10-2017

Optimal Remedies for Patent Infringement

Keith N. Hylton
Boston University School of Law

Mengxi Zhang
Boston University

BOSTON UNIVERSITY

OPTIMAL REMEDIES FOR PATENT INFRINGEMENT

Mengxi Zhang
Boston University, Department of Economics

Keith N. Hylton
Boston University School of Law

Optimal Remedies for Patent Infringement

Mengxi Zhang and Keith N. Hylton[*]

November 8, 2015

Abstract

This paper derives optimal remedies for patent infringement, examining damages awards and injunctions. The fundamental optimality condition that applies to both awards and injunctions equates the marginal static cost of intellectual property protection with the marginal "dynamic" benefit from the innovation thereby induced. We find that the optimal damages award may be greater than (or less than) the standard "lost profits" measure, depending on the social value of the innovation. When the social value of the patent is sufficiently high, the optimal award induces socially efficient investment by giving the innovator the entire social value of her investment.

Keywords: Optimal Patent Damages, Patent Infringement, Optimal Injunction Scope

JEL Classifications: D42, K13, L40, O31, O34

1 Introduction

In *Octane Fitness v. Icon Health*,[1] the U.S. Supreme Court overturned a rule adopted by the Federal Circuit restricting the conditions under which a prevailing litigant in a patent infringement lawsuit could collect attorney's fees from the opposing party. In essence, the Court made it easier for a prevailing patent litigant to collect attorney's fees. For patent holders, the decision increases expected damages, at least for strong infringement claims. More recently, the Supreme Court granted certiorari in two cases, *Halo Electronics, Inc. v. Pulse Electronics, Inc.*,[2] and *Stryker Corp. v. Zimmer, Inc.*,[3] seeking to extend its *Octane Fitness* standard to the matter of enhanced damages for willful infringement, on the ground that the test the Federal Circuit has used to determine whether patent infringement was willful makes it too difficult for patentees to win enhanced damages. *Octane Fitness*, *Halo Electronics*, and *Stryker* generate the normative question of what prevailing plaintiffs would receive in an optimal patent infringement damages regime. We examine this question here.

Prevailing patent infringement plaintiffs typically receive compensatory damages, described as "lost profits."[4] Specifically, the prevailing plaintiff is compensated for the profit loss suffered as a result of the infringement. If the plaintiff is unable to prove his loss in profits, he is at least awarded a reasonable royalty or licensing fee for the period of the infringement. In addition, the patent statute permits courts to treble the compensatory award in a case of willful infringement.

Compensatory damages are the norm in general tort litigation. But patent litigation is distinguishable from tort litigation, on several grounds. In the patent context, unlike tort litigation, the infringement not only injures the patentee but benefits society by subjecting the patented innovation to competition. In addition, the plaintiff has invested in innovation before the infringement occurs. Patent infringement reduces the reward to the patentee's investment in innovation. In tort litigation, by contrast, the injurer harms the plaintiff without generating any payoff to society, or necessarily impacting the rewards from some specific investment.[5]

---

[1] 134 S.Ct. 1749 (2014).

[2] S.Ct. No. 14-1513

[3] S. Ct. No. 14-1520

[4] Dam (1994) notes that before the Supreme Court's decision in Aro Manufacturing Co. v. Convertible Top Replacement Co., 377 US 476 (1964), prevailing infringement plaintiffs received either lost profits or the gain to the infringer. The Court in *Aro* interpreted a 1946 amendment to the patent statute to limit damages to the patentee's losses.

[5] Of course, one could argue that all injuries due to torts reduce the value of investments in health, etc. But if torts are relatively rare occurrences unmotivated by financial gain, as seems likely, few will fear becoming the victim of a tort on a regular basis, and even fewer will reduce their investments in health because of such a fear.

We find that optimal patent damages may exceed the lost profits measure. The optimal damages rule, when the social value of the patent is sufficiently high, awards lost profits and the expected consumer surplus from the patent net of the expected litigation and precautionary costs borne by the infringer. Moreover, this supercompensatory award provides an upper bound on optimal damages; optimal damages may be less than this sum but will never be greater.

We discuss the implications of these findings for the level of damages awarded for patent infringement. The Patent Act (Section 258) permits courts to award attorneys fees and even supercompensatory awards (treble damages) in exceptional cases. The model here has implications for the conditions under which courts should award attorneys' fees or treble damages for infringement.

2 Related Literature

This is the first paper to explore optimal damages for patent infringement,[6] though it builds on three separate strands of literature. The longest tradition is the literature on optimal patent protection (scope and length) beginning with Nordhaus (1969), and more recently Klemperer (1990), Gilbert and Shapiro (1990), and Gallini (1992). This literature has explored the tradeoff between optimizing innovation incentives and minimizing the static monopolization cost of intellectual property. Our paper shows how the ideas developed in this literature apply to the question of patent damages. Since we examine a single instrument, the damages measure, our analysis is simpler and more focused than the optimal protection literature. We distinguish the static scenario without investment in innovation from the "dynamic" scenario which includes investment, and compare optimal damage measures for the two scenarios.

The second strand is a series of papers on optimal tort damages, beginning with Polinsky and Rubinfeld (1988). Polinsky and Rubinfeld examined the tradeoff between deterrence and litigation costs in the design of an optimal tort damages scheme, in a model in which the care exercised by an injurer determines the likelihood of an accident. If care is sufficiently

---

[6]In the academic legal literature, at least two authors, Golden (2008) and Heald (2010), have noted the dearth of normative analysis of optimal infringement damages. Golden proposes several general norms that should govern optimal damages. Heald argues that damages should be designed to minimize transaction costs. Ayres and Klemperer (1999), building informally on Klemperer (1990), argue that optimal infringement damages should be constrained in order to minimize the social loss for every dollar of patentee profit. Blair and Cotter (1998) examine the economics of standard damages remedies (lost profits, reasonable royalty, restitution) within the property rules/liability rules framework, but do not derive the optimal award from a social welfare objective function.

productive, they showed, it may be optimal to award supercompensatory damages; when care is relatively unproductive it may be optimal to award subcompensatory damages. These results were extended in Hylton (1990), which showed that the optimal award would never exceed the level that compensates the victim for his loss and for his cost of litigation. The finding that the optimal tort award could range from subcompensatory to a maximum equal to the sum of compensatory damages plus the cost of litigation was later confirmed in richer models in Hylton (2002) and Polinsky and Shavell (2013).

The torts literature is highly relevant to this setting because patent infringement is analogous to an ordinary tort. In the tort scenario, the injurer's level of care determines the likelihood of an injury. In the patent infringement scenario, the infringer's level of care in searching previously patented inventions determines the likelihood of infringement.

The third strand of literature studies the incentive effects of damages for patent infringement. Schankerman and Scothmer (2001) examine the innovation incentive effects of various damages measures (lost profits versus unjust enrichment) and injunctions for patent holders who intend to license their innovations. Anton and Yao (2007) examine competition and innovation incentives under the lost-profits measure in a duopoly model, analyzing incentives to infringe. Our paper differs in that we are trying to determine optimal remedies for infringement rather than examining incentive effects of established remedies.

3   Model

There are two risk neutral players, the innovator/victim (she) and the infringer (he). If patent infringement does not occur, the innovator will become a monopoly and earn v, and society gets residual consumer surplus W. The variable v is assumed to be random, described by density function h and cumulative density function H, with support $[0, \infty)$. $W > 0$ is a constant. If patent infringement occurs, the innovator will earn 0, and society gets surplus $S > W$.

The monopoly rent v is variable because of the possibility of *post-patent efficiency gains*, which reduce the innovator-monopolist's cost of supplying the market. The post-patent efficiency gains can be realized only if the innovator gets a monopoly.[7]

---

[7]There are examples consistent with this structure. First, obtaining a patent can reduce the cost of capital for the firm by reducing uncertainty and informational asymmetry (Hegde and Mishra, 2014). Second, a patent can provide security that incentivizes a firm to make continuous enhancements in process and quality. In the drug industry, for example, a pioneer patent holder will continue to improve its production facilities and the quality of its product only if it retains its exclusive market protected by a

The risk of loss to the innovator can be reduced by the exercise of caution by the potential infringer, and it is costly for the infringer to take care. Caution in this setting, comes in the form of care exercised in searching for the existence of the innovator's patent.[8] Let x denote the cost of taking care, p denote the probability of loss from infringement if potential infringers do not take care, and q the probability of loss if infringers do take care. Clearly $x > 0$ and $p > q > 0$.[9]

Although this model may seem to focus on accidental infringement, willful infringement can be viewed as a special case where p approaches 1 and q approaches 0. The cost of taking care can then be interpreted as the profit the infringer forgoes by not infringing the patent.[10]

If patent infringement occurs, the innovator can choose to sue. Let c_v denote the litigation cost borne by a victim, and c_i denote the litigation cost by an injurer in defending himself against a claim. If the innovator's suit is successful, the court will award the innovator $v + \triangle$. The *damages additur* $\triangle$ is an adjustment to compensatory damages (lost profits) that can be positive or negative in this analysis.

In comparison to previous studies on optimal damages (Polinsky and Rubinfeld (1988) and related literature), we modify the analysis by incorporating investment incentives. To be specific, the innovator needs to determine whether to invest after learning the investment

---

patent; once the firm loses its market to a generic it no longer pursues such improvements (Blair and Cotter, 2002). Our model assumes the post-patent efficiencies are on the cost side.

[8]Once a search has revealed the existence of a valid patent, the infringer can take steps to reduce the likelihood of infringement, for example by redesigning his product, seeking a license, or refraining from entering the covered market (perhaps entering a different market) until the patent expires. Alternatively, a search might lead the potential infringer to conclude that the likelihood of infringement is too small to justify a change in strategy.

[9]This model of "accidental" infringement is sufficiently broad to incorporate a wide range of scenarios. It encompasses the problem of poor notice and uncertainty regarding the scope of rights in important parts of the patent system, stressed by Bessen and Meurer (2008) and Lemley and Shapiro (2005). In this model, the infringer can observe the monopoly rent that may be destroyed by his activity, but may not be able to tell whether it is supported by a valid patent or the precise scope of the patent. Consider, for example, Kodak's decision to enter into the instant photography market in competition with Polaroid. Kodak took care by searching, and attempting to design around Polaroid's patents, but was still found guilty of infringement (Bessen & Meurer, 2008, 50-51). Even the relatively clear rights observed in the pharmaceutical sector may, in some cases, be consistent with this model. A potential infringer may observe the monopoly rent and the existence of patent claims, but conclude that the validity of some or all of the claims in light or prior art is highly uncertain.

[10]However, uncertainty over patent validity or scope is still a necessary component, unless some other transaction-cost barrier would prevent a license. Bessen and Meurer (2008, at 50) note that the cost of searching in some industries may be so high that no rational infringer would conduct a search. Our model includes this scenario too. Some infringers may conduct a preliminary assessment of the cost of search, and upon finding it too high, decide not to search. Still, even in this case the infringer faces a decision to proceed knowing that there is a risk of infringement that could be avoided if it did not go forward or if it chose an alternative method. The cost of forgoing its planned method is then the cost of taking care.

cost. The investment cost k is assumed to be random, distributed with density function ϕ and cumulative density function Φ, with support $[0, \infty)$.

The timeline of the game is as follows:

- Stage 0: *Investment.* The innovator observes the investment cost k and decides whether to invest.

- Stage 1: *Exercise of Caution.* The value of patent v is realized. The injurer decides whether to take care. If patent infringement does not occur, the game ends. The innovator earns v and society gets W.

- Stage 2: *Litigation.* If patent infringement occurs, the innovator decides whether to sue. If she does not sue, the game ends. The innovator gets 0 and the society gets S. If the innovator sues, the two parties pay litigation costs c_v and c_i respectively. Then the innovator gets $v + \triangle$, the injurer pays $v + \triangle$, and society gets S.

To obtain neater results, we impose the following assumptions.

Assumption 1. $(c_v + c_i)(p - q) > x$.

The above assumption states that the cost of taking care is less than the decrease in the total litigation cost. This assumption implies that if the injurer knows the victim is going to sue when infringement occurs, he will take care.[11] In section 6, we extend the model to incorporate variable cost of care. We also impose the following assumptions on the cost and value distributions.

Assumption 2. *a)* $\frac{\phi(k)}{\Phi(k)}$ *is decreasing in k.*
b) $(p - q)v + q\frac{1 - H(v)}{h(v)}$ *is increasing in v.*

The first part of this assumption requires the distribution of the cost variable to be *not too convex.* All logconcave distributions (which includes most commonly used distributions, such as uniform, logistic, Chi-square, and exponential) meet this restriction. The second requires the distribution of monopoly value to be *not too concave.*[12] These assumptions

---

[11]The victim sues only if $v + \triangle > c_v$. Given that the victim is going to sue, the benefit of taking care is $(p - q)(c_i + \triangle + v) > (p - q)(c_v + c_i) > x$, where x is the cost of taking care. So the injurer will always take care if he knows the victim is going to sue.

[12]When $q = 0$, all distributions meet this requirement. When $q = p$, any distributions less concave (more convex) than the exponential distribution meet this requirement. The basic idea is, when $\frac{p}{q}$ goes to infinity, most distributions meet this requirement; for example, the uniform distribution meets this requirement when $\frac{p}{q} > 2$. When $\frac{p}{q}$ goes to 0, fewer distributions satisfy this requirement, but any distribution less concave than exponential always meets this requirement regardless of $\frac{p}{q}$.

make comparative statics analysis simpler and are not crucial for our main results. Loosely speaking, these assumptions are used to ensure the marginal effect of damages on the innovator's incentive increases slower/decreases faster than society's marginal cost.

4   Optimal Patent Damages

In this section, we consider the optimal patent damage. We analyze this problem by backward induction.

$t = 2$: If the victim sues, she will get $v + \triangle$. Given any value of $\triangle$, the victim will sue if and only if $\triangle + v > c_v$, or equivalently, $v > c_v - \triangle$.[13]

$t = 1$: By assumption 1, we know that the injurer will take care if and only if he expects the victim to sue at $t = 2$. That is, he will take care if and only if $v > c_v - \triangle$.

$t = 0$: The innovator will invest when the value of patent $V(\triangle) > k$, so the probability of investment is $\Phi(V(\triangle))$.

4.1   Static Case

In this section, we first consider the static setting, where there is no investment concern.

For any $\triangle$, let $V(\triangle)$ denote the value of the patent to the innovator - that is, the *private value of the patent*. The innovator will sue only when the value of her innovation is large enough. Thus, the private value of the patent can be expressed as

$$
\begin{aligned}
V(\triangle) \;=\; & (1-p)H(c_v - \triangle)E(v \mid v \leq c_v - \triangle) \\
& + [1 - H(c_v - \triangle)]\left[E(v \mid v > c_v - \triangle) + q(\triangle - c_v)\right].
\end{aligned}
$$

The first term reflects the case where the innovation has such a low value to the innovator that she will not sue for infringement. In this case the innovation has positive value to the innovator only if infringement does not occur. The second term reflects the case where the innovation has sufficient value to induce the innovator to sue to protect it.

Note that when $\triangle \geq c_v$, the above equation becomes

$$
V(\triangle) = E(v) + q(\triangle - c_v).
$$

Thus, when suit is certain to be brought, because the damages additur exceeds the plaintiff's cost of litigation, the value of the patent is the sum of the expected profit from the

---

[13]To break ties, we assume the victim will not sue (the injurer will not take care) if he is indifferent.

innovation plus the expected net reward from litigation above and beyond compensation for lost profits. Litigation occurs with frequency q because all potential infringers take care when suit is certain to be brought. For this case, a larger damage amount always benefits the innovator.[14]

Proposition 1. *The private value of the patent, $V(\triangle)$, is strictly increasing in the damages additur $\triangle$.*

Proof. In the case where $\triangle \geq c_v$, it is trivial that $V'(\triangle) = q > 0$. When $\triangle < c_v$, we have

$$
\begin{aligned}
V'(\triangle) &= -(1-p)(c_v - \triangle)h(c_v - \triangle) + (c_v - \triangle)h(c_v - \triangle) \\
&\quad + q\left[1 - H(c_v - \triangle)\right] + q(\triangle - c_v)h(c_v - \triangle) \\
&= (c_v - \triangle)h(c_v - \triangle)(p - q) + q\left[1 - H(c_v - \triangle)\right] > 0
\end{aligned}
$$

So $V(\triangle)$ is always strictly increasing in $\triangle$. $\square$

Similarly, we use $M(\triangle)$ to denote the *social value of the patent* given any value of $\triangle$:

$$
\begin{aligned}
M(\triangle) &= H(c_v - \triangle)\left\{pS + (1-p)\left[W + E(v \mid v \leq c_v - \triangle)\right]\right\} \\
&\quad + \left[1 - H(c_v - \triangle)\right]\left\{qS + (1-q)\left[W + E(v \mid v > c_v - \triangle)\right]\right\} \\
&\quad - \left[1 - H(c_v - \triangle)\right]\left[x + q(c_v + c_i)\right].
\end{aligned}
$$

The social value of the patent is the sum of expected innovation surplus and patentee profit, less the sum of the cost of taking care to avoid infringement and the expected total cost of litigation. Note that when $\triangle \geq c_v$, the above equation becomes

$$
M(\triangle) = qS + (1-q)\left[W + E(v)\right] - x - q(c_v + c_i),
$$

which is the social value of the patent when infringement litigation is certain.

It follows that although the private value of the patent is always positive, the social value is not necessarily positive in the static setting. The cost of taking care to avoid infringement and the cost of infringement litigation may exceed the innovation surplus from the patent. Indeed, given that the social value of the patent in a regime that bars patent infringement litigation, $pS + (1-p)[W + E(v)]$, may be greater than the social value of the patent when litigation is certain, it is not clear that infringement litigation enhances social welfare in the static scenario. The intuitive reason is straightforward: in the absence

---

[14]This statement is not necessarily true in the case with variable cost of care. See section 5.1 for details.

8

of substantial post-patent efficiencies, infringement necessarily enhances the innovation surplus.

Returning to $M(\triangle)$, it is clear that when $\triangle \geq c_v$, the social value of the patent is independent of $\triangle$, because the innovator will always sue for any $\triangle \geq c_v$, and $\triangle$ is simply a transfer.

When $\triangle < c_v$, the effect of a change in $\triangle$, on the patent's social value is

$$M'(\triangle) = h(c_v - \triangle)\{(p-q)\left[(c_v - \triangle) + (W-S)\right] - x - q(c_v + c_i)\}$$

The first term in brackets, $c_v - \triangle$, the only positive term in this expression, reflects that increasing the damage award shifts the marginal patent from one that will not be defended to one that will be defended. But this is at least partially offset by the negative effects: the reduction in consumer surplus, and the increase in the cost of taking care and cost of litigation.

According to the above first order condition, when $\triangle \leq c_v + W - S - \frac{q(c_v + c_i) + x}{p-q}$, M is increasing in $\triangle$; otherwise M is decreasing in $\triangle$. In other words, given $\triangle < c_v$, $M(\triangle)$ is maximized at $\triangle_{st} = c_v + W - S - \frac{q(c_v + c_i) + x}{p-q}$, or equivalently, when $v + \triangle_{st} - c_v = v + W - S - \frac{q(c_v + c_i) + x}{p-q}$. It is easy to check that $\triangle_{st}$ is also the global maximizer. Since $W - S - \frac{q(c_v + c_i) + x}{p-q} < 0$, this implies the following.

Proposition 2. *The optimal static damages additur, $\triangle_{st}$, is less than the plaintiff's cost of litigation, c_v.*

These results can be easily verified in the following graph of $M(\triangle)$.[15]

---

[15]Note that throughout this paper, the graphs are used to illustrate the general trend rather than the exact shape of variables. For example, we know $M(\triangle)$ always initially increases to a maximum, then decrease, and finally becomes a constant as shown in Figure 1. But $M(\triangle)$ may not be concave as in Figure 1. The same applies to all other figures.
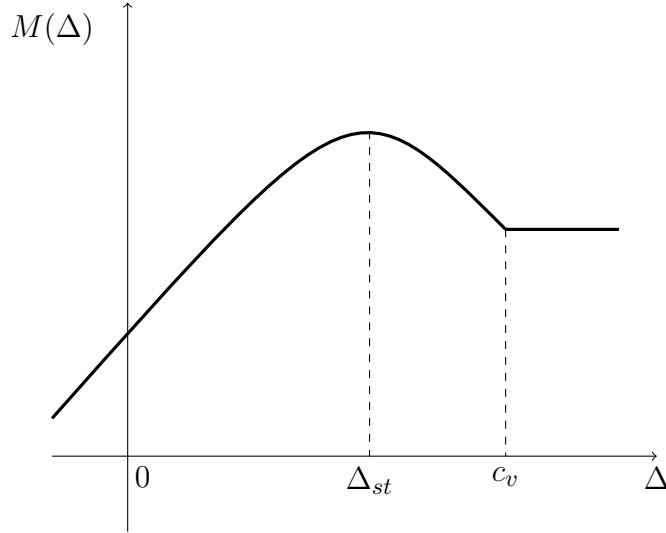
Figure 1: Illustration of $M(\Delta)$

Since infringement provides a payoff to society by expanding the innovation surplus, social welfare may be optimized by discouraging some patent infringement litigation. Thus, in the static setting it is not in society's interest to encourage every patent infringement victim to sue. Moreover, since the optimal damages additur may be negative, social welfare may be enhanced by setting the compensation award at a level less than lost profits.

Of course, one may ask, why not provide full encouragement to infringement in the static setting by dropping the additur to a level that completely eliminates the incentive to bring suit? The answer is that post-patent efficiencies may justify some level of protection from infringement. If the post-patent efficiencies (secured by the threat of infringement litigation) are sufficiently strong, they may outweigh the social loss that would otherwise result from deterring infringement.

The optimal static damages additur declines as the total surplus (S) increases relative to the residual surplus (W), which implies that as the consumer harm ($S - W$) from patent-based monopolization increases, other things held fixed, the optimal static additur declines. Thus, the optimal static additur operates in part like a penalty that internalizes consumer harm (Becker, 1968). The optimal static additur also declines as the cost of care (x) and the cost of litigation ($c_v + c_i$) increase relative to the productivity of care ($p - q$), which is consistent with the results of studies of optimal damages in the general tort setting (Polinsky and Rubinfeld, 1988, Hylton, 1990, Polinsky and Shavell, 2013).

Note that when $W = S = 0$, the current problem appears to reduce to the standard tort litigation problem examined in Polinsky and Rubinfeld and related literature, and the

optimal damages additur simplifies to $c_v - \frac{q(c_v + c_i) + x}{p - q}$. One consistent result of the torts literature is that the optimal tort damages additur could be as large as c_v, but might be smaller if the productivity of care is not high. This suggests that the model examined here is one in which the productivity of care is not high. Why? The reason care is not highly productive here is that the infringer decides whether to take care after observing v in this model, while he acts before observing v in the torts setting. The decision to take care just affects the marginal patent here, while the same decision affects all potential victims in the torts context. However, even if this model were changed to adopt the same assumption as the torts literature,[16] the optimal static additur would still be less than the optimal tort damages additur because of the negative externality W - S.

From the foregoing, $M - V$ is the *social externalilty from innovation*. Recall that the private value of the patent does not take into account the innovation surplus going to consumers, the cost of taking care to avoid infringement, and the litigation cost of the infringer. The social value does take these factors into account. It follows that the social externality can be positive or negative depending on whether the sum of the cost of taking care and the cost of litigation exceeds the innovation surplus. However, the effect of an increase in damages on the social externality is relatively easy to determine.

Proposition 3. *The social externality from innovation is strictly decreasing in the damages additur, $\triangle$.*

Proof. Proof. This statement is trivially true when $\triangle \geq c_v$. When $\triangle < c_v$, we have

$$
\begin{aligned}
M(\triangle) - V(\triangle) &= [1 - H(c_v - \triangle)]\,[qS + (1-q)W - qE(v \mid v > c_v - \triangle)] \\
&\quad + H(c_v - \triangle)\,[pS + (1-p)W] \\
&\quad - [1 - H(c_v - \triangle)]\,[x + q(\triangle + c_i)].
\end{aligned}
$$

The first derivative is

$$
\begin{aligned}
M'(\triangle) - V'(\triangle) &= h(c_v - \triangle)\,[(p-q)(W-S) - x - q(c_v + c_i)] \\
&\quad - q\,[1 - H(c_v - \triangle)] \\
&< 0.
\end{aligned}
$$

$\square$

---

[16] The model could be changed so that the potential infringer observes only $E(v)$ rather than v, which would make it equivalent to the torts model when $W = S = 0$. Thus, in a setting of even greater uncertainty, where the potential infringer cannot determine the size of the monopoly rent destroyed by his entry, one might find that the damages award has a stronger impact on social welfare.

For expositional purposes, from here onwards, we use $\triangle_{ex}$ to denote the solution to $M(\triangle) - V(\triangle) = 0$. At $\triangle_{ex}$, the damages additur is the level where the social externality is driven to zero.

4.2 Dynamic Case

Now we modify the analysis by incorporating the innovation incentive. The problem of interest is to choose $\triangle$ to maximize society's objective

$$\Pi(\triangle) \;=\; \Phi(V(\triangle))M(\triangle) - \int_0^{V(\triangle)} k\phi(k)dk,$$

where the first term is the expected social benefit from innovation, and the final term subtracts off the cost of investment in innovation.

The first order condition for the social objective function is

$$\begin{aligned}
\Pi'(\triangle) &= \phi(V(\triangle))V'(\triangle)M(\triangle) + \Phi(\triangle)M'(\triangle) - V(\triangle)\phi(\triangle)V'(\triangle) \\
&= 0.
\end{aligned}$$

Rearranging, we obtain

$$M'(\triangle) + \frac{\phi(\triangle)}{\Phi(\triangle)}V'(\triangle)M(\triangle) = \frac{\phi(\triangle)}{\Phi(\triangle)}V'(\triangle)V(\triangle).$$

Damages for patent infringement should be increased until the marginal social benefit from an additional dollar (left hand side, LHS) equals its marginal social cost (right hand side, RHS). The marginal social benefit reflects the social loss from expanding protection from infringement (the static cost of intellectual property protection) and the marginal innovation value to society of expanding protection (which may be positive or negative depending on the social value of the patent). The marginal social innovation value is shown to be the product of the elasticity of innovation with respect to an increase in damages ($\frac{\phi(\Delta)}{\Phi(\Delta)}V'(\Delta) > 0$) and the social value of the patent. Turning to the right hand side of the equation, the marginal social cost of increasing damages is unambiguously positive and equal to the product of the innovation elasticity and the private value of the patent. The marginal social cost is positive because a dollar increase in infringement damages generates additional investment based on the private value of the patent.

The first order condition can be rewritten as:

$$-M'(\triangle) = \frac{\phi(V(\triangle))}{\Phi(V(\triangle))}V'(\triangle)[M(\triangle)-V(\triangle)]$$

which reflects a balance between the static and dynamic effects of increasing damages for patent infringement. The right hand side reflects the dynamic benefit of enhancing infringement damages. Increasing damages generates more innovation, as reflected in the positive elasticity term. Thus, if the social externality from innovation (M - V) is positive, increasing infringement damages enhances social welfare, and should be encouraged up to the point where the marginal benefit is balanced off by the marginal social cost of intellectual property protection.

Even from this preliminary analysis it is clear that the optimal static damages additur, $\triangle_{st}$, is unlikely to be socially optimal when investment is sensitive to damages. If the social first order condition above is evaluated at $\triangle_{st}$, then $M'(\triangle_{st}) = 0$, but the condition is unlikely to be satisfied because the social and private values of the patent are generally unequal when evaluated at $\triangle_{st}$.

For expository purposes, let

$$LHS(\Delta) = -M'(\Delta)$$

and

$$RHS(\triangle) = \frac{\phi(V(\Delta))}{\Phi(V(\Delta))}V'(\Delta)[M(\Delta)-V(\Delta)].$$

We know the shape of $LHS(\Delta)$, the marginal static cost curve, based on Figure 1. The graphs in Figures 2 through 4 illustrate solutions to the optimal damages problem. In each of the three graphs below, $LHS(\Delta)$, has the same general shape; crossing the horizontal axis at the point of the optimal static damages additur $\triangle_{st}$, and then lying on the horizontal axis for levels of the additur above the plaintiff's cost of litigation. The reason $LHS(\Delta)$ is flat for $\Delta \geq c_v$ is that all patentees sue and all infringers take care once $\Delta = c_v$, so the social cost of protection is invariant to additional increases in the additur. The different outcomes in the three graphs below can therefore be explained entirely by $RHS(\Delta)$, the marginal dynamic benefit. We will emphasize intuition here and leave proofs for the appendix.

The first graph (Figure 2: Case 1) shows the case where the social externality from the innovation is negative when suit is certain ($\Delta = c_v$) and positive in the neighborhood of the optimal static additur. Here the optimal damages additur is definitely less than the victim's litigation cost. For the particular solution illustrated, litigation is encouraged

relative to the static scenario, but not so much as to lead all victims of patent infringement to bring suit.

In the second graph (Figure 3: Case 2), the social externality from the patent is positive *and* the marginal dynamic benefit exceeds the marginal static cost when suit is certain. The high social externality shown in the graph corresponds to instances where the expected redisual innovation surplus substantially exceeds the sum of expected litigation and infringement avoidance costs and the innovation elasticity is relatively large. For this case, the optimal damages additur is found where the social externality is driven to zero, and therefore the private and social values of the patent are equal. The damages additur that achieves this outcome is

$$\triangle_{ex} = \bar{\triangle} \equiv S - E(v) - c_i + \frac{(1-q)W - x}{q}.$$

Note that

$$
\begin{aligned}
V(\bar{\triangle}) &= E(v) + q(\bar{\triangle} - c_v) \\
&= qS + (1-q)[W + E(v)] - x - q(c_v + c_i)
\end{aligned}
$$

which equals the social value from the innovation, $M(\bar{\triangle})$. Thus the optimal damages remedy in this case, $\bar{\triangle}$, induces the innovator to make the socially efficient investment decision by giving her the entire social value of her investment. For this outcome to hold, the marginal dynamic benefit - the elasticity of innovation multiplied by the innovation externality - must exceed the marginal static cost when suit is certain.

The third graph (Figure 4: Case 3) illustrates the more complicated case where the social externality is positive when suit is certain, but the marginal dynamic benefit is less than the marginal static cost of increasing damages when suit is certain. Now there are two potential solutions to the optimal damages problem. The lowest one is between the optimal static additur and c_v. The other solution is $\bar{\triangle}$. Because of the discontinuity in marginal static cost, it is impossible to say *a priori* which solution is globally preferable.

There is actually a fourth case to be considered, really a special case of the first, not shown but intuitively obvious, and that is where the social externality is negative even when evaluated at the optimal static additur. In this case the optimal additur is unambiguously less than the optimal static additur. The social value of the patent is sufficiently negative that it is socially preferable to discourage infringement lawsuits even more than in the static setting, in part to discourage the act of innovation itself.

To elaborate on this counterintuitive scenario, recall that in the static case, the optimal

14

damages amount is smaller than that for the usual tort. One would usually think, because the innovator does not internalize the full social benefit of innovation, we should give her more than in the static case to encourage investment. This is true when $\triangle_{st} < \triangle_{ex} < c_v$. But it could be true that $\triangle_{ex} < \triangle_{st}$, in which case the optimal damages award is not only smaller than that for the usual tort, but even smaller than that for the static case. The social planner would discourage rather than encourage innovation. The innovator does not care about the society's benefit from innovation and so may under-invest; at the same time, he does not care about the society's cost in protecting the patent (c_i and x), so he also has incentives to over-invest. When the external benefit is small and the external cost is large, the social planner may want to lower damages to discourage the investment. This can also be easily seen from this simple example:

Example: Suppose v is uniformly distributed on $[0, 10]$. $S = 1$, $W = 0$, $c_v = c_i = 15$, $x = 5$, $p = 1$, $q = 0$. So $\triangle_{st} = 9$. It is relatively cheaper to take care, society's loss in welfare is relatively low, and the litigation cost is high. Given innovation that has already occurred, the social planner finds it optimal to induce the injurer to take care provided that v is not too small ($v > 6$). The innovator will earn a rent of 3.2 and thus will choose to invest when cost is, say 3. However, the society has to bear an additional expected cost 2 of taking care, only to get an expected benefit of 0.4. So the social value of this innovation is 1.6, less than the innovation cost. Such an investment should be discouraged, rather than encouraged.
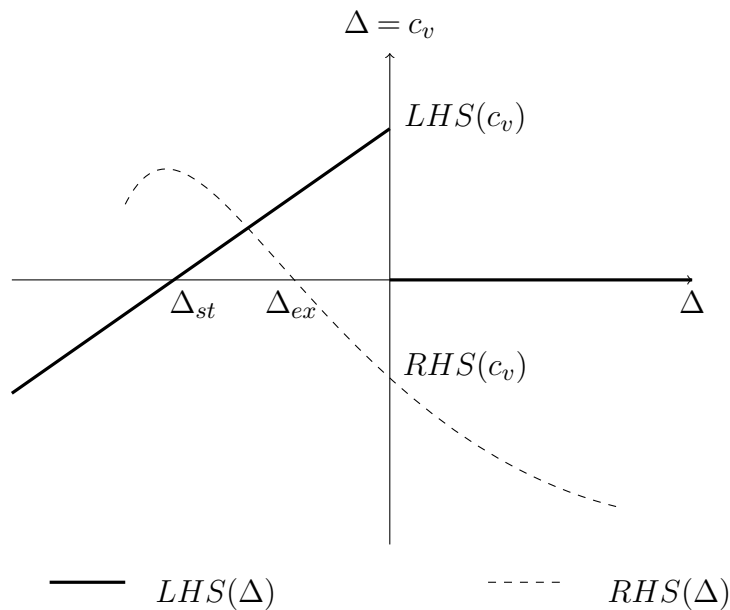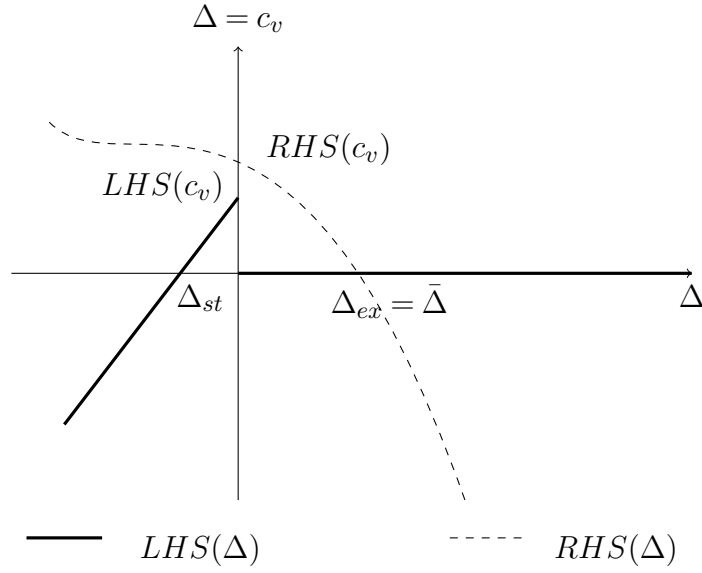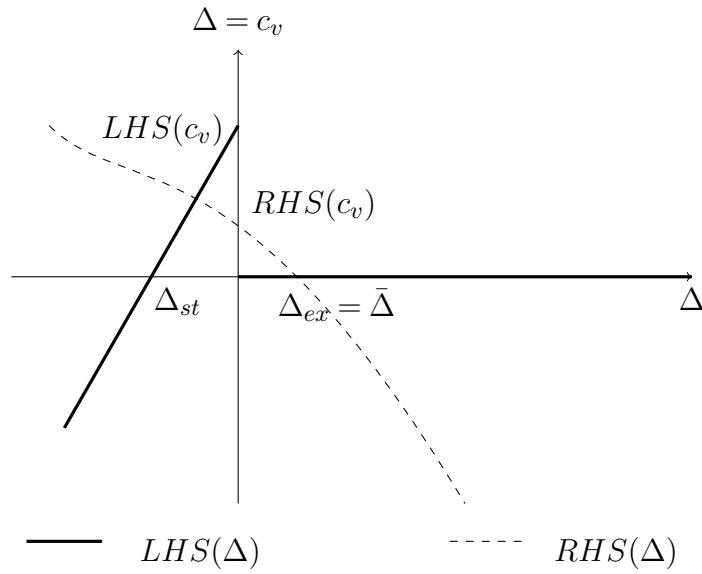

Figure 2: Case 1

15

Figure 3: Case 2

]

Figure 4: Case 3

The foregoing graphs show that the optimal damages additur will either balance static cost and dynamic benefit on the margin, or, if the social externality from innovation is sufficiently large, drive the marginal dynamic benefit to zero by equating the social and

16

private value of innovation. For a given private valuation, the social externality from innovation increases as the innovation surplus to consumers increases relative to the sum of the cost of taking care and the cost of litigation. The practical implication of these graphs is the following statement.

Proposition 4. *The optimal damages additur is always smaller than or equal to the greater of the plaintiff's cost of litigation and the amount that internalizes the social benefit from the innovation - i.e., $\triangle^* \max\{c_v, \bar{\triangle}\}$. Moreover, the optimal additur lies between the optimal static additur and the amount that internalizes the social benefit from the innovation - i.e. $\min\{\triangle_{st}, \triangle_{ex}\} \leq \triangle^* \leq \max\{\triangle_{st}, \triangle_{ex}\}$.*

4.3 Comparative Statics

The comparative statics are straightforward and intuitive. In the case where the patent has a high social value, and the optimal additur is unambiguously larger than c_v (Case 2), it is clear that the optimal damages additur increases with S and W, decreases with c_i and x, and is unaffected by changes in c_v. The reason follows from the function of the damages additur in this case, which is to internalize the social benefit from investment. Since c_v is already internalized in the innovator's investment decision, the optimal additur is independent of it. In the case where the patent has a comparatively low social value, and the optimal additur is unambiguously less than c_v (Case 1), the analysis is more tricky, because when $\triangle_{ex} < \triangle_{st}$, there may exists multiple local maximum of Π. However, if we restrict attention to the case where there exists a unique local maximum, the comparative statics results are similar, with the only difference observed in the effect of c_v.

Proposition 5. *If the optimal damages additur is less than c_v, then an increase in x, c_i will lead to a decrease in the optimal damages additur, and an increase in W will lead to an increase in the optimal additur. An increase in c_v has an ambiguous effect on the optimal additur.*

The effect of an increase in S on the optimal damages additur is also ambiguous because M' is decreasing in S and $M - V$ is increasing in S. An upward shift of the distribution of the cost function in general has an ambiguous effect. When $\triangle_{st} < \triangle^* < \triangle_{ex}$ - that is, when the social planner feels the need to encourage investment - an upward shift of the cost function leads to an increase in optimal $\triangle$. When $\triangle_{ex} < \triangle^* < \triangle_{st}$ - that is, when the social planner feels the need to discourage investment-an upward shift of the cost function leads to a decrease in optimal $\triangle$.

5   Extensions

5.1   Variable Cost of Care

In this section, we extend the model to incorporate the case where the injurer's cost of taking care varies. Specifically, we assume that x is distributed according to some distribution with pdf g and cdf G, with support $x \in [0, \infty)$. Introducing variable care changes the model mainly be potentially amplifying the effect of an increase in the damages additur on the degree to which potential infringers take care. Still, we hope to show that most of the conclusions of the previous part remain reliable. We have relegated some details to the appendix.

In this case, the injurer will take care only if (1) the innovator will sue and (2) $x < (p-q)(v+\triangle+c_i)$. So conditional on the innovator credibly threatening to sue, the infringer will take care with probability $G((p-q)(v+\triangle+c_i))$.

The general optimality condition, balancing the marginal static cost against dynamic benefit, still holds, with the same intuition. Letting $\hat{V}$ denote the private value of the patent,

$$
\begin{aligned}
\hat{V}(\triangle) \;=\; & (1-p)\int_0^{c_v-\triangle} vh(v)dv + \int_{c_v-\triangle}^{\infty} vh(v)dv \\
& + q(\triangle - c_v)\int_{c_v-\triangle}^{\infty} G((p-q)(v+\triangle+c_i))h(v)dv \\
& + p(\triangle - c_v)\int_{c_v-\triangle}^{\infty} [1 - G((p-q)(v+\triangle+c_i))]h(v)dv
\end{aligned}
$$

When $\triangle < c_v$, this is increasing in $\triangle$. When $\triangle \geq c_v$,

$$
\begin{aligned}
\hat{V}'(\triangle) \;=\; & q + (p-q)\int_0^{\infty} g((p-q)(v+\triangle+c_i))h(v)dv \\
& \times \left\{ \frac{\int_0^{\infty}[1-G((p-q)(v+\triangle+c_i))]\,h(v)dv}{\int_0^{\infty} g((p-q)(v+\triangle+c_i))h(v)dv} - (p-q)(\triangle - c_v) \right\}.
\end{aligned}
$$

Unlike in the baseline model, the patent's private value may not be strictly increasing in the damages additur for values of the additur greater than the plaintiff's cost of litigation.[17] This is explained by two revenue effects: (a) the patentee gets more revenue from every case of infringement (direct effect), and (b) some potential infringers switch from not taking care to taking care (indirect effect), which reduces revenue. If $\triangle$ gets sufficiently large and

---

[17]In the appendix, we provide an example where $\hat{V}'(\triangle)$ eventually becomes negative.

if G has a sufficiently fat tail, the indirect revenue effect caused by switching may dominate the direct effect (see appendix for an example). It follows that, unlike the baseline model, there may be a privately optimal damages additur that maximizes the value of the patent to the innovator.

The social value of the patent is also more complicated than in the baseline model. The patent's social value is now given by

$$
\begin{aligned}
\hat{M}(\triangle) &= (1-p)[E(v)+W]+p(S-c_v-c_i) \\
&+ \int_0^\infty G((p-q)(v+\triangle+c_i))(p-q)[(v+W)-(S-c_v-c_i)]h(v)dv \\
&- \int_0^\infty \left[\int_0^{(p-q)(v+\triangle+c_i)} xg(x)dx \right] h(v)dv
\end{aligned}
$$

This can be interpreted as the social value when potential infringers do not take care (first line) plus the expected welfare increment that results from being induced to take care. The first derivative is

$$
\begin{aligned}
\hat{M}'(\triangle) &= h(c_v-\triangle)\{(p-q)(c_v-\triangle+W-S)G((p-q)(c_v+c_i)) \\
&- \int_0^{(p-q)(c_v+c_i)} xg(x)dx - (c_v+c_i)[p-(p-q)G((p-q)(c_v+c_i))]\} \\
&+(p-q)^2 \int_{c_v-\triangle}^\infty \left[(c_v-\triangle+W-S)g((p-q)(v+\triangle+c_i)) \right] h(v)dv
\end{aligned}
$$

Suppose the static externality is zero, that is, $W = S = 0$, which enables comparison with Polinsky and Rubinfeld (1988), Hylton (1990), and Polinsky and Shavell (2013). In this case, note that setting $\triangle = c_v$ may be a local solution to the first order condition. But it is unlikely to be a global solution, in contrast to the torts literature, because the derivative is likely negative as soon as you reduce $\triangle$ below c_v. Recall that unlike the torts literature, the infringer observes v before deciding whether to take care, which limits the productivity of care in comparison to the torts context.[18]

It is impossible to get a closed form solution for the optimal additur in this more complicated version, but we can show that the main results of the baseline model still hold. The first term of the first derivative of $\hat{M}$ crosses 0 only once, at

$$
\triangle_1 = c_v+W-S- \frac{p-(p-q)G((p-q)(c_v+c_i))}{(p-q)G((p-q)(c_v+c_i))}(c_v+c_i)
$$

---

[18]As noted earlier, if we assume instead that the infringer observes only $E(v)$ and not v, then care would be more productive and the result of the tort damages literature would emerge as a special case.

as $(p-q)(c_v - \triangle + W - S)G((p-q)(c_v + c_i)) - \int_0^{(p-q)(c_v+c_i)} xg(x)dx - (c_v + c_i)[p - (p - q)G((p-q)(c_v+c_i))])$ is decreasing in $\triangle$. So is the second term, and it crosses 0 at

$$\triangle_2 = c_v + W - S$$

Clearly $\triangle_1 < \triangle_2$. Though we don't know if the solution to $\hat{M}'(\triangle) = 0$ is unique, we do know that any solution should lie between $\triangle_1$ and $\triangle_2$. Moreover, the social value of patent is always decreasing in $\triangle$ beyond c_v. So the static optimal damage $\hat{\triangle}_{st}$ must be smaller than c_v. Finally, the social externality of the patent $\hat{V}$-$\hat{M}$ is always decreasing, as in the baseline model (we refer interested readers to the appendix for the proof).

From here it is not difficult to compare the results of this version with the baseline model. Returning to Figures 2 through 4, the graph now changes so that the static cost curve, $LHS(\triangle)$, is upward sloping even after the point $\triangle = c_v$. If the shape of the dynamic cost curve, $RHS(\triangle)$, is as regular as shown, then the optimal additur will be less than the greater of c_v and the amount that internalizes the social benefit from innovation, as in the baseline model.

5.2 Optimal Injunction

Much of the preceding analysis can be applied to the determination of an optimal injunction, which we consider here. The timeline of the game is very similar to that of the optimal damages case. The only difference is that at stage 2, if the innovator sues, an injunction will be issued. We use $\theta \in [0, 1]$ to denote the *scope of the injunction* (i.e, extent or probability). If a injunction of scope θ is issued, the innovator gets θv, the injurer pays 0, the society gets $\theta S + (1 - \theta)W$. For example, θ could be interpreted simply as the duration of the injunction. If the injunction has the full duration, $\theta = 1$, then the innovator obtains the full profits promised by the patent grant, and consumers receive none of the benefits from infringement. We assume that an injunction issues immediately and prevents the infringement from occurring.

To simplify, we will use a modified version of Assumption 1 to ensure that the injurer will take care provided that the victim is going to sue when infringement occurs.

Assumption 3. $(p-q)c_i > x$.

Then the innovator will sue if and only if

$$\theta v > c_v.$$

For any $\theta > 0$, the above equation can be rewritten as,[19]

$$v > \frac{c_v}{\theta}.$$

Given any θ, the private value of the patent to the innovator is

$$
\begin{aligned}
V(\theta) &= (1-p)H(\frac{c_v}{\theta})E(v \mid v \leq \frac{c_v}{\theta}) \\
&+[1-H(\frac{c_v}{\theta})]\left\{[1-q(1-\theta)]E(v \mid v > \frac{c_v}{\theta})-qc_v\right\}
\end{aligned}
$$

and its first derivative is

$$
V'(\theta) = h(\frac{c_v}{\theta})(\frac{c_v}{\theta^2})(p-q)\frac{c_v}{\theta} + q\int_{\frac{c_v}{\theta}}^{\infty} vh(v)dv > 0
$$

Hence, V is increasing in θ. The innovator will only invest when $V(\theta) > c$, so the probability of investment is $\Phi(V(\theta))$.

The social value of the patent is

$$
\begin{aligned}
M(\theta) &= H(\frac{c_v}{\theta})\left\{pS+(1-p)[W+E(v \mid v \leq \frac{c_v}{\theta})]\right\} \\
&+[1-H(\frac{c_v}{\theta})]\left\{q(1-\theta)S+[1-q(1-\theta)][W+E(v \mid v > \frac{c_v}{\theta})]\right\} \\
&-[1-H(\frac{c_v}{\theta})][x+q(c_v+c_i)]
\end{aligned}
$$

and its first derivative is

$$
\begin{aligned}
M'(\theta) &= h(\frac{c_v}{\theta})(\frac{c_v}{\theta^2})\left\{[p-q(1-\theta)](\frac{c_v}{\theta}+W-S)-x-q(c_v+c_i)\right\} \\
&+q[1-H(\frac{c_v}{\theta})]\left[W-S+E(v \mid v > \frac{c_v}{\theta})\right].
\end{aligned}
$$

The above equation is always positive as θ goes to 0, and $\frac{M'(\theta)}{h(\frac{c_v}{\theta})}$ is decreasing in θ. So it either crosses 0 once or remains positive till $\theta = 1$. Thus we know either M is always increasing, or first increases to a maximum and then decreases. The result slightly differs from the optimal damages case because we cannot let θ go to infinity as $\triangle$. So at full injunction, the society's marginal benefit of protection may still be positive.

---

[19]It is not hard to verify that the first order derivative is positive at $\theta = 0$, so $\theta = 0$ is never optimal.

The social externalilty from the innovation is

$$M(\theta) - V(\theta) = H(\frac{c_v}{\theta})[pS + (1-p)W]$$
$$+[1 - H(\frac{c_v}{\theta})]\{q(1-\theta)(S-W) + W - x - qc_i\}.$$

Again, we derive the first derivative of the above equation

$$M'(\theta) - V'(\theta) = (\frac{c_v}{\theta^2})h(\frac{c_v}{\theta})\{(W-S)[p - q(1-\theta)] - x - qc_i\}$$
$$-q(S-W)[1 - H(\frac{c_v}{\theta})]$$
$$< 0.$$

So we know $M - V$ is decreasing in θ.

From this point, much of the preceding analysis can be replicated. The social optimality condition balances marginal static cost and dynamic benefit

$$-M'(\theta) = \frac{\phi(V(\theta))}{\Phi(V(\theta))}V'(\theta)[M(\theta) - V(\theta)].$$

Again, the marginal dynamic benefit is the product of the innovation elasticity and the social externality from the patent. If it is positive, the scope of the injunction should be increased to the point that it is equal to the marginal static cost of intellectual property protection. Obviously, a corner solution of full scope will be observed when the marginal dynamic benefit exceeds the marginal static cost at the full protection level.

We conclude the section by showing that a combination of damages and an injunction may strictly dominate using either damages or injunction alone. Suppose v is uniformly distributed on $[0, 10]$, $W = 4$, $S = 10$, $p = 1$, $q = 0.5$, $x = 0.25$, $c_v = c_i = 1$. To better illustrate the idea, we assume the investment cost is fixed at $c = 3.5$. If the court uses only damages, then the optimal damages additur is $\triangle^* = -2$, at which the expected social welfare is 5.8. Similarly, if the court uses only an injunction, the optimal injunction is 0.61, at which the expected social welfare is 5.33. The following rule, full injunction if $v > 6$ and $\triangle = -2$ when $v < 6$, gives the innovator a value of 3.9. So the innovator will invest. It also gives a social welfare of 6.15. The intuition for this result is simple. When v is large, monopolization is efficient for the society, so we should choose injunction. On the contrary, when v is small, monopolization is bad for the society, the court only protects the patent for the investment incentive, so it is more efficient to use damage to compensate the innovator and keep a competitive market.

6 Discussion

While the literature on optimal patent scope and term has posed the social planner's objective as minimizing the deadweight loss for a given level of reward for innovation, our approach has been to set up a simple objective of maximizing the social gain from innovation net of the cost of investment. Patent protection imposes a static monopolization cost but also induces the investment that creates the very market the innovator will monopolize if patent protection is sufficiently strong. The only instrument of interest here is the level of damages for patent infringement. As the level of damages increases, the effective degree of patent protection also increases. Our model delivers an intuitive optimality condition that balances, on the margin, the static cost of intellectual property protection against the "dynamic" benefit from encouraging innovation. Moreover, this model yields a specific formulation for the dynamic benefit: the product of the innovation elasticity and the social externality from the patent.

We find that the optimal damages award for patent infringement exceeds the sum of lost profits and the plaintiff's litigation cost if the social value of the patent - that is, the expected social surplus net of the expected costs of litigation and of taking care to avoid infringement - is sufficiently high. This is interesting for several reasons. First, in comparison to the literature on tort damages, it shows that optimal damages for infringement of intellectual property may exceed the upper bound on optimal damages for ordinary torts (negligence cases), which is equal to the sum of compensatory damages and the plaintiff's litigation cost.

The upper bound on optimal damages for patent infringement derived here is the sum of lost profits and the amount that internalizes to the innovator the expected social gain from her investment in innovation. The baseline model examined (in which the cost of care is sufficiently low that a credible threat to sue for infringement always induces care), delivers a simple closed-form solution in which the upper bound on optimal damages is the sum of lost profits and the expected consumer surplus from the patent net of the expected litigation and precautionary costs borne by the infringer. This latter amount can easily exceed the plaintiff's litigation cost if the value of the innovation is high.

For the law, the results here provide some support for the Supreme Court's decision in *Octane Fitness*, which overturned rules adopted by the Federal Circuit Court of Appeals making it difficult for a prevailing litigant to obtain an award of attorney's fees in a patent infringement lawsuit. The previous rule, from *Brooks Furniture v. Dutailier*,[20] required an objectively baseless lawsuit brought in subjectively bad faith in order to receive attorney's

---

[20]393 F.3d 1378 (Fed. Cir. 2005).

fees. The Court held instead that an unusually weak legal case on the opposing side, based on the facts or the law, could justify attorney's fees. More importantly, the results here lend support to the broader argument that the legal standard for enhanced damages should be relaxed in a similar fashion.[21]

This model suggests that the patent's social value should play a role in determining whether supercompensatory patent infringement awards - damages exceeding lost profits - are appropriate. In particular, under this model, supercompensatory damages may be optimal in the case of infringement of a patent with a high social value. An example might be a patent on a life-saving medical innovation. For such innovation, the externality to society is large, even under monopoly. Bessen and Meurer (2008) find that the private value for pharmaceutical patents generally exceeds expected litigation costs by a substantial margin, which implies that the many such patents have sufficiently high social value to justify awarding infringement damages greater than lost profits. On the other hand, for areas where the social externality from innovation is likely to be low and infringement difficult to determine (meaning high costs of litigation and taking care to avoid infringement), this model suggests that damages less than lost profits may be optimal.[22] In many of these cases the patentee will lose his infringement claim anyway.

Treble damages are permitted under the Patent Act (Section 285) in cases of willful infringement. Willful infringement corresponds in our model to instances where the cost of taking care is simply the profit forgone by not infringing the patent, and the probability of infringement is nearly one when the firm does not take care and nearly zero when the firm does take care. If infringement induces follow-on entrants, the profit forgone by not infringing the patent will be modest. Our baseline model easily includes this intentional infringement scenario. Thus, when the damages additur that internalizes the social value of innovation is greater than twice lost profits, the optimal damages award will exceed the treble damages cap.

---

[21]For a discussion of the issues, see Halo Electronics, Inc. v. Pulse Electronics, Inc., 780 F.3d 1357 (Fed. Cir. 2015). In rejecting Halo's request for a full circuit review of a panel decision rejecting its motion for enhanced damages, the court noted that the patent statute by its terms gives enormous discretion to judges on the matter of enhanced damages.

[22]It should be clear that if infringement offers greater benefits than assumed in this model, the case for reducing damages only strengthens. The model can be extended to incorporate such concerns. For example, if infringement enhances additional innovation, then it would be desirable to encourage more infringement. Such a case might be presented where innovation is both sequential and complementary, as in Bessen & Maskin (2009).

7 Conclusion

Patent infringement remedies may provide the best measure of the degree of intellectual property protection offered by the legal system. A government can award a patentee an infinite term and all-encompassing scope, but if infringement remedies are feeble, the effective level of protection will be quite low.

We derive an intuitively appealing condition that governs optimal damages, balancing on the margin the static monopolization cost against the dynamic innovation benefit. If the social value of the patent is sufficiently high, the optimal damages award may exceed the lost profits standard, up to a limit that internalizes the net external benefit from the innovation.

Appendix

Optimal Damage

Recall the first derivative of $\Pi(\triangle)$ is

$$\Pi'(\triangle) \;=\; \phi(V(\triangle))V'(\triangle)[M(\triangle)-V(\triangle)]+\Phi(V(\triangle))M'(\triangle). \tag{1}$$

As in the text, we use $\triangle_{st}$ to denote the solution to

$$M'(\triangle)=0,$$

and use $\triangle_{ex}$ to denote the solution to

$$M(\triangle)-V(\triangle)=0.$$

We have already show that both solutions are unique. Moreover, $M'(\triangle)$ is strictly negative for $\triangle < \triangle_{st}$ and is positive otherwise; $M(\triangle)-V(\triangle)$ is strictly positive for $\triangle < \triangle_{ex}$ and negative otherwise. So if $\triangle_{ex}=\triangle_{st}$, the optimal damage is trivially $\triangle_{st}$. Otherwise, a solution to the first order condition always exists and must lie between $\triangle_{st}$ and $\triangle_{ex}$.

To find the optimal damage $\triangle^*$, we consider three cases.

Case 1: $qS+(1-q)W-qE(v) < x+q(c_v+c_i)$.

The above inequality implies that

$$M(c_v)-V(c_v)<0,$$

or equivalently, $\triangle_{ex} < c_v$.

From earlier analysis, we also know that , $\triangle_{st}$ is always smaller than c_v. So $\Pi'(\triangle)$ is negative for all $\triangle > c_v$. The optimal damage $\triangle^*$ must be smaller than c_v. This is the case represented by Figure 2.

In what follows, we show when $\triangle_{st} < \triangle_{ex}$, there exists a unique solution to equation 1, which maximizes Π. We also illustrate why this may not be true when $\triangle_{st} > \triangle_{ex}$.

For illustrative purpose, for $\triangle < c_v$, let

$$
\begin{aligned}
\mathcal{L}(\triangle) \;&=\; \frac{\Pi'(\triangle)}{\Phi(V(\triangle))h(c_v-\triangle)} \\
&=\; \frac{\frac{\phi(V(\triangle))}{\Phi(V(\triangle))}V'(\triangle)[M(\triangle)-V(\triangle)]}{h(c_v-\triangle)}+\frac{M'(\triangle)}{h(c_v-\triangle)}.
\end{aligned} \tag{2}
$$

As any solution to $\Pi'(\triangle) = 0$ must lie in $(\triangle_{st}, \triangle_{ex})$. To show $\Pi'(\triangle) = 0$ has a unique solution, it suffices to show the above equation is strictly decreasing when $\triangle < \triangle_{ex}$. Recall that in section 4.2, we defined the following notations:

$$LHS(\Delta) = -M'(\Delta),$$

and

$$RHS(\Delta) = \frac{\phi(V(\Delta))}{\Phi(V(\Delta))} V'(\Delta)[M(\Delta) - V(\Delta)].$$

So equation 2 can be rewritten as

$$\mathcal{L}(\triangle) \;=\; \frac{RHS(\Delta)}{h(c_v - \Delta)} - \frac{LHS(\Delta)}{h(c_v - \Delta)}$$

In the above equation,

$$
\begin{aligned}
\frac{RHS(\Delta)}{h(\Delta - c_v)} &= \frac{\frac{\phi(V(\Delta))}{\Phi(V(\Delta))} V'(\Delta)[M(\Delta) - V(\Delta)]}{h(c_v - \Delta)} \\
&= \frac{\phi(V(\Delta))}{\Phi(V(\Delta))} \times [M(\Delta) - V(\Delta)] \\
&\quad \times \left[(c_v - \Delta)(p - q) + q\frac{1 - H(c_v - \Delta)}{h(c_v - \Delta)} \right]
\end{aligned}
$$

In the above equation, both $\frac{\phi(V(\Delta))}{\Phi(V(\Delta))}$ and $(c_v - \Delta)(p-q) + q\frac{1-H(c_v-\Delta)}{h(c_v-\Delta)}$ are decreasing in $\triangle$ by assumption 2 $.M(\Delta) - V(\Delta)$ is strictly decreasing. Finally, all three terms are positive when $\triangle_{st} < \triangle < \triangle_{ex} < c_v$. Then $\frac{RHS(\Delta)}{h(\Delta - c_v)}$ is decreasing in $\triangle$ when $\triangle < c_v$.

In addition, we have Now consider

$$
\begin{aligned}
\frac{-LHS(\Delta)}{h(c_v - \Delta)} &= \frac{M'(\Delta)}{h(c_v - \Delta)} \\
&= (p - q)[(c_v - \Delta) + W - S] - x - q(c_v + c_i))
\end{aligned}
$$

which is clearly strictly decreasing in $\triangle$. So $\mathcal{L}$ is decreasing on $(-\infty, \triangle_{ex})$ as desired. Indeed, the foregoing arguments imply that $\mathcal{L}$ is decreasing when $\triangle < \min\{c_v, \triangle_{ex}\}$. We will use this implication in the following parts. Thus the solution to the first order condition is unique.

Use $\triangle^*$ to denote the solution. The last step is to check the second order condition:

$$\Pi''(\triangle^*) < 0.$$

27

By foregoing arguments, we have

$$\frac{\partial \mathcal{L}^*}{\partial \triangle^*} = \frac{\Pi''(\triangle^*)\Phi(V(\triangle^*))h(c_v - \triangle^*) + \Pi'(\triangle^*)\frac{\partial \Phi(V(\triangle)))h(c_v - \triangle)}{\partial \triangle}|_{\triangle = \triangle^*}}{[\Phi(V(\triangle^*))h(c_v - \triangle^*)]^2}$$
$$< 0$$

and $\Pi'(\triangle^*) = 0$. So it directly follows that $\Pi''(\triangle^*) < 0$. So $\triangle^*$ is indeed a global maximizer of Π.

The previous analysis does not apply to the case where $\triangle_{ex} < \triangle_{st}$ because although $M(\triangle) - V(\triangle)$ is still decreasing, it is negative on $(\triangle_{ex}, \triangle_{st})$. So we can no longer conclude that $\mathcal{L}$ is always strictly decreasing on $(\triangle_{ex}, \triangle_{st})$.

Case 2& Case 3: Suppose $qS + (1-q)W - qE(v) \geq x + q(c_v + c_i)$, then we have

$$\triangle_{ex} = \bar{\triangle} \equiv S - E(v) - c_i + \frac{(1-q)W - x}{q} \geq c_v.$$

Note that the above inequality implies that $\triangle_{st} < c_v \leq \triangle_{ex}$. It is easy to show that $\Pi'(\triangle) > 0$ for $c_v < \triangle < \bar{\triangle}$, $\Pi'(\triangle) < 0$ for $\triangle > \bar{\triangle}$. So $\bar{\triangle}$ is the only local maximizer on $[c_v, \infty)$.

Then we need to consider two subcases.

Case 2:$qS + (1-q)W - qE(v) \geq x + q(c_v + c_i)$ and $RHS(c_v) > LHS(c_v)$.

Since $\mathcal{L}$ is decreasing when $\triangle < \min\{c_v, \triangle_{ex}\} = c_v$, we then have $\mathcal{L}(\triangle) > 0 \iff \Pi'(\triangle) > 0$ for all $\triangle \leq c_v$. So there is no local maximizer on $(-\infty, c_v]$. Then the optimal damage must be

$$\bar{\triangle} = S - c_i + \frac{(1-q)W - x}{q}.$$

This is the case represented by Figure 2.

Case 3: $qS + (1-q)W - x \geq q(c_v + c_i)$ and $RHS(c_v) \leq LHS(c_v)$.

Then we have $\Pi'(c_v) = h(0)\mathcal{L}(c_v) \leq 0$. Moreover, $\frac{LHS(\triangle)}{h(c_v - \triangle)} \to -\infty$ and $\frac{RHS(\triangle)}{h(c_v - \triangle)} \to \infty$ as $\triangle \to -\infty$. Then there exists y such that $\Pi'(\triangle) = h(c_v - \triangle)\mathcal{L}(\triangle) > 0$ for all $\triangle < y$. $\mathcal{L}$ is strictly decreasing when $\triangle < c_v$ and continuous, so there exists a unique $\tilde{\triangle}$ which is smaller than c_v and solves $\Pi'(\triangle) = 0$. It is easy to check that the corresponding second derivative

$$\Pi''(\tilde{\triangle}) = \mathcal{L}'(\tilde{\triangle})h(c_v - \triangle)\Phi(V(\triangle) + \mathcal{L}(\tilde{\triangle})[h(c_v - \triangle)\Phi(V(\triangle)]'$$
$$< 0.$$

So $\tilde{\triangle}$ is indeed a local maximizer. However, as shown above, $\bar{\triangle} > c_v$ is also a local maximizer. In this case, we need to compare $M(\triangle_a)$ and $M(\triangle_b)$ to find out the damage amount that maximizes the society's benefit. This is the case illustrated by Figure 4.

Comparative Statics (Proposition 5)

Here we derive comparative statics for Case 1 $(qS + (1-q)W - qE(v) < x + q(c_v + c_i))$, the optimal damage is unambiguously smaller than c_v), assuming unique local maximum.

Use $\triangle^*$ to denote the optimal damage. Then it follows that $\mathcal{L}^* \equiv \mathcal{L}(\triangle^*) = 0$ and $\frac{\partial \mathcal{L}^*}{\partial \triangle^*} < 0$. Moreover,

$$\frac{\partial \mathcal{L}^*}{\partial x} = \frac{\partial \left\{ \frac{\phi(V(\triangle^*))}{\Phi(V(\triangle^*))} \frac{V'(\triangle^*)}{h(c_v - \triangle^*)} [M(\triangle^*) - V(\triangle^*] + \frac{M'(\triangle^*)}{h(c_v - \triangle^*)} \right\}}{\partial x}.$$

In the above equation, $\frac{M(\triangle^*) - V(\triangle^*)}{h(c_v - \triangle^*)}$ and $\frac{M'(\triangle^*)}{h(c_v - \triangle^*)}$ are decreasing in x, $\frac{\phi(V(\triangle^*))}{\Phi(V(\triangle^*))} V'(\triangle^*)$ is constant in x and positive. So $\frac{\partial L^*}{\partial x} < 0$. Then we can use the implicit function theorem and conclude

$$\frac{\partial \triangle^*}{\partial x} = -\frac{\frac{\partial \mathcal{L}^*}{\partial x}}{\frac{\partial \mathcal{L}^*}{\partial \triangle^*}} < 0$$

That is, the optimal damage decreases when x increases.

Similarly, one can show $\frac{\partial \mathcal{L}^*}{\partial W} > 0$ and $\frac{\partial \mathcal{L}^*}{\partial c_i} < 0$. And thus $\frac{\partial \triangle^*}{\partial W} > 0$ and $\frac{\partial \triangle^*}{\partial c_i} < 0$. That is, the optimal damage increases when W or/and c_i increases.

Now we consider the effect of change in c_v. Note that

$$\begin{aligned} V(\triangle^*) &= (1-p)H(c_v - \triangle^*)E(v \mid v \le c_v - \triangle^*) \\ &\quad + [1 - H(c_v - \triangle^*)][E(v \mid v > c_v - \triangle^*) + q(\triangle^* - c_v)]. \\ &= (1-p)E(v) + [1 - H(c_v - \triangle^*)][pE(v \mid v > c_v - \triangle^*) + q(\triangle^* - c_v)] \end{aligned}$$

is decreasing in c_v. So in $\mathcal{L}^*$, $\frac{\phi(V(\triangle))}{\Phi(V(\triangle))}$ is increasing in c_v. Moreover,

$$\frac{V'(\triangle^*)}{h(c_v - \triangle^*)} = (p-q)(c_v - \triangle^*) + q \frac{1 - H(c_v - \triangle^*)}{h(c_v - \triangle^*)}$$

is increasing in c_v, $M(\triangle^*) - V(\triangle^*)$ is increasing in c_v and is positive if $\triangle_{ex} > \triangle_{st}$. The last term $(p-q)[S - W - c_v + \triangle^*] + x + q(c_v + c_i)$ is certainly increasing in c_v when $p > 2q$, otherwise ambiguous. So when $\triangle_{ex} > \triangle_{st}$ and $p > 2q$, we have $\frac{\partial L^*}{\partial c_v} > 0$ which implies an

increase in c_v results in a decrease in $\triangle^*$. This refer to the circumstances where innovation needs to be encouraged and taking care is relatively effective. Otherwise, the effect of change of c_v on $\triangle^*$ is ambiguous.

Lastly, we investigate how a shift of cost distribution affects $\triangle^*$. To be precise, we replace ϕ with ϕ_ε such that

$$
\begin{aligned}
\phi_\varepsilon(k) &= \phi(k - \varepsilon) \\
\Phi_\varepsilon(k) &= \Phi(k - \varepsilon).
\end{aligned}
$$

A positive ε represents an upward shift and a negative ε represents an downward shift of the cost distribution . The new first derivative becomes

$$
\Pi'_\varepsilon(\triangle) = \phi_\varepsilon(V(\triangle))V'(\triangle)[M(\triangle) - V(\triangle)] + \Phi_\varepsilon(V(\triangle))M'(\triangle)
$$

And we denote

$$
\begin{aligned}
\mathcal{L}_\varepsilon(\triangle) &= \frac{\Pi'_\varepsilon(\triangle)}{h(c_v - \triangle)} \\
&= \frac{\frac{\phi_\varepsilon(V(\triangle))}{\Phi_\varepsilon(V(\triangle))}V'(\triangle)[M(\triangle) - V(\triangle)] + M'(\triangle)}{h(c_v - \triangle)} \\
&= \frac{\frac{\phi(V(\triangle) - \varepsilon)}{\Phi(V(\triangle) - \varepsilon)}V'(\triangle)[M(\triangle) - V(\triangle)] + M'(\triangle)}{h(c_v - \triangle)}
\end{aligned}
$$

When $\varepsilon > 0$, $\frac{\phi(V(\triangle^*) - \varepsilon)}{\Phi(V(\triangle^*) - \varepsilon)} > \frac{\phi(V(\triangle^*))}{\Phi(V(\triangle^*))}$. If $\triangle_{ex} > \triangle_{st}$, then $M(\triangle^*) - V(\triangle^*) > 0$, which implies $\mathcal{L}_\varepsilon(\triangle^*) > 0 \Rightarrow \Pi'_\varepsilon(\triangle^*) > 0$ at $\triangle^*$. So the new optimal damage must be greater than $\triangle^*$. Otherwise, the new optimal damage is smaller than $\triangle^*$.

So an upward shift of ϕ result in an increase of optimal damage when $\triangle_{ex} > \triangle_{st}$, and result in a decrease of optimal damage otherwise. The opposite conclusion holds for a downward shift of ϕ.

Variable Cost of Care

The private value of the patent is

$$\hat{V}(\triangle) = (1-p)\int_0^{c_v-\triangle} vh(v)dv + \int_{c_v-\triangle}^{\infty} vh(v)dv$$

$$+q(\triangle - c_v)\int_{c_v-\triangle}^{\infty} G((p-q)(v+\triangle+c_i))h(v)dv$$

$$+p(\triangle - c_v)\int_{c_v-\triangle}^{\infty} [1-G((p-q)(v+\triangle+c_i)]h(v)dv$$

which can be rewritten

$$\hat{V}(\triangle) = (1-p)E(v) + \int_{c_v-\triangle}^{\infty} \{pv - [p-(p-q)G((p-q)(v+\triangle+c_i))](c_v - \triangle)\}h(v)dv.$$

Note that the innovator will sure only if $v > c_v - \triangle$. So when $\triangle < c_v$, in the second term, $pv - [p-(p-q)G((p-q)(v+\triangle+c_i))](c_v - \triangle) > pv + p(\triangle - c_v) > 0$. Moreover, both $p - (p-q)G((p-q)(v+\triangle+c_i))$ and $c_v - \triangle$ are positive and decreasing in $\triangle$. So $pv - [p-(p-q)G((p-q)(v+\triangle+c_i))](c_v - \triangle)$ is increasing in $\triangle$. And thus the second term is also increasing in $\triangle$. Thus, when $\triangle < c_v$, the private value of the patent is increasing in $\triangle$. As we note in the text, the private value is not necessarily increasing for $\triangle \geq c_v$. For example, suppose, $q = 0$, $p = 1$ and $g \sim \exp(\lambda_g)$, then $\frac{g(x)}{1-G(x)} = \lambda_g$ for all x. In this case, when $\triangle \geq c_v$,

$$\hat{V}'(\triangle) = \int_0^{\infty} g(v+\triangle+c_i)h(v)dv \times \left\{ \frac{\int_0^{\infty}[1-G(v+\triangle+c_i)]h(v)dv}{\int_0^{\infty} g(v+\triangle+c_i)h(v)dv} - (\triangle - c_v) \right\}$$

$$= \int_0^{\infty} g(v+\triangle+c_i)h(v)dv \left[\frac{1}{\lambda_g} - (\triangle - c_v) \right]$$

which turns negative as $\triangle \to \infty$.

When $\triangle \geq c_v$, the social value equation becomes

$$\hat{M}(\triangle) = (1-p)[E(v)+W] + p(S - c_v - c_i)$$

$$+ \int_0^{\infty} G((p-q)(v+\triangle+c_i))(p-q)(v+W+c_v+c_i-S)h(v)dv$$

$$- \int_0^{\infty} \left[\int_0^{(p-q)(v+\triangle+c_i)} xg(x)dx \right] h(v)dv.$$

Then

$$\hat{M}'(\triangle) = (p-q)^2 \int_0^\infty g((p-q)(v+\triangle+c_i))(W-S+c_v-\triangle)h(v)dv$$
$$< 0$$

for all $\triangle > c_v$.

The social externality is

$$\hat{M}(\triangle)-\hat{V}(\triangle) = \int_{c_v-\triangle}^\infty G((p-q)(v+\triangle+c_i))[(1-q)W+q(S-v-\triangle-c_i)]h(v)dv$$
$$+\int_{c_v-\triangle}^\infty [1-G((p-q)(v+\triangle+c_i))][(1-p)W+p(S-v-\triangle-c_i)]h(v)dv$$
$$-\int_{c_v-\triangle}^\infty \left[\int_0^{(p-q)(v+\triangle+c_i)} xg(x)dx\right]h(v)dv + H(c_v-\triangle)[pS+(1-p)W]$$

The first derivative of $M-V$ is

$$\hat{M}'(\triangle)-\hat{V}'(\triangle) = h(c_v-\triangle)G((p-q)(c_v+c_i))(p-q)(W-S)$$
$$+h(c_v-\triangle)(c_v+c_i)[G((p-q)(c_v+c_i))(p-q)-p]$$
$$-h(c_v-\triangle)\left[\int_0^{(p-q)(c_v+c_i)} xg(x)dx\right]$$
$$+\int_{c_v-\triangle}^\infty (p-q)^2 g((p-q)(v+\triangle+c_i))(W-S)h(v)dv$$
$$-q\int_{c_v-\triangle}^\infty G((p-q)(v+\triangle+c_i))h(v)dv$$
$$-p\int_{c_v-\triangle}^\infty [1-G((p-q)(v+\triangle+c_i))]h(v)dv$$

Note that $W-S<0$, and $G((p-q)(c_v+c_i))(p-q)-p<0$, so clearly all terms in the above equation are negative. Thus $M'-V'$ is always negative and $M-V$ is always decreasing.

Optimal Injunction
The society's objective function is

$$\Pi(\theta) = \Phi(V(\theta))M(\theta) - \int_0^{V(\theta)} c\phi(c)dc.$$

The first order derivative is

$$\Pi'(\theta) = \phi(V(\theta))V'(\theta)[M(\theta)-V(\theta)]+\Phi(V(\theta))M'(\theta).$$

As in the analysis for optimal damage, let

$$\mathcal{L}(\theta) = \frac{\phi(V(\theta))}{\Phi(V(\theta))} \times [M(\theta)-V(\theta)] \times \frac{V'(\theta)}{h(\frac{c_v}{\theta})} + \frac{M'(\theta)}{h(\frac{c_v}{\theta})}.$$

Clearly, $\mathcal{L}(\theta)$ and $\Pi'(\theta)$ always have the same sign. In the above equation, the second term

$$\frac{M'(\theta)}{h(\frac{c_v}{\theta})} = \left\{ \left[\frac{c_v}{\theta^2}(p-q)+q\frac{c_v}{\theta} \right] (\frac{c_v}{\theta}+W-S]-x-q(c_v+c_i) \right\} $$
$$+q\frac{1-H(\frac{c_v}{\theta})]}{h(\frac{c_v}{\theta})} \left[W-S+E(v \mid v > \frac{c_v}{\theta}) \right]$$

is decreasing in θ. So if $M'(1) > 0$, that is,

$$M'(1) = h(c_v)c_v \{p(c_v+W-S)-x-q(c_v+c_i)\}$$
$$+q[1-H(c_v)][W-S+E(v \mid v > c_v)].$$
$$> 0$$

$M'(\theta)$ is always positive on $[0,1]$. Otherwise there exists $0 < \theta_{st} < 1$ such that $M'(\theta)$ is strictly positive for $\theta < \theta_{st}$ and negative otherwise.

For the first term, $\frac{\phi(V(\theta))}{\Phi(V(\theta))}$ and $\frac{V'(\theta)}{h(\frac{c_v}{\theta})}$ are always positive and $M(\theta)-V(\theta)$ is decreasing in θ. So if $M(1)-V(1) > 0$, that is,

$$M(1)-V(1) = H(c_v)[pS+(1-p)W]$$
$$+[1-H(c_v)]\{W-x-qc_i\}$$
$$> 0,$$

then $M'(\theta)$ is always positive on $[0,1]$. Otherwise there exists $0 < \theta_{ex} < 1$ such that $M(\theta)-V(\theta)$ is strictly positive for $\theta < \theta_{ex}$ and negative otherwise.

So if $M'(1) > 0$ and $M(1)-V(1) > 0$, then full injunction is always optimal.

If $M'(1) < 0$ and $M(1)-V(1) < 0$, then full injunction is never optimal. The optimal injunction level lies in between θ_{st} and θ_{ex}.

If $M'(1) > 0$ $M(1)-V(1) < 0$, the optimal injunction level is in $(\theta_{ex},1]$.

If $M'(1) < 0$ and $M(1)-V(1) > 0$, the optimal injunction level lies in $(\theta_{st},1]$.

References

[1] James J. Anton & Dennis A. Yao, Finding "Lost" Profits: An Equilibrium Analysis of Patent Infringement Damages, Journal of Law, Economics and Organization, vol. 23, 186-207 (2007).

[2] Ian Ayres & Paul Klemperer, Limiting Patentees' Market Power Without Reducing Innovation Incentives: The Perverse Benefits of Uncertainty and Non-Injunctive Remedies, Michigan Law Review, vol. 97, 985-1033 (1999).

[3] Gary Becker; Crime and Punishment: An Economic Approach, Journal of Political Economy, vol. 76, 169-217 (1968).

[4] James Bessen & Eric Maskin, Sequential Innovation, Patents, and Imitation, Rand Journal of Economics, vol. 40, 611-635 (2009).

[5] James Bessen & Michael J. Meurer, Patent Failure: How Judges, Bureaucrats, and Lawyers Put Innovators at Risk, Princeton University Press (2008).

[6] Roger D. Blair & Thomas F. Cotter, Are Settlements of Patent Disputes Illegal Per Se?, Antitrust Bulletin, vol. 47, 491-539 (2002).

[7] Roger D. Blair & Thomas F. Cotter, An Economic Analysis of Damages Rules in Intellectual Property Law, William & Mary Law Review, vol. 39, 1585-1694 (1998).

[8] Kenneth W. Dam, The Economic Underpinnings of Patent Law, Journal of Legal Studies, vol. 23, 247-271 (1994).

[9] Nancy T. Gallini, Patent Policy and Costly Imitation, Rand Journal of Economics, vol. 23, 52-63 (1992).

[10] Richard Gilbert & Carl Shapiro, Optimal Patent Length and Breadth, Rand Journal of Economics, vol. 21, 106-112 (1990).

[11] John M. Golden, Principles for Patent Remedies, Texas Law Review, vol. 88, 505-592 (2010).

[12] Paul J. Heald, Optimal Remedies for Patent Infringement: A Transactions Cost Approach, Houston Law Review, vol. 45, 1165-1200 (2008).

[13] Shantaram P. Hegde & Dev R. Mishra, Do Patented Innovations Affect Cost of Equity Capital? (January 18, 2014), available at http://ssrn.com/abstract=2385062.

[14] Keith N. Hylton, The Influence of Litigation Costs on Deterrence under Strict Liability and under Negligence, International Review of Law and Economics, vol. 10, 161–171 (1990).

[15] Keith N. Hylton, Welfare Implications of Costly Litigation under Strict Liability, American Law and Economics Review, vol. 4, 18–43 (2002).

[16] Paul Klemperer, How Broad Should the Scope of Patent Protection Be? RAND Journal of Economics, vol. 21, 113-130 (1990).

[17] Mark A. Lemley & Carl Shapiro, Probabilistic Patents, Journal of Economic Perspectives, vol. 19, 75-98 (2005).

[18] William Nordhaus, An Economic Theory of Technological Change, American Economic Review, vol. 59, 18-28 (1969).

[19] A. Mitchell Polinsky & Daniel L. Rubinfeld, The Welfare Implications of Costly Litigation for the Level of Liability, The Journal of Legal Studies, vol. 17, 151-164 (1988).

[20] A. Mitchell Polinsky & Steven Shavell, Costly Litigation and Optimal Damages, International Review of Law and Economics, vol. 37, 86-89 (2014).

[21] Mark Schankerman & Suzanne Scotchmer, Damages and Injunctions in Protecting Intellectual Property, Rand Journal of Economics, vol. 32, 199–220 (2001).