1, 5, 10, 15, 20, and 25 years were 6, 8, 7.9, 7.8, 7.7 and 7.6%, respectively, the "summed five-year width" during that quarter would be 2.4%. This measure will systematically tend to overstate the width of the entire curve.)

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[224] They are reproduced in their entirety in Robert J. Shiller, The Term Structure of Interest Rates, 1 Handbook of Monetary Economics 627, 672-715 tbls. 13.A.1-3 (Benjamin M. Friedman & Frank H. Hahn eds., 1990).

[225] Id. at 672.

[226] Id.

"Average Five-Year Width": The sum of the absolute values of the differences between yields to maturity at each observed interval of the curve, divided by one-fifth of the lengthiest outstanding maturity, during each quarter surveyed. The divisor is five, except during quarters when the lengthiest outstanding maturity was less than 25 years.

"Average Maximum Five-Year Width": The average of the absolute values of the differences between yields to maturity at that interval on the curve during each quarter that exhibited the greatest width. (The entry that would have been selected during the quarter used to exemplify the computation of the "Summed Five-Year Width" would have had a value of two.)

"Rolling Average Change, One Year Rate": The difference between the one-year rate in any quarter and the one-year rate during the fourth preceding quarter, beginning with March 1948, averaged over the entire period.

The values of these items are summarized below, based on the 160-quarter sample over the 40-year period. (Sample standard deviations are in parentheses.)

	Zero Coupon	*Coupon Carrying*
Width		
Average Value	1.161% (0.741)	1.052% (0.651)
Maximum Value	4.410%	3.430%
Number > 4%	1	0
Number > 3%	3	1
Number > 2%	18	11
Number > 1.5%	44	36
Width (1-5)		
Maximum Value	2.900%	2.340%
Maximum Value (if positive)	2.100%	2.340%
Width (1-5) = Maximum 5-Year Width		
Number	107	102
Average Value	0.634% (0.486)	0.623% (0.478)
Summed Five-Year Width	1.267% (0.801)	1.135% (0.758)
Average Five-Year Width	0.300% (0.176)	0.272% (0.172)
Average Maximum Five-Year Width	0.702% (0.475)	0.687% (0.525)
Rolling Average Change, One-Year Rate	1.199% (1.165)	1.207% (1.169)

Appendix B

This Appendix develops several propositions advanced in the text relating to the computation of present values, the valuation of debt and the term structure of interest rates.

1. *Interest Rate Volatility of a Pure Discount Bond*

The original issue discount provisions of the Code prescribe the computation of present values and yields to maturity by dividing time into discrete intervals. They use a present value function of the form

$$PV(r,n) = \frac{FV}{(1+r)^n} \tag{1}$$

where *FV* is the amount whose present value is to be computed, *r* is the periodic interest rate, *n* is the number of compounding periods, and *PV*(,) denotes that present value is a function of *r* and *n*. The Code prescribes a default compounding interval of six months,[227] but the interval may be varied by the parties to a loan as long as it does not exceed one year.[228]

In general, the choice of compounding frequency for computing present values is arbitrary. It is a well-known fact from elementary calculus that, as the choice of compounding interval is made arbitrarily small, so that compounding becomes arbitrarily frequent, the present value function in expression (1) takes the form of the exponential function:

$$PV(r,t) = FVe^{-rt} \tag{2}$$

where the (now continuous) passage of time is denoted by *t*, and *r* is the instantaneous rate of interest.[229]

For a pure discount bond of remaining duration *t* that pays $1 at maturity, the bond price is given by expression (2) (setting FV = 1). For all values of *r* and *t*, this expression is positive. To determine how the sensitivity of the bond price to fluctuations in the interest rate varies as its maturity changes, one first examines how the bond price changes as the interest rate changes by calculating the first partial derivative of the present value function with respect to the interest rate:

$$\frac{\partial PV}{\partial r} = -te^{-rt} \tag{3}$$

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227 IRC § 1272(a)(5).

228 Prop. Reg. § 1.1272-1(d).

229 E.g., 1 Richard Courant, Differential and Integral Calculus 179-80 (2d ed. 1988).

This derivative gives the instantaneous rate of change of the bond price as the interest rate changes. Since both t and e^{-rt} are always positive, the value of the derivative in (3) is always less than zero, confirming the familiar fact that, as interest rates rise, the value of a debt instrument declines.

The expression for the derivative in (3) involves t, indicating that the value of the derivative varies with the instrument's maturity. The nature of that dependency can be found by differentiating expression (3), this time with respect to t, to see more precisely how the *sensitivity* of the bond price to changes in interest rates *itself* changes as the maturity of the instrument varies. That derivative is given by

$$(4) \qquad \frac{\partial^2 PV}{\partial r \, \partial t} = -e^{-rt} + rte^{-rt} = [rt - 1]e^{-rt}$$

The derivative in (4) tells a slightly more complicated story than simply that the price of a debt instrument becomes increasingly volatile (and, hence, increasingly "risky") in the face of interest rate fluctuations with increasing maturity. Since the derivative in (3) takes on only negative values, a negative value for the derivative in (4) indicates that volatility increases (in absolute value) with the instrument's maturity (t), consistent with the proposition that long-term debt is more volatile than short-term debt.

The derivative in (4) will in fact take on negative values whenever the quantity rt is less than 1. There is a continuous range of combinations of relatively low interest rates and/or relatively short maturities for which this will be so. For example, for an instrument with ten years remaining to maturity (so that $t = 10$), rt will be less than 1 (and the value of the derivative in (4) will be negative) for any interest rate less than .10. Nevertheless, at long maturities or high interest rates, rt will be greater than 1 and the value of the derivative in (4) will be positive, indicating that, in those ranges, volatility in fact *decreases* with increasing maturity.

There is some intuition behind this conclusion. At long maturities or high interest rates, the present value of a debt instrument is substantially depressed (and approaches zero as t approaches infinity). In such circumstances, changes in the interest rate necessarily will have a diminishing incremental influence on present value: As present value approaches zero, there is less "room" for it to move as interest rates change. Consequently, the observation that the interest rate volatility of a debt instrument increases with maturity, while generally true for moderate maturities and interest rates at levels typically encountered, is not universally true.[230]

Finally, noting that the derivative in (3) also involves r, we can differentiate it a second time with respect to r, to ascertain how the interest rate sensitivity of the bond price changes as *interest rates* change:

$$(5) \qquad \frac{\partial^2 PV}{\partial r^2} = t^2 e^{-rt}$$

Since both e^{-rt} and t^2 are always positive, the value of this derivative is unambiguously positive, indicating that, at any maturity, the sensitivity of the bond price to fluctuations in interest rates is lower at *higher* interest rates. That is, since expression (3) is negative, the positive value for expression (5) indicates that the *absolute value* of expression (3) is approaching zero.

2. Duration, Rates and the Influence of the Term Structure

Differentiating expression (2) with respect to t, that partial derivative is given by:

$$(6) \qquad \frac{\partial PV}{\partial t} = -re^{-rt}$$

This derivative gives the instantaneous change in the bond price as maturity changes. In other words, it gives the accrual of interest, equal to $re^{-rt} = rPV$, in continuous time.[231] Expression (2) can be rewritten so as to reflect the fact that the applicable interest rate changes with maturity—that is, to reflect a term structure of interest rates—indicating that r is a function of maturity with the notation:

$$(7) \qquad r = r(t)$$

Using this notation—which assumes nothing in particular about the *shape* of the yield curve—and again setting $FV = 1$, expression (2) becomes:

$$(8) \qquad PV(r(t), t) = e^{-tr(t)}$$

[230] It is, however, true that the responsiveness of the bond price to interest rate fluctuations in *percentage* terms is always increasing with maturity. Expression (3) can be rewritten as an elasticity, by multiplying through by t and dividing through by e^{-rt}, to get:

$$\epsilon_r = -tr,$$

the derivative with respect to t of which is simply $-r < 0$. Since both the elasticity and its derivative are always less than zero, the *absolute value* of the elasticity is always increasing as the instrument's duration grows.

[231] The value of this derivative is negative, reflecting the fact that as t grows, present value becomes smaller. The passage of time, however, is denoted by *declining t*: As t declines, present value grows.

If expression (8) is differentiated with respect to t the result is:

$$\frac{\partial PV}{\partial t} = -r(t)e^{-tr(t)} - te^{-tr(t)} \frac{dr(t)}{dt} \tag{9}$$

The derivative in (9), like that in expression (6), specifies generally the accrual of interest over time to a pure discount bond, but now taking the yield curve into account. The first term in this derivative is simply the accrual of interest as time passes, identical to the derivative in (6). The second term reflects the fact that, as time passes and an instrument's maturity moves along the yield curve, the applicable *interest rate* changes, affecting the accrual of interest. It is important to note that this second term has exactly the same form as the derivative of the present value function with respect to the interest rate, as given in (3), multiplied by the change in the interest rate along the yield curve. This indicates that changes in interest rates as an instrument moves along the yield curve affect the instrument's present value in the same fashion as changes in interest rates generally. The change in interest rate along the yield curve—captured by $dr(t)/dt$, the "time derivative" of the yield curve—is just the slope of the yield curve.

As with the derivative in (6), the negative value for the first term in (9) indicates that, as time passes and maturity (t) declines, present value grows, reflecting the accrual of interest. The second term in (9) will also have a negative sign if we assume more about the yield curve, specifically that its slope—given by $dr(t)/dt$—is positive, and conversely. That is, a yield curve with a positive slope generally will hasten the accrual of interest, whereas a negative slope generally will retard it.

3. *Risk-Neutral Valuation of a Pure Discount Bond under Uncertainty*

From expression (5), the second derivative of the present value function with respect to the interest rate is greater than zero. Thus, holding t constant and treating present value solely as a function of the interest rate, it is what is known as a (strictly) "convex" function. It then follows, from a proposition known as "Jensen's inequality," that if future interest rates are uncertain, the mathematical expected value (or "expectation") of the present value, computed over the uncertain interest rate, exceeds the present value computed using the expected value of the uncertain interest rate.[232] That is, using $\tilde{r}$ to denote the fact that r is uncertain and E [ ] to denote taking mathematical expectations, Jensen's inequality says that

[232] E.g., 2 William G. Feller, An Introduction to Probability and Statistics at 152-55 (2d ed. 1971).

(10) $$E\,[e^{-\tilde{r}t}] > e^{-E[-\tilde{r}]\,t}$$

The import of this is that a risk-neutral investor—that is, an individual who simply maximizes expected value instead of having preferences characterized by a concave (or convex) utility function—will prefer the uncertain present value to a certain present value computed using the mean value of the uncertain interest rate. The point is both mathematically and verbally subtle, and perhaps can be better conveyed through an example, which for simplicity will be constructed using discrete rather than continuous compounding. Suppose an investor is considering an investment in an instrument that will pay $1 in one year. Suppose also that, at the time the investment must be made, the interest rate that will prevail during that year is uncertain. It will be fixed on the day after the investment is made, and it is known in advance that with equal probability the interest rate will be either 0% or 20%. The expected (or mean) value of the interest rate is 10%, and $1 discounted for one year at 10% is worth $1/1.1 or $0.9091. This is the present value computed using the *expected interest rate.*

The *expected present value* of the investment is computed by using each of the two values the interest rate may take on. One dollar discounted for one year at 0% is worth $1, whereas $1 discounted for one year at 20% is worth $0.8333. Since each of these two outcomes is equally likely, the *expected present value* is their average, or ($1 + $0.8333)/2 = $0.9167. Thus, given a choice between the two, a risk-neutral investor, intent on maximizing his expected (discounted) return, would regard the $1 discounted at 10% as being less valuable than the gamble on $1 discounted at 0% or 20% with equal probability. Jensen's inequality, captured in expression (10) for the present value function, asserts that this inequality always holds for a risk-neutral investor.

An individual whose utility function is described by the natural logarithm of their wealth—a concave function—is risk averse, and will exhibit constant relative risk aversion with a coefficient of relative risk aversion equal to 1.[233] If the wealth of such an individual is expressed as a present value—so that it depends on *r* and *t* and will be denoted *W(r, t)* —it then follows, since the logarithm is not merely a concave function, but the inverse of the exponential function, that:

(11) $$ln\,[W(r,t)] = ln\,(e^{-rt}) = -rt$$

That is, utility for such an individual becomes linear in *r*, so that the individual's utility as a function of the interest rate is neither convex nor concave. Consequently, an individual whose utility is given by the natural logarithm, if maximizing expected utility when his wealth consists of

[233] E.g., Olivier J. Blanchard & Stanley Fischer, Lectures on Macroeconomics 43-44 (1989).

a present value and interest rates are uncertain, will *act* as though he was risk neutral in the sense that, for a given probability distribution of interest rates, he would be indifferent between the utility of the expected present value and the utility of the present value computed using the expected interest rate. An individual whose utility is described by a function that is any less concave than the natural logarithm, so that the composition of his utility function with the present value function remains convex, should (like the strictly risk-neutral investor) prefer the uncertain present value to a present value computed using the expected value of the uncertain interest rate.

4. *Adjusting the OID Provisions for a Term Structure of Interest Rates Attributable to Risk*

The computation of present values can be adjusted if the term structure is assumed to be attributable to risk, with different discount rates applicable at each interval of maturity. To take a simple example, if the annual rate applicable to a one-year pure discount instrument is denoted by r_1, and the annual rate applicable to the first year of a two-year instrument is denoted by r_2 (with the rate applicable to the second or final year being r_1), the present value of the instrument can be written, using a version of expression (1), as:

$$PV(r_1, r_2) = \frac{FV}{(1 + r_1)(1 + r_2)}$$

More generally, for an instrument with a maturity of n years and a distinct interest rate applicable to each interval to maturity, the present value function can be written as

$$PV(r_1, \ldots, r_n) = \frac{FV}{(1 + r_1) \ldots (1 + r_n)} \tag{12}$$

where r_i ($i = 1, \ldots, n$) is the rate that applies at the margin during the year when there are i years remaining to maturity. (For an instrument valued in this way, the single "yield to maturity" conventionally computed for the instrument would be given by $[(1 + r_1) \cdots (1 + r_n)]^{1/n} - 1$.)

If the term structure of interest rates implied by the expression in (12) remained constant from year to year, the increase in the market valuation of an instrument valued in that environment between any year (year n) and the next (when the instrument's remaining maturity would have declined by one year to $n - 1$ and its present value accordingly would have increased) would be given by the difference between its present value in year $n - 1$ and its value in year n, or

$$\frac{FV}{(1 + r_1) \ldots (1 + r_{n-1})} - \frac{FV}{(1 + r_1) \ldots (1 + r_n)} =$$

$$= \frac{FV(1 + r_n - 1)}{(1 + r_1) \ldots (1 + r_n)}$$

$$(13) \qquad = \frac{FVr_n}{(1 + r_1) \ldots (1 + r_n)}$$

The expression in (13) is simply the instrument's present value at the beginning of year *n*, as given in (12), multiplied by the interest rate (r_n) applicable to an instrument with *n* years remaining to maturity. That is, the increase in value can be computed by simply multiplying the instrument's present value at the beginning of year *n* by the interest rate that prevails along the yield curve for instruments during their *n*th year from maturity. It can easily be shown by induction that this conclusion holds for any *n*.