3-2019

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Boston University School of Law
Law & Economics Series Paper No. 19-5

March 2019

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## Abstract

This paper examines the welfare tradeoff between patent and antitrust law. Since patent and antitrust law have contradictory goals, the question that naturally arises is how one should choose between the two in instances where there is a conflict. One sensible approach to choosing between two legal standards, or between proof standards with respect to evidence, is to consider the relative costs of errors. The approach in this paper is to consider the ratio of false positives to false negatives in patent antitrust. We find that the relevant error cost ratio for patent antitrust is the proportion of the sum of the monopoly profit and the residual consumer surplus to the deadweight loss. This error cost ratio, for a wide range of deterministic demand functions, ranges from infinity to a low of roughly three. This suggests that patent antitrust law should err on the side of protecting innovation incentives.

*JEL Classifications*: K21, L43, O31, O34

*Keywords*: Patent Antitrust, Patent Monopoly, False Positives, False Convictions, False Negatives, False Acquittals, Error Costs, Ratio Tests

# 1 Introduction

This paper examines the welfare tradeoff between patent and antitrust law. Patent law enables the patent holder to obtain and exploit a monopoly lawfully. Antitrust law regulates the acquisition, maintenance, and, to some degree, the exploitation of monopoly power.

Since patent and antitrust law have superficially contradictory goals, the question that naturally arises is how one should choose between the two in instances where there appears to be a conflict. There are methods of exploiting patent monopolies that have been treated as antitrust violations. For example, the Federal Trade Commission recently sued Qualcomm, a manufacturer of communications technology, on the theory that the firm had abused its patent monopoly by adopting a two-part pricing scheme in the licensing and sale of its smartphone semiconductor chips.

One sensible approach to choosing between two legal standards, or two proof standards with respect to evidence, is to consider the relative costs of errors. Overenforcement generates "false positives," cases where the regulated firm is punished or prohibited from taking a certain action when society should prefer that the action be taken. Underenforcement generates "false negatives," cases where society should prefer that the excused firm be punished or prohibited from taking action.

The approach in this paper is to consider the ratio of false positives to false negatives in the patent antitrust area. A high error cost would imply that antitrust should be reluctant to restrain patentees. Moreover, proof standards and legal doctrines should be biased toward protecting innovation incentives.

We find that the relevant error cost ratio for patent antitrust is the ratio of the sum of the monopoly profit and the residual consumer surplus to the deadweight loss. This is different from the profit-deadweight loss (reward-social loss) ratio advanced in the literature (Kaplow, 1984; Gilbert and Shapiro, 1990; Klemperer, 1990). The reward-loss ratio is a doubtful measure of the patent-antitrust welfare tradeoff because it does not take into account the residual surplus to consumers, and thereby underweights the social value of innovation.

We find that the error cost ratio, for a wide variety of deterministic demand functions, ranges from infinity to a low of roughly three. This supports a bias, when designing legal standards and proof standards in patent antitrust law, in favor of the patent holder.

# 2 Baseline Model

We assume patent protection gives the innovator a monopoly in a market that exists only because of a prior investment by the innovator. In the first period, the prospective patentee invests in innovation, and in the second period, the innovator is awarded a patent (with probability one) that guarantees a monopoly in the market created by the innovation.[1] The costs of innovation are sunk

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[1] An alternative version of innovation, more consistent with process inventions, assumes that innovation consists of reducing the cost of producing some good in an existing market. The version here, where innovation creates a new market, and the aforementioned alternative are the same when the process innovation reduces cost to such a degree that the innovator gets the entire market to himself - that is, completely drives out of business the inefficient firms -

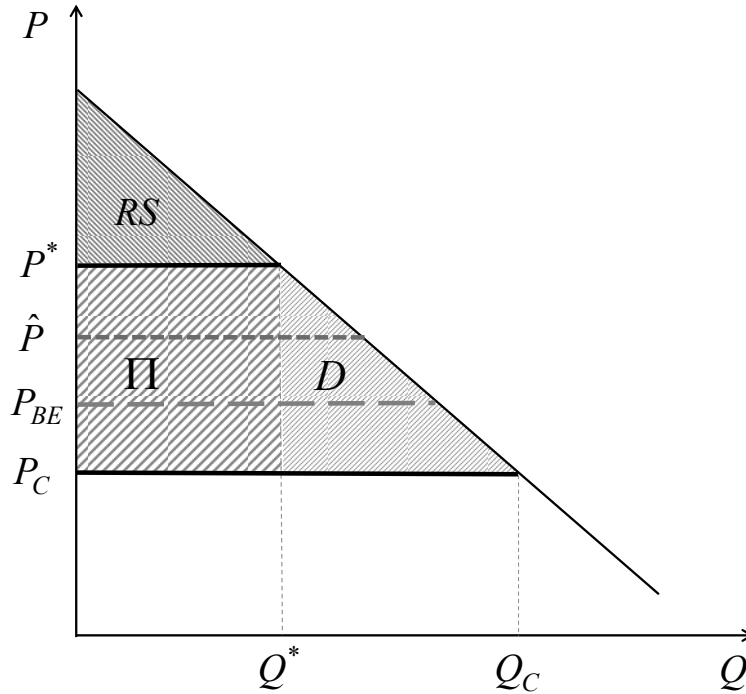Electronic copy available at: https://ssrn.com/abstract=3365034

Figure 1: Error Cost Ratio

when the second period arrives.

The patentee-monopolist faces a downward-sloping inverse demand curve $p = p(q)$ and constant marginal cost of $c$. The firm's profit maximization problem is

$$\max_{q} : \pi(q) = p(q)q - cq.$$

The monopolistic output $q^\star$ satisfies the familiar optimality condition $p'(q^\star)q^\star + p(q^\star) = c$. Denoting demand elasticity at a price $p$ by $\epsilon(p) = -\frac{q'(p)}{q/p}$, the monopoly price $p^\star$ satisfies $p^\star \left[1 - \frac{1}{\epsilon(p^\star)}\right] = c$. The patentee anticipates all of this when he invests in innovation in the first period.

Under competition, price would marginal cost, $p^c = c$, and the competitive quantity $q^c = p^{-1}(p^c)$. Figure 1 illustrates the standard monopoly outcome, with profit denoted by $\boldsymbol{\Pi}$, the residual surplus denoted $\boldsymbol{RS}$, and deadweight loss, from constraining output below the competitive level, $\boldsymbol{D}$.

# 3 Error Cost Ratio

In this part, we consider the welfare tradeoffs of antitrust enforcement in the intellectual property area. Although we focus on patents, the model applies equally to many other types of intellectual property (copyrights, trade secrets, trademarks).

Figure 1 illustrates our basic argument. In the figure, $p_{BE}$ is the break-even price necessary for

even when charging the monopoly price.

the innovator to recover the fixed (sunk) costs of innovation. If the price the firm expects to receive in the second period is less than the break-even price, the firm will not invest in innovation (research and development, R&D).

We incorporate antitrust, in Figure 1, as a mechanism that operates as a price regulation, $\hat{p}$. This is different from the more traditional economic model of antitrust operating as a penalty imposed on monopolizing firms.[2] Here, antitrust operates as an injunction, constraining the firm from choosing its preferred price-output combination along the demand curve.

If the anticipated antitrust-regulated price cap is greater than the break-even price, the firm will innovate and charge up to the price cap. If the anticipated antitrust price cap is less than the break-even price, the firm will not invest in the first period, and no entry will occur.

The most stringent antitrust regime is equivalent to a price cap equal to marginal cost. In this case, antitrust is so effective that the innovator will be forced to charge the competitive price in the second period. Obviously, investment will occur in the first period, in this case, only if the break-even price is equal to marginal cost. That will occur only if research and development is essentially costless, which is likely to be rare. An example of an injunctive policy that would implement a price cap equal to marginal cost is a rule denying enforceability to the patent, thus opening the market to competition and driving price down to marginal cost.

The most relaxed antitrust regime would set the effective price cap at the monopoly price $p^\star$. With an antitrust-regulated price cap greater than or equal to the monopoly price, the innovator would never be deterred by the threat of antitrust regulation from investing.

Perfect antitrust, in this model, reduces to setting the antitrust price cap equal to the break-even price. With the antitrust cap set at the break-even price, the innovator will invest, and society will get the benefit from innovation with the smallest possible deadweight loss.

If the antitrust price cap is less than the break-even price, society loses the gain from innovation. The firm will not invest in innovation and the *minimum* social loss is the sum of consumer's surplus and the firm's profit in the unconstrained regime,

$$RS + \pi = \int_0^{q^\star} [p(q) - c] dq.$$

Society does not lose the potential welfare captured by area $D$ in Figure 1 because this portion of the potential surplus from innovation would never have been available to society in the unconstrained regime.[3]

In contrast, if the antitrust-regulated price cap is greater than the break-even price, the *maximum*

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[2]See Becker (1968), Landes (1983), Hylton and Lin (2014).

[3]All of this assumes, of course, that the patent award is based on an innovation rather than a fraud on the patent office, or corruption in the patent system. In the latter case, no new surplus is created, and the social loss from setting the price cap below the break-even price would be zero. Consider, for example, a patent for playing cards, as in Darcy v. Allein, 74 Eng. Rep. 1131 (K.B. 1603). As a general matter one could introduce a measure of the probability of "real" innovation (in contrast to fraudulent) and multiply the sum of profit and residual surplus by such a "validity probability" to arrive at a measure of the social loss. The analysis here assumes implicitly that the validity probability is one.

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society will lose is the deadweight loss $D$, given by

$$D = \int_{q^\star}^{q^c} [p(q) - c]dq.$$

Errors, in this framework, are deviations from perfect antitrust enforcement. An error in the direction of excessive enforcement, where the antitrust price cap is below the break-even price, results in "false conviction" or "false positive" costs. Errors in the direction of too little enforcement, where the antitrust price cap is greater than the break-even price, result in "false acquittal" or "false negative" costs. For enforcement authorities, the *error cost ratio* - that is, the ratio of false positive costs to false negative costs - provides guidance on the optimal direction of any bias due to errors in enforcement. For adjudicatory tribunals, the error cost ratio provides guidance on the standard of proof that should govern in disputes over whether a firm has violated antitrust law. More generally, the error cost ratio also serves as a measure of the welfare tradeoff relevant to any conflict between the scope of the patent laws and the scope of antitrust laws. This measure of the welfare tradeoff differs from the ratio test (reward-to-social-loss ratio) advanced in parts of the patent antitrust literature.

Given the foregoing, the most conservative measure of the error cost ratio is represented by

$$\rho \doteq \frac{RS + \pi}{D}.$$

This quotient provides a lower-bound measure, taking account the different levels of stringency in antitrust enforcement, of the ratio of the cost of excessive enforcement (false convictions) to the cost of inadequate enforcement (false acquittals) in the patent antitrust area.

Under traditional decision theory arguments (Kaplan, 1968; Burtis, Gelbach, Kobayashi, 2018), an error cost ratio equal to one would justify a balanced approach to the standard of proof, such as the preponderance standard. Such a standard would treat prospective errors in the direction of excessive enforcement as equally costly as prospective errors in the direction of lenient enforcement. On the other hand, a ratio of one third would justify an approach that favors patent challengers (implementers), say by adopting a rebuttable presumption of guilt in patent-antitrust cases. A ratio considerably higher than one, such as three, would justify a proof standard favoring the innovator.

These implications of the error cost ratio should be considered while also taking into account that it is a *minimum* estimate. Thus, if the ratio presented above is greater than one, given a particular market structure, the actual ratio in applications to specific regulatory interventions within the particular market typically will be even greater. If the ratio is greater than one, then the case for adopting a proof standard favoring the innovator is even stronger than implied by the numerical value of the ratio.

Simply looking at the ratio formula, however, it is impossible to tell what the value of it might be. It seems plausible, initially, that some market structures might generate a high ratio, and others might generate a low ratio. In the abstract, a demand function could take a shape that could

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generate almost any positive numerical value as a ratio estimate. However, we limit the range of possibilities below by examining plausible and widely used functional forms.

We analyze the error cost ratio first by a linear demand function. Then we relax the assumption to show that the qualitative results are robust over several commonly used demand assumptions.

## 3.1 Linear Demand

Assume the demand function takes the form $p = A - bq$, where $A > 0$ *and* $b > 0$. The basic results are summarized in Proposition 1. Later, in the next subsection, we show that the qualitative result holds under more robust assumptions on the demand function.

**Proposition 1 (Linear Demand).** *If the demand curve is linear and marginal cost is constant, then the error cost ratio for patent antitrust is*

$$\rho \doteq \frac{RS + \pi}{D} = 3.$$

Under linear demand, the error cost ratio is a constant value of three. If the error cost ratio happened to be one, there would be a credible argument for adopting a balanced approach to patent antitrust where the risk of excessive enforcement is equated with the risk of inadequate enforcement. In terms of proof standards for adjudication, this would be equivalent to a preponderance test. However, an error cost ratio of three is more consistent with a biased approach to the risk of excessive enforcement, where the bias favors leniency toward the regulated party (Burtis, Gelbach, Kobayashi, 2018). Alternatively, the ratio of three suggests in the adjudication setting a "clear and convincing" standard of proof. We consider below whether this implication is also valid for other common representations of consumer demand.

## 3.2 Power-law Demand

In this part we consider power-law demand functions, such as the isoelastic, algebraic, and exponential forms. The advantage of these forms over the linear is that they better represent demand in settings of wealth inequality or where a relatively small number of consumers bid intensively for the good (e.g, medical care).

Consider the algebraic demand form, $p = \alpha q^\beta - \sigma$, $\beta \in (-1, 0)$. After that, we will examine the results under isoelastic and exponential functional form. Linear and isoelastic demand functions are special cases of the algebraic form. In particular, if $\alpha = -b$, $\beta = 1$, and $\sigma = -A$, the algebraic demand function turns to be a linear demand function. If $\sigma = 0$, the algebraic demand function becomes an isoelastic demand function, $p = \alpha q^\beta$.

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**Proposition 2 (Algebraic Demand).** *If the demand curve is algebraic $p = \alpha q^\beta - \sigma$, $\beta \in (-1, 0)$, and marginal cost is a constant $c$, then the minimum false conviction cost is $RS + \pi = (\sigma + c) \left[ \frac{-\beta(\beta+2)}{(\beta+1)^2} \right] \left[ \frac{(\sigma+c)}{\alpha(\beta+1)} \right]^{1/\beta}$ and the maximum false acquittal cost is $D = (\sigma + c) \left[ \frac{-\beta}{\beta+1} \right] \left[ \frac{\sigma+c}{\alpha} \right]^{1/\beta} - (\sigma + c) \left[ \frac{-\beta(\beta+2)}{(\beta+1)^2} \right] \left[ \frac{(\sigma+c)}{\alpha(\beta+1)} \right]^{1/\beta}$.[4] Therefore, the error cost ratio is*

$$\rho \doteq \frac{RS + \pi}{D} = \frac{1}{\frac{(\beta+1)^{1+1/\beta}}{\beta+2} - 1},$$

*which ranges, as the elasticity of demand increases, from $+\infty$ to a lower bound of $\frac{1}{e/2-1}$.*

The algebraic demand form does not generate a constant elasticity. Elasticity is $\epsilon(p) = -\frac{p}{\beta(p+\sigma)}$, so introducing the parameter $\epsilon$, where $\epsilon \equiv -\frac{1}{\beta}$, allows us to examine the behavior of the error cost ratio as demand elasticity goes to infinity.[5] As demand becomes more inelastic, the error cost ratio approaches infinity. As demand becomes more elastic, the error cost ratio falls to its lower bound of $\frac{1}{e/2-1} \approx 2.8$.

In more intuitive terms, Proposition 2 says that for relatively uncompetitive markets - where the elasticity of demand is still above but close to one - the error cost ratio is extremely high. For such markets, the sum of profit and consumer surplus is very large relative to deadweight loss, and society loses much more than one dollar for each dollar of deadweight loss avoided through excessive antitrust regulaiton. The error cost ratio falls toward its lower bound of roughly 2.8 as the market moves toward perfect competition.

**Proposition 3 (Isoelastic Demand).** *If the demand curve is isoelastic $p = \alpha q^\beta$, $\beta \in (-1, 0)$, and marginal cost is a constant $c$, then the minimum false conviction cost is $RS + \pi = c \left[ \frac{-\beta(\beta+2)}{(\beta+1)^2} \right] \left[ \frac{c}{\alpha(\beta+1)} \right]^{1/\beta}$ and the maximum false acquittal cost is $D = c \left[ \frac{-\beta}{\beta+1} \right] \left[ \frac{c}{\alpha} \right]^{1/\beta} - c \left[ \frac{-\beta(\beta+2)}{(\beta+1)^2} \right] \left[ \frac{c}{\alpha(\beta+1)} \right]^{1/\beta}$. Therefore, the error cost ratio is*

$$\rho = \frac{1}{\frac{(\beta+1)^{1+1/\beta}}{\beta+2} - 1}.$$

This case delivers the same result as the algebraic demand case, and again the error cost ratio ranges with the elasticity of demand from positive infinity to a limiting lower bound of $\frac{1}{e/2-1}$. Figure 2 shows the relationship between the error cost ratio and the elasticity of demand for the isoelastic case.[6]

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[4]We deliberately express this as the difference of two areas ($RS + \Pi + D$ - ($RS + \Pi$)) to facilitiate comparison of the areas under the demand curve.

[5]Since the dependent variable of interest is the error cost ratio, which assumes optimization by the monopolist, we could just as well examine the point elasticity at the optimal output level. Substituting the monopoly price, $\epsilon(p^*) = -\frac{c}{\beta(c+\sigma)} + \frac{\sigma}{c+\sigma}$, so that the parameter $\epsilon \equiv -\frac{1}{\beta}$ closely tracks the relevant point elasticity measure. If $\sigma$ is small relative to $c$, then the parameter is a nearly precise measure of point elasticity at the privately optimal quantity.

[6]In Figure 2, the error cost ratio is equal to 3 when elasticity is equal to 2, equal to 4 when elasticity is roughly 1.3, and equal to 6 when elasticity is roughly 1.1.

**Proposition 4 (Exponential Demand).** *If the demand curve is exponential form $q = \gamma e^{-\beta p}$, $\beta > 0$, and marginal cost is a constant $c$, then the minimum false conviction cost is $RS + \pi = \frac{1}{\beta}\gamma e^{-\beta c}\frac{2}{e}$, the maximum false acquittal cost is $D = \frac{1}{\beta}\gamma e^{-\beta c}(1 - \frac{2}{e})$, and the error cost ratio is*

$$\rho = \frac{1}{\frac{e}{2} - 1}.$$

Like the linear case, exponential demand delivers an error cost ratio that is independent of market structure as measured by the elasticity of demand. Of the demand functions considered in this part, the exponential generates the lowest error cost ratio, which is a constant value of $\frac{1}{e/2-1} \approx 2.8$.[7] The linear and the exponential also share the feature that there is a maximum price that consumers are willing to pay, above which demand falls to zero. However, in most markets, there is always some consumer willing to bid up the price of a scarce item. In particular, in markets characterized by wealth inequality among consumers, the wealthiest consumers can bid up the price of a scarce and highly desirable good (e.g., housing) to a level that is quite well beyond affordability for the average consumer. Given this, the algebraic probably best captures the features of real markets.
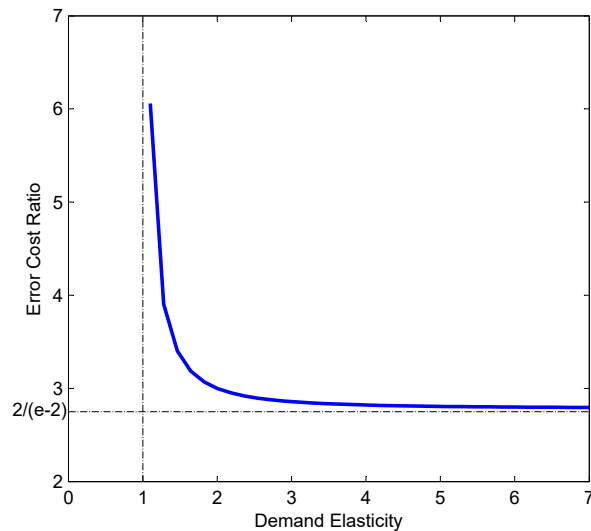


Figure 2: Error Cost Ratio and Demand Elasticity

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[7]That the error cost ratio for the exponential is less than that of the linear is consistent with the feature that the ratio of the monopolistic output to the competitive output is lower in the exponential case than in the linear case. The monopolistic output level is half of the competitive output level in the linear case, whereas in the exponential case the monopolistic level is equal to the competitive level divided by the natural base $e$. Since the deadweight loss from monopoly is generally greater under exponential demand, the error cost ratio associated with antitrust intervention is lower.

# 4 Model Extension

This section presents an extended model incorporating the probability of violating antitrust law to explicitly model the firm's stage-1 investment decision and the error cost ratio. The reason is to separate the effect of market structure on the probability of entry (investment) and on the error cost ratio. By separating these two effects, we can capture both the static and dynamic effects of changes in market structure on the error cost ratio.

At stage 1, the firm observes the R&D investment cost $K$ and decides whether to invest in innovation. The choice variable at stage 1 is denoted as $r \in \{0, 1\}$, where $r = 1$ denotes the firm invests in R&D and *enters* the subgame of stage 2, and $r = 0$ denotes the firm dose not enter and the game ends. If $r = 1$, the firm incurs the fixed cost of R&D and enters into stage 2. At stage 2, the firm is possibly faced with antitrust challenge. Let $v$ denote the probability that the firm is held violating the antitrust law and gets zero economic profit in stage 2.

This framework differs from the previous part by treating antitrust enforcement as "all or nothing". As a result, the error cost ratio is the same as before, though this time it represents the consistent value, given a specific demand curve, rather than the minimum taking account different levels of enforcement stringency. However, the probability of enforcement affects the incentive to invest and the likelihood of deadweight loss arising.

## 4.1 Stage-1 Choice

Assume that the probability of antitrust legal enforcement is fixed at a level $v$, $v \in (0, 1)$. The innovation investment cost $K$ follows cumulative density function $F$, which is differentiable and its derivative $F'(K) \doteq f(K)$ is positive everywhere. Assume the firm is risk neutral.

We solve the firm's entry game by backward induction. From our proceeding result in the previous section, if the firm continues to stage 2 as a monopoly, it charges monopoly price $p^*$ and gets monopoly profit $\pi^*$. Given that $v$ is the probability that the firm is held in violation of the antitrust law, the expected stage-2 utility is

$$\mathbb{E}\left[U_2(r = 1)\right] = (1 - v)\pi^*$$

where $r = 1$ means that the firm invests in R&D in stage 1, $U_2$ denotes the firm's utility at stage 2. If the firm does not invest in R&D, it earns zero. It optimizes by choosing whether to invest or not in stage 1. Thus the firm's stage 1 maximal utility is

$$U_1 = \max\left\{\mathbb{E}\left[U_2(r = 1)\right] - K, 0\right\}$$

Therefore, the firm invests in R&D to continue in stage 2, if and only if $\mathbb{E}\left[U_2(r = 1)\right] - K \geq 0$, that is,

$$r = 1 \text{ if and only if } (1 - v)\pi^* \geq K$$

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Let $\theta$ denote the probability that the firm invests in R&D, and is called probability of entry in later analyses.[8] We have the following expression for $\theta$,

$$\theta \doteq \mathrm{Probability}\{(1-v)\pi^* \geq K\} = F\left[(1-v)\pi^*\right]$$

To study the effect of market structure on firm's R&D investment decision, we study how market structure affects the monopoly profit $\pi^*$ which then determines the firm's entry decision.

$$\frac{\partial \theta}{\partial \pi^*} = f\left((1-v)\pi^*\right) \cdot (1-v) > 0 \tag{1}$$

The condition in Equation (1) indicates, in accord with intuition, that the firm's investment decision depends positively on the monopoly profit expected at stage 2.

**The Probability of Entry and the Adjusted Error Cost Ratio**   As analyzed in the proceeding part, if the firm does not invest in R&D, the society loses $RS + \pi$. If the firm invests in R&D and monopolizes in stage 2, the society loses monopoly deadweight loss $D$. Taking the firm's R&D probability into consideration, the adjusted error cost ratio is

$$\tilde{\rho} \doteq \frac{(1-\theta)(RS+\pi)}{\theta D} = \frac{1-\theta}{\theta}\rho$$

The effect of entry probability on error cost is captured by the partial derivative of $\tilde{\rho}$ with regard to $\theta$, which is given by

$$\frac{\partial \tilde{\rho}}{\partial \theta} = -\frac{\rho}{\theta^2} < 0$$
$$\frac{\partial^2 \tilde{\rho}}{\partial \theta^2} = \frac{2\rho}{\theta^3} > 0$$

Thus, the error cost ratio decreases as the likelihood of innovation investment increases, and this negative relationship grows at an increasing rate. Other things equal, excessive enforcement is less harmful to society where there are strong incentives to invest because of high expected monopoly profits or low research and development costs. In addition, a little increase in the investment probability has a larger downward effect on the error cost ratio in an industry where firms have more and frequent R&D than in an industry with less R&D intensity.

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[8]Because R&D investment is a fixed amount, the optimality decision for the patentee is straightforward, unlike the Nordaus (1967) model where the innovator chooses the amount to invest in innovation and therefore equates the marginal cost of investment with its marginal private benefit. Treating R&D investment as a lump sum seems defensible, since the end goal of the innovator is some definite new product in this model. The inventor cannot invest half of the required amount in R&D and get some fraction of the payoff. The variation in $K$ reflects the implicit assumption that the cost of the required investment is greater for some innovators than for others.

## 4.2 Effect of Market Structure

The price elasticity of demand is an important feature of market structure, and usually varies across industries (Johnson and Helmberger, 1967). A number of industry-specific characteristics contribute to the pattern of demand elasticity, such as the degree of substitutability between goods. If a product does not have substitutes, such as some drugs and treatments for rare diseases, the product's demand is likely to be inelastic. Path dependence (switching costs) is another factor giving rise to inelastic demand (Klemperer, 1987). Path dependence is a common feature observed in the demand for high tech products, such as online platforms and operating systems. Because consumers take time to adapt to a new technology, they tend to continue using related products from the same firm and are willing to pay a price premium for doing so.[9]

### 4.2.1 Negative relationship between elasticity and profit.

Generally speaking, a low demand elasticity reveals the consumer's high tolerance to price increases, and that the monopolist can exploit this high tolerance to earn more profits. In this sense, an industry with low demand elasticity is favorable for a monopoly firm (Kamien and Schwartz (1970)). Of course, demand elasticity varies along the demand curve in most cases. To study the effect of demand elasticity on proft, we will first have to construct a parameter that tracks the elasticity measure at all points along the demand curve. By examining the relationship between such a measure of elasticity and profit, we can draw inferences on the relationship between market structure and profitability.

We start by considering the power demand functions: isoelastic, algebraic, and exponential. For the isoelastic form $p = \alpha q^\beta$, $\beta < 0$, the price elasticity of demand is simply $\epsilon = -\frac{1}{\beta}$, which obviously tracks elasticity at all points along the demand curve. The monopoly profit under isoelastic demand can be expressed as a function of demand elasticity

$$\pi^* = \left(\frac{\epsilon - 1}{c}\right)^{\epsilon-1} \left(\frac{\alpha}{\epsilon}\right)^\epsilon,$$

where

$$\frac{\partial \pi^*}{\partial \epsilon} < 0 \tag{2}$$

Generally, the elasticity of demand for each of the power demand functions can be expressed as $\epsilon(p) \equiv \epsilon f(p)$, where the $\epsilon$ is a parameter that tracks the elasticity of demand. In the algebraic case $\epsilon(p) = -\frac{p}{\beta(p+\sigma)}$, so that $\epsilon = -\frac{1}{\beta}$ and $f(p) = \frac{p}{(p+\sigma)}$. In the isoelastic case $\epsilon = -\frac{1}{\beta}$ and $f(p) = 1$. In the exponential case, $\epsilon(p) = -\beta p$, so that $\epsilon = \beta$ and $f(p) = p$. For this class of demand functions, (2) holds.

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[9]One example is Android system or iOS. After a consumer purchases the first Apple product and gets used to the iOS system, the iOS shapes the consumer's habit of using smartphones. If she changes to the Android system, it takes her time to adjust.

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Although the parameterization is not as straightforward, a similar decomposition can be accomplished with linear demand, where the same negative relationship between profit and the elasticity tracking parameter holds.[10]

### 4.2.2 Dynamic Effect of Market Structure on Error Cost Ratio.

Recall that in section 3, we demonstrated that the probability of investment (entry) increases with monopoly profit. Combining this with the negative relationship between profit and demand elasticity, we expect that the inelastic market demand generates higher investment and entry probability.

$$\frac{\partial \theta}{\partial \epsilon} = \frac{\partial \theta}{\partial \pi} \cdot \frac{\partial \pi}{\partial \epsilon} < 0 \tag{3}$$

To consider the effect of market structure on the error cost ratio, we analyze the derivative of the error cost ratio with respect to elasticity, i.e.,

$$\frac{\partial \tilde{\rho}}{\partial \epsilon} = \frac{\partial \frac{(1-\theta)}{\theta} \rho}{\partial \epsilon} = \underbrace{\frac{1-\theta}{\theta} \frac{\partial \rho}{\partial \epsilon}}_{\text{static effect}} + \left[ \underbrace{-\frac{\rho}{\theta^2} \frac{\partial \theta}{\partial \epsilon}}_{\text{dynamic effect}} \right] \tag{4}$$

where the first part in Equation (4), which is negative, is the static effect of elasticity on the error cost ratio, discussed in section 2. The second term, which is positive, captures the dynamic effect of the demand elasticity on the error cost ratio. As the post-patent award market becomes more competitive, because of the greater availability of substitutes, investment and entry are less likely, increasing the error cost ratio. On one hand, low elasticity leads a higher monopoly profit and residual surplus relative to deadweight loss, leading to a larger error cost as the direct impact, which we call the static effect in this model. On the other hand, the higher monopoly profit expected in stage 2 encourages entry, partially offsetting the static effect.

The conflicting static and dynamic effects suggest that the relationship between demand elasticity and error cost may not be negative as suggested in some of our earlier analyses of demand functions. The limiting ratio derived for the algebraic and isoelastic cases is larger than $\frac{1}{e/2-1}$ when the entry/investment effect is taken into account. Indeed, for the linear and exponential cases, where the static effect is zero, increasing demand elasticity (making the market more competitive) generates only a dynamic effect, raising the limiting error cost ratio.

### 4.2.3 Example: Isoelastic Demand and Uniform Distribution.

As an illustration, assume demand is isoelastic and that the probability distribution that determines $\theta$ is uniform, i.e., the R&D investment cost $K$ follows a uniform distribution on $(0, \bar{K}]$, where $\bar{K}$

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[10]One could, for example, express elasticity as a function of price in the linear case as $\epsilon(p) \equiv \left(\frac{1}{A}\right)\left(\frac{p}{1-\left(\frac{p}{A}\right)}\right)$, with elasticity tracking parameter defined as $\epsilon = \frac{1}{A}$.

denotes the upper bound of investment cost. It follows that if expected profit is greater than the upper bound on investment cost, the firm is sure to enter. If expected profit is below the upper bound, then the entry probability is determined by the cumulative distribution function of $K$. Formally, we have

$$\theta = \begin{cases} 1 & \text{if } (1-v)\pi^* \geq \bar{K} \\ \frac{(1-v)\pi^*}{\bar{K}} & \text{if } (1-v)\pi^* < \bar{K} \end{cases}$$

The adjusted error cost ratio taking into account the dynamic effect of enforcement can now be expressed as a function of demand elasticity

$$\begin{aligned}\tilde{\rho} &= \frac{1-\theta}{\theta}\rho \\ &= \begin{cases} \frac{\bar{K}-(1-v)\pi^*}{(1-v)\pi^*}\left[(1-\frac{1}{\epsilon})^{1-\epsilon}(2-\frac{1}{\epsilon})^{-1}-1\right]^{-1} & \text{if } (1-v)\pi^* < \bar{K} \\ 0 & \text{if}(1-v)\pi^* \geq \bar{K} \end{cases}\end{aligned}$$

where $\pi^* = \left(\frac{\epsilon-1}{c}\right)^{\epsilon-1}\left(\frac{\alpha}{\epsilon}\right)^{\epsilon}$. Differentiating with respect to elasticity,

$$\begin{aligned}\frac{\partial\tilde{\rho}}{\partial\epsilon} = \Bigg\{ &(\frac{\bar{K}}{(1-v)\pi^*}-1)\left[-\rho(1+\rho)\right]\left[\ln(\frac{\epsilon}{\epsilon-1})-\frac{1}{\epsilon-\frac{1}{2}}\right] \\ &\frac{\bar{K}}{(1-v)\pi^*}\rho\left[\ln(\frac{\epsilon}{\epsilon-1})-\ln\frac{\alpha}{c}\right]\Bigg\}\end{aligned} \tag{5}$$

The first line in equation (5) reflects the static effect, which is negative,[11] while the second line is that of the dynamic effect, which is positive.[12] Whether the static effect dominates the dynamic effect depends on the comparison of the absolute values of the two lines in equation (5). Thus, the

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[11]The term $\frac{\bar{K}}{(1-v)\pi^*}-1$ is positive, if the adjusted ratio is positive. The term $-\rho(1+\rho)\left[\ln(\frac{\epsilon}{\epsilon-1})-\frac{1}{\epsilon-\frac{1}{2}}\right]$ is the derivative of the error cost ratio with respect to $\epsilon$. We have shown in Section 3 that the error cost ratio $\rho$ decreases with $\epsilon$ and also from appendix **A2**, we have $\ln(\frac{\epsilon}{\epsilon-1})-\frac{1}{\epsilon-\frac{1}{2}} > 0$. Combining these, we have

$$-\tilde{\rho}(1+\rho)\left[\ln(\frac{\epsilon}{\epsilon-1})-\frac{1}{\epsilon-\frac{1}{2}}\right] < 0.$$

This illustrates our argument that the static effect takes negative sign.

[12]As the expected profit, investment cost and the error cost ratio are all positive, we have $\frac{\bar{K}}{(1-v)\pi^*}\rho > 0$. From the negative relationship between elasticity and monopoly profit under isoelastic demand (appendix A3), we have that $\ln(\frac{\epsilon}{\epsilon-1})-\ln\frac{\alpha}{c} > 0$. Combine these results, we have that

$$\frac{\bar{K}}{(1-v)\pi^*}\rho\left[\ln(\frac{\epsilon}{\epsilon-1})-\ln\frac{\alpha}{c}\right] > 0$$

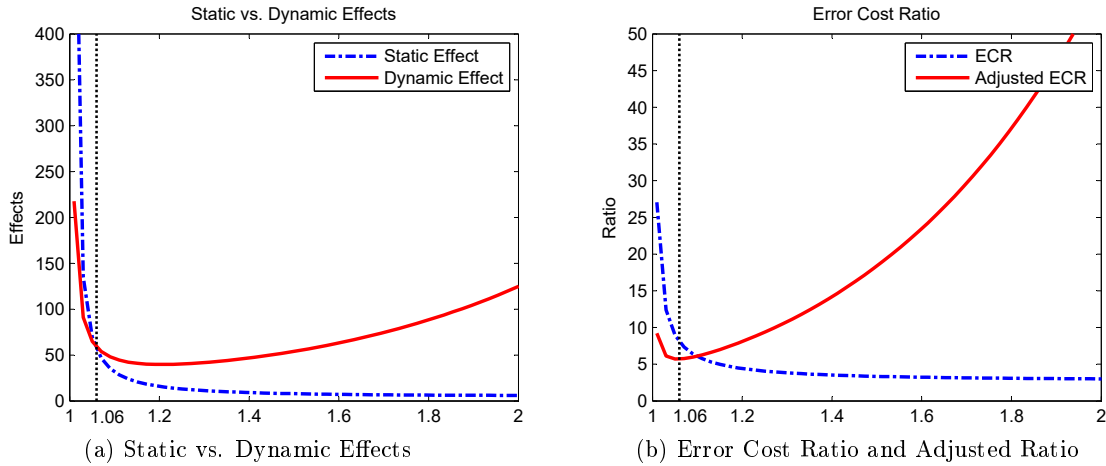This implies a positive dynamic effect of demand elasticity on the error cost ratio.

(a) Static vs. Dynamic Effects         (b) Error Cost Ratio and Adjusted Ratio

Figure 3: Adjusted Error Cost Ratio

static effect is dominant if and only if

$$\tilde{\rho}(1+\rho)\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}\right] \tag{6}$$
$$> \frac{\bar{K}}{(1-v)\pi^*}\rho\left[\ln(\frac{\epsilon}{\epsilon-1}) - \ln\frac{\alpha}{c}\right]$$

If this condition holds (6), the static effect is dominant and the adjusted error cost ratio decreases with elasticity. Otherwise, the error cost ratio *increases* with elasticity, the opposite of our result in Section 2.

Figure 3 shows a simulation of the relationship between static and dynamic effects, and of the relationship between the adjusted and static error cost ratios. Both figures assume a modest antitrust enforcement probability of 0.2.[13] As shown in Figure 3a, the dynamic effect overtakes the static effect after the elasticity becomes greater than 1.06.

Figure 3b compares the error cost ratio and the adjusted error cost ratio curves. For relatively low elasticity values, the adjusted error cost ratio is below the error cost ratio. For relatively high elasticity levels, the adjusted curve is above the static error cost ratio, eventually going to infinity. The intuition behind this pattern is the following. For high elasticity values, the market is relatively competitive, and profit expectations are low. As a result, the rate of entry/investment is low, and approaching zero as the market becomes perfectly competitive. Because entry is so low, and the expectant deadweight loss from monopoly pricing therefore low, the adjusted error cost ratio steadily

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[13]Other parameter assumptions are $\bar{K}=2$, $c=8$, $\alpha=2$. The same parameter values are assumed in Appendix B which shows adjusted ratios under other enforcement probabilities.

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goes to infinity.

Conversely, when the demand elasticity is relatively low, so that the market is not competitive, firms expected to earn relatively large profits. Now the likelihood of entry and investment is high, reducing the adjusted error cost ratio below its static counterpart.

Though the adjusted error cost ratio dips below the (static) error cost ratio for low elasticity values (i.e., uncompetitive markets), note that it is always well above the asymptotic limit of $\frac{1}{e/2-1}$ for the isoelastic demand case. Indeed, the minimum value of the adjusted error cost ratio in this simulation is greater than five. In the appendix we consider additional simulations with higher probabilities of enforcement, one intermediate with the probability enforcement set at 0.5 and the other a high-enforcement regime with a probability of 0.8. In both of the additional simulations the adjusted error cost ratio is greater, at each elasticity value, than in modest enforcement regime simulated in Figure 3.

Although we believe the assumptions in these simulations are reasonable, we do not intend to suggest that they provide a representation of the relationship of the error cost ratio to the adjusted error cost ratio for every conceivable demand function or investment cost distribution. It is possible to generate an example where the adjusted ratio falls below one. Indeed, in the case of exponential demand, where the error cost ratio has a constant value, the adjusted ratio could start from a level below the error cost ratio (for low elasticity) before going to infinity.[14] Given this, a measure of the average value of the adjusted error cost ratio over a wide range of elasticity values might offer an alternative single measurement.

# 5    Implications

The foregoing analysis has examined the welfare tradeoffs of antitrust enforcement in the innovation setting. We have focused on antitrust regulation of patentees, though the issues addressed here apply to any area where firms make investments that create new markets or substantially enhance existing markets. The tradeoffs examined here have implications for many facets of law enforcement. First, society must determine the level of resources to pour into the antitrust enforcement effort against patent holders. In other words, what is the optimal probability of enforcement when monopolists have obtained their status through innovation? Second, society must determine an optimal legal standard - for example, whether to apply a per se prohibition, a per se legality rule, or a rule of reason test. Third, society must choose the optimal standard of proof in trials, where the occurrence of an antitrust violation under the operable legal standard is uncertain. For each of these questions, the tradeoff between the costs of excessive enforcement and the costs of inadequate enforcement should be considered in determining the features of an optimal enforcement system.

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[14]For high enforcement probabilities, the adjusted error cost ratio curve will always be above the error cost ratio curve for all elasticity parameter values. However, for low enforcement probabilities it is possible to get a range of low elasticity values where the adjusted error cost curve is below the error cost curve under certain parameter values, but this requires an assumption that the upper bound on investment cost in the uniform probability model is modest relative to expected profit. Such an assumption would appear to go against intuition.

In this paper's model, we begin with a perfect enforcement ideal, and consider the cost of deviating from the ideal toward excessive enforcement, and the cost of deviating from the ideal toward inadequate enforcement. The ratio of these two costs, which we label the error cost ratio, constitutes the appropriate welfare ratio test for determining optimal antitrust enforcement in the patent context. The perfect enforcement ideal is met when antitrust enforcement enables the innovating monopolist to recover its research and development costs, and also prevents the firm from imposing unnecessary deadweight loss on society.

Using the foregoing approach to defining perfect enforcement, we find that the error cost ratio for patent antitrust, or alternatively the appropriate welfare tradeoff ratio for patent antitrust, is equal to the sum of the monopoly profit and the residual consumer surplus all divided by the deadweight loss. This finding contradicts the view suggested in the literature (Kaplow, 1984; Gilbert and Shapiro, 1990; Klemperer, 1990) that the relevant welfare ratio for patent antitrust is the profit divided by the deadweight loss (reward-loss ratio).

The reward-loss ratio appears to be invalid for assessing the welfare tradeoffs in patent antitrust because it accords inadequate weight to the residual surplus going to consumers, and in doing so undercounts the social value of innovation. One immediate implication is that prescriptions for patent antitrust law based on the reward-loss framework should be reconsidered from a perspective that gives greater consideration to the social incentive to innovate.

We find that the error cost ratio is genearlly well above one, and declines as function of market competitiveness as measured by the elasticity of demand. The minimum value of the ratio is roughly equal to three under commonly used demand functions. When the effects of enforcement on innovation are taken into account, the adjusted error cost ratio is likely to be even greater, and tends toward infinity as the elasticity of demand increases. These results suggest that society should show a greater concern for the costs of excessive enforcement than the costs of inadequate enforcement of antitrust in the patent context. The law should err on the side of protecting innovation incentives.

These implications have immediate practical relevance. There are novel theories of antitrust being applied to patentees currently. In an ongoing lawsuit against Qualcomm, the FTC claims that the firm's patent licensing fees are an abusive exercise of monopoly power. Such efforts to introduce antitrust regulation into areas that had until recently been controlled almost entirely by patent law should be assessed under a consideration of the associated error costs. An attack on patent pricing as a form of monopoly abuse is equivalent to introducing price regulation through antitrust. Determination of the welfare-maximizing antitrust price cap, which encourages innovation and at the same time avoids unnecessary deadweight loss, is subject to uncertainty. Such an effort should err on the side of protecting innovation incentives, given the high ratio of false-positive costs to false-negative costs.

In other areas of litigation, courts must determine whether to apply a per se legality test, per se illegality test, or rule of reason test to alleged antitrust violations by patent holders.[15] For example,

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[15]On error costs and legal tests in antirust, see Katsoulacos and Ulph (2009).

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settlements of patent infringement lawsuits were until recently examined under antitrust according to a per se legality test. Such settlements have typically involved the patent holder transferring a share of the patent revenue to the challenger, which traditionally has been deemed within the power of a patent holder. The Supreme Court overturned the per se legality rule and replaced it with a rule of reason test, for pharmaceutical patent infringement settlements, in *FTC v. Actavis*.[16] The rule of reason test has led to numerous lawsuits against pharmaceutical patent holders for entering into settlements with generic drug makers. The error cost ratios examined here suggest that the *Actavis* analysis should be conducted in a manner that takes into consideration the high error cost ratio for patent antitrust.[17]

Lastly, there is the question of the appropriate standard of proof in antitrust challenges of patent holders. The error cost ratio, when applied to this question, would support a high burden of proof, such as requiring clear and convincing evidence to support antitrust theories.

# 6    Conclusion

The error cost ratio - that is, the ratio of false-positive to false-negative costs - for patent antitrust is equal to the sum of the monopoly profit and residual consumer surplus all divided by the deadweight loss. We find that this ratio ranges from infinity, in uncompetitive markets (no substitutes to the patent), to a low of roughly three, in competitive markets, for commonly used demand functions. When we extend the analysis to take enforcement's effect on entry into account, we find that the range of values for the ratio is even higher under reasonable assumptions. This implies that patent antitrust rules, from substantive law to proof standards, should tilt generally in favor of patentees.

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[16]570 U.S. 136 (2013).

[17]Another example of the choice over how to implement the legal standard is observed in the area of predatory innovation claims under Section 2 of the Sherman Act, which prohibits monopolization. The choice here is between the rule of reason test described in United States v. Microsoft, 253 F.3d 34 (D.C. Cir. 2001), or the innovator-favoring version of the test articulated, specifically for predatory innovation claims, in Allied Orthopedic v. Tyco Healthcare, 592 F.3d 991 (9th Cir. 2010).

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# Appendix

## Appendix A: Model Results

## A1. Error Cost Ratio under Common Demand Function Assumptions

### Proposition 1.

The demand curve is linear $p = A - bq$ and the marginal cost is a constant $c$. Optimal output is $q^\star = \frac{A-c}{2b}$ and monopolistic price $p^\star = \frac{A+c}{2}$. The competitive price is $p^c = c$ and output is $q^c = \frac{A-c}{b}$. The minimum social loss under antitrust law that deters entry is $RS + \pi = \frac{3(A-c)^2}{8b}$, the maximum social loss if the firm is allowed to enter the market is $D = \frac{(A-c)^2}{8b}$. The error cost ratio is therefore $\rho \equiv \frac{RS+\pi}{D} = 3$.

### Proposition 2.

The demand curve is algebraic $p = \alpha q^\beta - \sigma$ and the marginal cost is a constant $c$, so the firm's profit maximization problem is $\max_q : (\alpha q^\beta - \sigma)q - cq$. Optimal output satisfies $\alpha(\beta+1)q^\beta = c + \sigma$, implying $q^\star = \left(\frac{\sigma+c}{\alpha(\beta+1)}\right)^{1/\beta}$ and monopoly price $p^\star = \frac{\sigma+c}{\beta+1} - \sigma$. The competitive output is $q^c = \left(\frac{\sigma+c}{\alpha}\right)^{1/\beta}$. The minimum social loss when antitrust law deters entry is given by

$$
\begin{aligned}
RS + \pi &= \int_0^{q^\star} [\alpha q^\beta - \sigma - c]dq \\
&= \frac{\alpha}{\beta+1} q^{\star \beta+1} - (\sigma+c)q^\star \\
&= (\sigma + c)\left[\frac{-\beta(\beta+2)}{(\beta+1)^2}\right]\left(\frac{\sigma+c}{\alpha(\beta+1)}\right)^{1/\beta},
\end{aligned}
$$

and the maximum social loss if the firm enters is

$$
\begin{aligned}
D &= \int_{q^\star}^{q^c} [\alpha q^\beta - \sigma - c]dq \\
&= (\sigma+c)\left[\frac{-\beta}{\beta+1}\right]\left(\frac{\sigma+c}{\alpha}\right)^{1/\beta} - (\sigma+c)\left[\frac{-\beta(\beta+2)}{(\beta+1)^2}\right]\left(\frac{\sigma+c}{\alpha(\beta+1)}\right)^{1/\beta}.
\end{aligned}
$$

Therefore, the error cost ratio is

$$
\begin{aligned}
\frac{RS+\pi}{D} &= \frac{(\sigma+c)\left[\frac{-\beta(\beta+2)}{(\beta+1)^2}\right]\left(\frac{\sigma+c}{\alpha(\beta+1)}\right)^{1/\beta}}{(\sigma+c)\left[\frac{-\beta}{\beta+1}\right]\left(\frac{\sigma+c}{\alpha}\right)^{1/\beta} - (\sigma+c)\left[\frac{-\beta(\beta+2)}{(\beta+1)^2}\right]\left(\frac{\sigma+c}{\alpha(\beta+1)}\right)^{1/\beta}} \\
&= \frac{1}{\frac{(\beta+1)^{1+1/\beta}}{(\beta+2)} - 1},
\end{aligned}
$$

with $\beta \in (-1, 0)$. Because $e = \lim_{n \to \infty} (1 + \frac{1}{n})^n$, it follows that $\lim_{\beta \to 0} (1 + \beta)^{1/\beta} = e$. Therefore,

$$\lim_{\beta \to 0} \frac{1}{\frac{(\beta+1)}{(\beta+2)} \cdot (1+\beta)^{\frac{1}{\beta}} - 1} = \frac{1}{\frac{1}{2} \cdot e - 1} \approx 2.8.$$

**Proposition 4.**

The demand curve is exponential $q = \gamma e^{-\beta p}$ and the marginal cost is a constant $c$. The inverse demand curve is $p = \beta^{-1} \ln \gamma - \beta^{-1} \ln q$. Optimal output satisfies $\beta^{-1} \ln \gamma - \beta^{-1} \ln q - \beta^{-1} = c$, so that $q^\star = \gamma e^{-(\beta c+1)}$ and $p^\star = \frac{\beta c+1}{\beta}$. The competitive price is $p^c = c$ and output is $q^c = \gamma e^{-\beta c}$. The social loss under antitrust law that deters firm's entry is

$$RS + \pi = \int_0^{q^\star} [-\frac{1}{\beta} \ln(\frac{q}{\gamma}) - c] dq$$
$$= -\frac{1}{\beta} [q^\star \ln(\frac{q^\star}{\gamma}) - q^\star] - cq^\star$$
$$= \gamma e^{-(\beta c+1)} \frac{2}{\beta},$$

and the social loss if the firm is allowed to enter the market is

$$D = \int_{q^\star}^{q^c} [-\frac{1}{\beta} \ln(\frac{q}{\gamma}) - c] dq$$
$$= q^c [\frac{1 - \ln(q^c/\gamma)}{\beta} - c] - q^\star [\frac{1 - \ln(q^\star/\gamma)}{\beta} - c]$$
$$= \gamma e^{-\beta c} \left[ \frac{1}{\beta} (1 - \frac{2}{e}) \right].$$

The error cost ratio is

$$\frac{RS + \pi}{D} = \frac{\gamma e^{-(\beta c+1)} \frac{2}{\beta}}{\gamma e^{-\beta c} \left[ \frac{1}{\beta} (1 - \frac{2}{e}) \right]} = \frac{\frac{2}{e}}{1 - \frac{2}{e}} \approx 2.8.$$

## A.2 Effect of Elasticity on Error Cost Ratio under Isoelastic Demand

From proposition 1 and 4, we have that the error cost ratio is a constant number when the demand function is linear or exponential. However, when the demand function is algebraic $p = \alpha q^\beta - \sigma$, or is isoelastic $p = \alpha q^\beta$, where $\beta \in (-1, 0)$, the error cost ratio is a function of demand elasticity. We claim that the ratio $\rho$ decreases with regard $\epsilon \doteq -\frac{1}{\beta}$.

According to proposition 2 and 3, the error cost ratio is given by

$$\rho = \frac{1}{\frac{(\beta+1)^{1+1/\beta}}{(\beta+2)} - 1}$$

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Since demand elasticity parameter is given by $\epsilon \doteq -\frac{1}{\beta}$, we have

$$\rho = \cfrac{1}{\cfrac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}} - 1}$$

To study the relationship between $\rho$ and $\epsilon$, we can first analyze the relationship between $\frac{1}{\rho}$ and $\epsilon$ by taking derivative of $\frac{1}{\rho}$ to $\epsilon$.

$$
\begin{aligned}
\frac{1}{\rho} &= \frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}} - 1 \\
\Rightarrow \frac{d(\frac{1}{\rho})}{d\epsilon} &= \frac{d\left(\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}} - 1\right)}{d\epsilon} \\
&= \frac{d\left(\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}}\right)}{d\epsilon} \\
&= \frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}} \frac{d\left(\ln\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}}\right)}{d\epsilon}
\end{aligned}
\tag{7}
$$

To derive $\dfrac{d\left(\ln\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}}\right)}{d\epsilon}$, we first compute $\ln\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}}$ and have

$$\ln\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}} = (1-\epsilon)\ln(1-\frac{1}{\epsilon}) - \ln(2-\frac{1}{\epsilon})$$

Then we have the derivative $\dfrac{d\left(\ln\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}}\right)}{d\epsilon}$ derived as follows

$$\frac{d\left(\ln\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}}\right)}{d\epsilon} = \ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}} \tag{8}$$

Then we have

**Claim 1.** $\forall \epsilon \in (1, +\infty)$,
$$\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}} > 0 \tag{9}$$

**Proof:** Since $\beta \in (-1, 0)$, we have $\epsilon \in (1, +\infty)$. Then we show that $\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}$ is monotonically decreasing with $\epsilon$ in $(1, +\infty)$.

We first derive the derivative of $\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}}$ with regard to $\epsilon$, as follows

$$\frac{d\left(\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}}\right)}{d\epsilon} = \frac{-\frac{1}{4}}{\epsilon(\epsilon-1)(\epsilon-\frac{1}{2})^2} < 0$$

The last equality is from $\epsilon \in (1, +\infty)$. This proves that $\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}}$ is monotonically decreasing with $\epsilon$ in $(1, +\infty)$, i.e., $\forall \epsilon \in (1, +\infty)$, $\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}} > \lim_{\epsilon \to +\infty}\left(\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}}\right)$

Then we have

$$\lim_{\epsilon \to +\infty}\left(\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}}\right) = 0$$

Thus, we have $\forall \epsilon \in (1, +\infty)$, $\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}} > 0$.

Plug the equation (8) back in equation 7, we have

$$\frac{d(\frac{1}{\rho})}{d\epsilon} = \frac{(1 - \frac{1}{\epsilon})^{1-\epsilon}}{2 - \frac{1}{\epsilon}}\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}}\right] \tag{10}$$

From Claim 1, we have $\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon - \frac{1}{2}} > 0$. Also from $\epsilon > 1$, we have

$$\frac{(1 - \frac{1}{\epsilon})^{1-\epsilon}}{2 - \frac{1}{\epsilon}} > 0$$

Thus, we have

$$\frac{d(\frac{1}{\rho})}{d\epsilon} > 0$$

Then we have

$$\frac{d\rho}{d\epsilon} = -\rho^2\frac{d(\frac{1}{\rho})}{d\epsilon} < 0. \tag{11}$$

## A3. Negative Relationship between Elasticity and Profit

**Lemma 2. (Negative Relationship between Elasticity and Profit)**   *The price elasticity functions for these demand representations are given by: (i) The isoelastic demand function takes a constant elasticity:*

$$\epsilon_{iso} = -\frac{1}{\beta}, \tag{12}$$

*(ii) The price elasticity of algebraic demand function is given by*

$$\epsilon_{alg}(p) = -\frac{p}{\beta(p + \sigma)},$$

*(iii) The price elasticity of exponential demand function is given by*

$$\epsilon_{exp}(p) = \beta p.$$

*Then, the profit is an decreasing function in price elasticity, i.e.,*

$$\frac{\partial \pi^*}{\partial \epsilon} < 0$$

*for the above three cases.*

**Case I: Isolastic Demand Function**   From the proceeding result in the Proposition 3, we have that, under isoelastic demand, the monopoly price and quantity are given by

$$p^* = \frac{c}{1+\beta},$$
$$q^\star = \left(\frac{c}{\alpha(1+\beta)}\right)^{1/\beta}.$$

Then we have the monopoly profit is given by

$$\pi^* \doteq (p^* - c)\, q^\star = \left(\frac{c}{1+\beta}\right)^{1+\frac{1}{\beta}} \left(\frac{1}{\alpha}\right)^{\frac{1}{\beta}} (-\beta). \tag{13}$$

Plug that $\epsilon \equiv -\frac{1}{\beta}$ in Equation (13), we have

$$\pi^* = \left(\frac{c}{\epsilon-1}\right)^{1-\epsilon} \left(\frac{\epsilon}{\alpha}\right)^{-\epsilon}$$
$$= c^{1-\epsilon}(\epsilon-1)^{\epsilon-1}\epsilon^{-\epsilon}\alpha^{\epsilon}.$$

Then we have

$$\ln\pi^* = (1-\epsilon)\ln c + (\epsilon-1)\ln(\epsilon-1) - \epsilon(\ln\epsilon - \ln\alpha) \tag{14}$$

We can back out the sign of derivative $\frac{\partial \pi^*}{\partial \epsilon}$ from the sign of $\frac{\partial(\ln\pi^*)}{\partial \epsilon}$ . Formally, from $\frac{\partial(\ln\pi^*)}{\partial \epsilon} = \frac{1}{\pi^*}\frac{\partial \pi^*}{\partial \epsilon}$ and $\pi^* > 0$, we have that the two derivatives $\frac{\partial \pi^*}{\partial \epsilon}$ and $\frac{\partial(\ln\pi^*)}{\partial \epsilon}$ have the same sign.

From Equation (14)

$$\frac{\partial (\ln\pi^*)}{\partial \epsilon} = -\ln c + \ln(\epsilon-1) - (\ln\epsilon - \ln\alpha)$$
$$= \ln\frac{\alpha}{c} + \ln(\frac{\epsilon-1}{\epsilon}). \tag{15}$$

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Then from the model primitive $\beta \in (-1, 0)$, we have

$$\beta \in (-1, 0) \Rightarrow \epsilon > 1$$
$$\Rightarrow 0 < \frac{\epsilon - 1}{\epsilon} < 1$$
$$\Rightarrow \ln(\frac{\epsilon - 1}{\epsilon}) < 0.$$

Assume $0 < \alpha < c$, then $\ln \frac{\alpha}{c} < 0$, then we have $\forall \epsilon \in (1, +\infty)$,

$$\frac{\partial (\ln \pi^*)}{\partial \epsilon} < 0 \Rightarrow \frac{\partial \pi^*}{\partial \epsilon} < 0.$$

**Case II: Algebraic Demand Function** From the proceeding result in the Proposition 2, we have that, $q^\star = \left( \frac{\sigma + c}{\alpha(\beta + 1)} \right)^{1/\beta}$ and monopolistic price is $p^\star = \frac{\sigma + c}{\beta + 1} - \sigma$.

Then we have the monopoly profit is given by

$$\pi^* \doteq (p^* - c) q^\star$$
$$= \left( \frac{-\beta\alpha}{1 + \beta} \right) \left( \frac{\sigma + c}{\alpha(1 + \beta)} \right)^{1 + \frac{1}{\beta}}$$
$$= \frac{-\beta\alpha^{-\frac{1}{\beta}}(\sigma + c)^{1 + \frac{1}{\beta}}}{(1 + \beta)^{1 + \frac{1}{\beta}}},$$

Plug that $\epsilon \equiv -\frac{1}{\beta}$ in the equation , we have

$$\pi^* = \frac{\alpha^\epsilon}{\epsilon} \frac{(\sigma + c)^{1-\epsilon}}{(1 - \frac{1}{\epsilon})^{1-\epsilon}}$$
$$= \alpha^\epsilon \epsilon^{-\epsilon}(c + \sigma)^{1-\epsilon}(\epsilon - 1)^{\epsilon - 1}$$

Then we have
$$\ln \pi^* = (\epsilon - 1)\ln(\epsilon - 1) + (1 - \epsilon)\ln(\sigma + c) + \epsilon \ln \alpha - \epsilon \ln \epsilon. \tag{16}$$

We can back out the sign of derivative $\frac{\partial \pi^*}{\partial \epsilon}$ from the sign of $\frac{\partial (\ln \pi^*)}{\partial \epsilon}$ . Formally, from $\frac{\partial (\ln \pi^*)}{\partial \epsilon} = \frac{1}{\pi^*} \frac{\partial \pi^*}{\partial \epsilon}$ and $\pi^* > 0$, we have that the two derivatives $\frac{\partial \pi^*}{\partial \epsilon}$ and $\frac{\partial (\ln \pi^*)}{\partial \epsilon}$ have the same sign.

From Equation (16)

$$\frac{\partial (\ln \pi^*)}{\partial \epsilon} = \ln(\epsilon - 1) - \ln \epsilon - \ln(\sigma + c) + \ln \alpha,$$

Then from the model primitive $\beta \in (-1, 0)$, we have

$$\beta \in (-1, 0) \Rightarrow \epsilon > 1$$
$$\Rightarrow \ln(\epsilon - 1) - \ln \epsilon < 0.$$

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Similarly as the analysis on isoelastic demand function, if $c + \sigma > \alpha$, we have that

$$\frac{\partial (\ln \pi^*)}{\partial \epsilon} < 0 \Rightarrow \frac{\partial \pi^*}{\partial \epsilon} < 0.$$

**Case III: Exponential Demand Function** From the proceeding result in the Proposition 4, the monopoly quantity, under exponential demand function, is $q^\star = \gamma e^{-(\beta c + 1)}$ and monopolistic price is $p^\star = \frac{\beta c + 1}{\beta}$. The monopoly profit is given by

$$\pi^* = (p^* - c) q^\star = \frac{\gamma}{\beta} e^{(-1 - \beta c)}.$$

From proceeding result, we have that the elasticity parameter $\epsilon_{exp}^A = \beta$. Thus, to analyze the effect of market elasticity parameter on the monopoly profit, we take the partial derivative of profit with regard to $\beta$, i.e.,

$$\frac{\partial \pi^*}{\partial \beta} = -\gamma e^{(-1 - \beta c)} - \frac{\gamma e^{(-1 - \beta c)}}{\beta^2} = \gamma e^{(-1 - \beta c)} \left[ -1 - \frac{1}{\beta^2} \right].$$

From the demand function's primitive setup, we have that $\gamma > 0$, and $\beta > 0$. Thus,

$$\frac{\partial \pi^*}{\partial \beta} < 0.$$

## A4. Isoelastic Demand and Uniform Distribution of Investment Cost

Assume the R&D investment cost follows uniform distribution on $(0, \bar{K}]$. To compute the adjusted error cost ratio, we first compute the probability of a firm with the firm's entry decision in period 1. Then we have the entry probability denoted as $\theta$ is given by

$$\theta = \frac{(1 - v)\pi^*}{\bar{K}} \tag{17}$$

Then we have the adjusted ratio is given by

$$\begin{aligned} \tilde{\rho} &= \frac{1 - \theta}{\theta} \rho \\ &= (\frac{\bar{K}}{(1 - v)\pi^*} - 1)\rho \end{aligned}$$

where the firm enters regardless of enforcement, if $(1 - v)\pi^* > \bar{K}$, thus,

$$\tilde{\rho} = \begin{cases} 0 & \text{if } (1 - v)\pi^* > \bar{K} \\ (\frac{\bar{K}}{(1-v)\pi^*} - 1)\rho & \text{if } (1 - v)\pi^* \leq \bar{K} \end{cases}$$

When elasticity is not small, such that $(1 - v)\pi^* \leq \bar{K}$. the adjusted error cost ratio taking into

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account the dynamic effect of enforcement can now be expressed as a function of demand elasticity

$$\tilde{\rho} = \frac{1-\theta}{\theta}\rho$$
$$= (\frac{\bar{K}}{(1-v)\pi^*} - 1)\left[(1-\frac{1}{\epsilon})^{1-\epsilon}(2-\frac{1}{\epsilon})^{-1} - 1\right]^{-1}$$

**Static and Dynamic Effects under Isoelastic Demand and Uniform Distribution of Investment Cost**

The static effect is defined by $\frac{1-\theta}{\theta}\frac{\partial\rho}{\partial\epsilon}$ , and hence it is given by

$$\frac{1-\theta}{\theta}\frac{\partial\rho}{\partial\epsilon} = (\frac{\bar{K}}{(1-v)\pi^*} - 1)\frac{\partial\rho}{\partial\epsilon}$$

From equation (11) and equation (10), we have

$$\frac{d\rho}{d\epsilon} = -\rho^2\frac{d(\frac{1}{\rho})}{d\epsilon}.$$
$$= -\rho^2\frac{(1-\frac{1}{\epsilon})^{1-\epsilon}}{2-\frac{1}{\epsilon}}\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}\right]$$
$$= -\rho^2(\frac{1}{\rho}+1)\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}\right]$$
$$= -\rho(1+\rho)\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}\right]$$
$$\text{where } \rho = \left[(1-\frac{1}{\epsilon})^{1-\epsilon}(2-\frac{1}{\epsilon})^{-1} - 1\right]^{-1}$$

Thus, the static effect is given by

$$\frac{1-\theta}{\theta}\frac{\partial\rho}{\partial\epsilon} = (\frac{\bar{K}}{(1-v)\pi^*} - 1)\left[-\rho(1+\rho)\right]\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}\right] \tag{18}$$

The dynamic effect is defined by $-\frac{\rho}{\theta^2}\frac{\partial\theta}{\partial\epsilon}$. Plug equation (17) in the expression of dynamic effect,

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we have

$$
\begin{aligned}
-\frac{\rho}{\theta^2}\frac{\partial\theta}{\partial\epsilon} &= -\rho\left(\frac{(1-v)\pi^*}{\bar{K}}\right)^{-2}\frac{\partial\theta}{\partial\pi^*}\frac{\partial\pi^*}{\partial\epsilon} \\
&= -\rho\frac{\frac{(1-v)}{\bar{K}}}{\left(\frac{(1-v)\pi^*}{\bar{K}}\right)^2}\frac{\partial\pi^*}{\partial\epsilon} \\
&= \frac{-\rho}{(\pi^*)^2}\frac{\bar{K}}{(1-v)}\frac{\partial\pi^*}{\partial\epsilon}
\end{aligned}
\tag{19}
$$

From proceeding result in Appendix A3, we have

$$
\frac{\partial\pi^*}{\partial\epsilon} = \pi^*\frac{\partial\ln\pi^*}{\partial\epsilon}
\tag{20}
$$

Combine equation (20) and equation (15) in Appendix A3, we have

$$
\frac{\partial\pi^*}{\partial\epsilon} = \pi^*\left[\ln\frac{\alpha}{c} + \ln(\frac{\epsilon-1}{\epsilon})\right]
\tag{21}
$$

Combine equation (21) and (19), we have the dynamic effect given by

$$
\begin{aligned}
-\frac{\rho}{\theta^2}\frac{\partial\theta}{\partial\epsilon} &= \frac{-\rho}{(\pi^*)^2}\frac{\bar{K}}{(1-v)}\pi^*\left[\ln\frac{\alpha}{c} + \ln(\frac{\epsilon-1}{\epsilon})\right] \\
&= \frac{-\rho}{\pi^*}\frac{\bar{K}}{(1-v)}\left[\ln\frac{\alpha}{c} + \ln(\frac{\epsilon-1}{\epsilon})\right]
\end{aligned}
\tag{22}
$$

We can compare the magnitude of static and dynamic effects by the absolute value of equation (18) and equation (22). The static effect dominates the dynamic effect if and only if

$$
(\frac{\bar{K}}{(1-v)\pi^*} - 1)\left[\rho(1+\rho)\right]\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}\right] > \frac{-\rho}{\pi^*}\frac{\bar{K}}{(1-v)}\left[\ln\frac{\alpha}{c} + \ln(\frac{\epsilon-1}{\epsilon})\right]
\tag{23}
$$

$$
\Leftrightarrow \tilde{\rho}(1+\rho)\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}\right] > \frac{\bar{K}}{(1-v)\pi^*}\rho\left[\ln(\frac{\epsilon}{\epsilon-1}) - \ln\frac{\alpha}{c}\right]
\tag{24}
$$

$$
\Leftrightarrow \left(\bar{K} - (1-v)\pi^*\right)(1+\rho)\left[\ln(\frac{\epsilon}{\epsilon-1}) - \frac{1}{\epsilon-\frac{1}{2}}\right] > \bar{K}\left[\ln(\frac{\epsilon}{\epsilon-1}) - \ln\frac{\alpha}{c}\right]
\tag{25}
$$

From equation $\rho = \left[(1-\frac{1}{\epsilon})^{1-\epsilon}(2-\frac{1}{\epsilon})^{-1} - 1\right]^{-1}$, we have

$$
1 + \rho = \frac{\epsilon^\epsilon}{\epsilon^\epsilon - (2\epsilon-1)(\epsilon-1)^{\epsilon-1}}
\tag{26}
$$

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Plug equation (26) back in the comparison inequality (23), we have that static effect dominates, if

$$\frac{\left(\bar{K} - (1-v)\pi^*\right)\epsilon^\epsilon}{\epsilon^\epsilon - (2\epsilon - 1)(\epsilon - 1)^{\epsilon - 1}}\left[\ln(\frac{\epsilon}{\epsilon - 1}) - \frac{1}{\epsilon - \frac{1}{2}}\right] > \bar{K}\left[\ln(\frac{\epsilon}{\epsilon - 1}) - \ln\frac{\alpha}{c}\right]$$

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## Appendix B. Simulations with Varying Enforcement Probability

In this appendix, we show how the adjusted error cost ratio changes according to the antitrust enforcement probability. In the main text, we show the adjusted error cost ratio and corresponding static and dynamic effects under a modest enforcement probability, $v = 0.2$. If the enforcement probability increases, the adjusted error cost ratio curve shifts up. So does the dynamic effect curves, while the change in the static effect is smaller than the change in the dynamic effect.

**Medium Enforcement Probability**  In Figure 4, we assume the enforcement probability is at the intermediate level, $v = 0.5$. The adjusted error cost ratio is higher than that under $v = 0.2$ (see Figure 3 in main text). The lower bound of the adjusted error cost ratio now is about 13, which occurs when the demand elasticity is about 1.09.
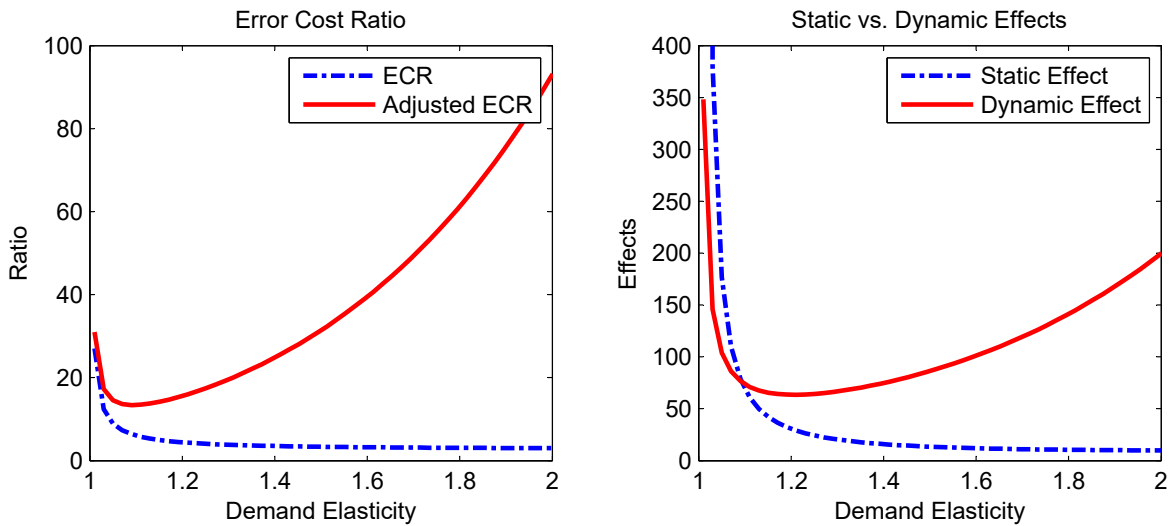


Figure 4: Medium Enforcement Probability $v = 0.5$

**High Enforcement Probability** In Figure 5, we assume the enforcement probability is high, $v = 0.8$. The adjusted error cost ratio curve shifts up further in Figure 5 than in Figure 4. The lower bound of the adjusted error cost ratio now is about 42, which occurs when the demand elasticity is about 1.11.
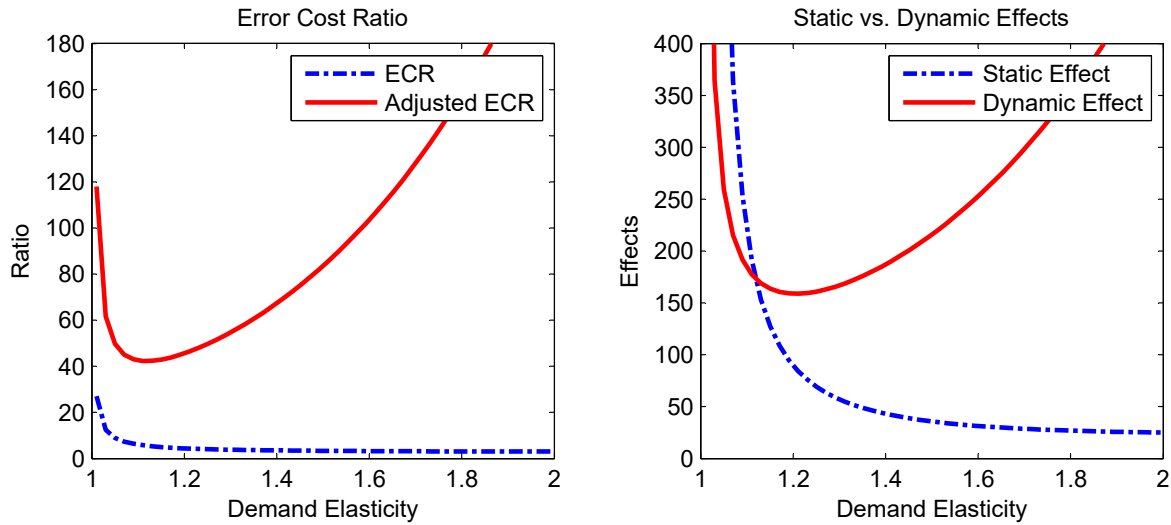


Figure 5: High Enforcement Probability $v = 0.8$

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