Boston University School of Law

Scholarly Commons at Boston University School of Law

Faculty Scholarship

7-27-2013

A Tale of Four Treatments: Preferential Taxation and Asset Valuation

Theodore S. Sims

Boston Univeristy School of Law

Follow this and additional works at: https://scholarship.law.bu.edu/faculty_scholarship

Part of the [Law and Economics Commons](#), and the [Tax Law Commons](#)

Recommended Citation

Theodore S. Sims, *A Tale of Four Treatments: Preferential Taxation and Asset Valuation*, in No. 13-42 Boston University School of Law, Law and Economics Research Paper Series (2013).

Available at: https://scholarship.law.bu.edu/faculty_scholarship/22

This Article is brought to you for free and open access by Scholarly Commons at Boston University School of Law. It has been accepted for inclusion in Faculty Scholarship by an authorized administrator of Scholarly Commons at Boston University School of Law. For more information, please contact lawlessa@bu.edu.

A TALE OF FOUR TREATMENTS: PREFERENTIAL TAXATION AND ASSET VALUATION

Boston University School of Law Working Paper No. 13-42
(August 28, 2013)

Theodore S. Sims
Boston University School of Law

This paper can be downloaded without charge at:

<http://www.bu.edu/law/faculty/scholarship/workingpapers/2013.html>

July 27, 2013

A Tale of Four Treatments:
Preferential Taxation and Asset Valuation^{*}

Theodore S. Sims^{**}

ABSTRACT: Conventional wisdom is that preferential taxation of property income elevates asset values above their values in the absence of a tax, with those values strictly increasing in the marginal rate of the holder. I show that preferential tax rates (such as the rate on realized long-term capital gains) do indeed have that property. Preferential timing on the other hand -- pure "tax deferral" -- does not. The value of an asset subject to pure deferral does increase with the holder's marginal rate, but only up to a point, at some marginal rate in excess of 50 percent. With increases in the marginal rate beyond that point, the value of the asset declines, approaching its value in the absence of a tax as the marginal rate approaches 100 percent. Disadvantageous timing has exactly the opposite effect. As far as I can tell these properties have not hitherto been noticed.

* Forthcoming, 68 Tax L. Rev. (2014).

** Professor of Law, Boston University (*sims@bu.edu*). Very special debts in helping to clarify my thinking about this paper are owing to AnTon Bui, Linda T. Bui, and Robert Guth. Invaluable comments on an earlier version from David Garlock, Daniel I. Halperin, James S. Halpern and Alvin C. Warren, Jr., and research assistance from David Bringleb, are gratefully acknowledged. Errors remain mine.

July 26, 2013

A Tale of Four Treatments:
Preferential Taxation and Asset Valuation
Theodore S. Sims

I. Introduction

The common understanding, in the setting of an income tax with graduated marginal rates, is that the tax benefit of an exclusion from or deduction in computing taxable income increases with a taxpayer's marginal tax rate. An extension of that insight is that deductions or exclusions that are "preferential," in the sense that they depart from some norm about what constitutes an appropriate tax base, confer benefits that likewise increase with the recipient's marginal rate. At one time that property -- that the benefit of "subsidies" delivered through the tax system increases with the recipient's marginal rate (and, presumably, their economic well-being) -- was a conspicuous feature of debates about the proper contours of the income tax.¹ Although the issue is no longer endemic to tax policy discussions, the impact of graduated marginal tax rates on the behavior of economic agents still deserves to be well understood; and estimates of the benefits conferred by preferential aspects of our income tax remain a systematic part of the Federal budget process, and a matter of some ongoing academic interest.²

¹ The view of preferential tax provisions as conferring subsidies that increase with the marginal rate of the recipients originates with the work of Stanley S. Surrey, Assistant Secretary of the Treasury during the Kennedy and Johnson administrations. Surrey's view was first publicly articulated in a 1967 speech, and given a fuller articulation after he left the Treasury in 1969 to return to Harvard Law School. *E.g.*, Stanley S. Surrey, Tax Incentives as a Device for Implementing Government Policy: A Comparison with Direct Government Expenditures, 83 Harv. L. Rev. 707, 720-723 (1970) (hereinafter "*Surrey (Tax Incentives)*"); Stanley S. Surrey, Federal Income Tax Reform: The Varied Approaches Necessary to Replace Tax Expenditures with Direct Governmental Assistance, 84 Harv. L. Rev. 352 (1970); Surrey, Pathways to Tax Reform (1973) (hereinafter "*Pathways*"). The resulting "upside-down" characteristic of the induced subsidies was a conspicuous feature of the criticisms levelled by Surrey and his colleagues at what were denominated (and have been referred to ever since) as "tax expenditures." *E.g.*, Stanley S. Surrey, The Tax Reform Act of 1969 -- Tax Deferral and Tax Shelters, 12 B.C.L.Rev. 307, 317-18 (1971) (hereinafter "*Surrey (Tax Deferral)*"); Paul R. McDaniel, Federal Matching Grants for Charitable Contributions: A Substitute for the Income Tax Deduction, 27 Tax. L. Rev. 377 (1972) (hereinafter "*McDaniel (Matching Grants)*"); *see* Paul R. McDaniel and Alan S. Kaplinsky, The Use of the Federal Income Tax System to Combat Air and Water Pollution: A Case Study in Tax Expenditures, 12 B.C.L.Rev. 351, 360-62 (1971). *See infra* Part II. A history of the tax expenditure notion may be found in Jonathan Barry Forman, Origins of the Tax Expenditure Budget, 30 Tax Notes 537 (1986).

² Estimates of tax expenditures are compiled annually by the Office of Management and Budget as part of each annual budget presentation, and by the Congressional Joint Committee on Taxation. *E.g.*, Analytical Perspectives on the 2013 Budget, Federal Receipts, Ch. 17 (Tax Expenditures); Joint Committee on Taxation, Estimates of Federal Tax Expenditures for Fiscal Years 2009-2013, JCS 1-10 (January 11, 2010). They are also prepared biannually by the Congressional Research Service of the Library of Congress for the Senate Budget Committee. *E.g.*, Tax Expenditures: Compendium of Background Material on Individual Provisions, S. Prt. 111-58 (111th cong. 2d. sess. December 2010). As discussed in Section II, the notion of accounting coherently for tax expenditures was challenged virtually from its inception and has been persistently controversial. *See, e.g.*, Boris I. Bittker, A "Comprehensive Tax Base"

It is not my objective to revisit the earlier debates. What I wish to investigate and to challenge is the assumption, routinely taken as true, that the benefits of preferential taxation do actually increase monotonically with the recipient's tax rate.³ To pursue that I must as a predicate provide a taxonomy for use in identifying the particular species of preferential tax provisions to which the analysis applies, and then define a norm by comparison with which to identify what is "preferential." I take up that endeavor in Part II. The central observations of that section are that the discussion that ensues is concerned with preferential aspects of property (or "capital") income taxation; that the norm I use to identify "preferential" departures from which is that of a pure accrual tax, as envisioned by the work of Robert Murray Haig and Henry Simons;⁴ one that, as defined more precisely by a theorem of Paul Samuelson, is neutral in its impact on asset values.⁵ In that context, I shall distinguish principally between provisions that confer a preferential tax rate on some subspecies of capital income, on the one hand, and those that confer only a timing advantage ("pure timing preferences"), on the other.

In practice, many preferential aspects of our income tax -- the most familiar of which is surely realization-based taxation of long-term capital gains -- involve both preferential rates and preferential timing.⁶ For analytic purposes, however, it will prove useful to disentangle the two. What will emerge from that exercise is that the benefit of a preferential rate does increase monotonically with the marginal rate that otherwise applies to the beneficiary. But the central insight of the paper is that the benefit of a pure timing preference does not. For such provisions the benefit of preferential taxation is (self-evidently) 0 at a marginal rate of 0; a bit less self-evidently,

as a Goal of Tax Reform, 80 Harv. L. Rev. 925 (1967) (hereinafter "Bittker (*CTB*)"); Boris I. Bittker, Accounting for Federal "Tax Subsidies" in the National Budget, 22 Nat'l Tax J. 244 (1969); for a recent treatment, including a canvas of the intervening literature, see J. Clifton Fleming & Robert J. Peroni, Can Tax Expenditure Analysis be Divorced from a Normative Tax Base? A Critique of the "New Paradigm" and its Denouement, 30 Va. Tax Rev. 135 (2010).

³ E.g. Pathways, *supra* note 1, at 36-39; Surrey (*Tax Incentives*), *supra* note 1, at 720-23 & n. 23; Surrey, The Tax Reform Act of 1969 -- Tax Deferral and Tax Shelters, 12 B.C.L.Rev. 307, 317-18 (1971); McDaniel (*Matching Grants*), *supra* note 1, at ____ (1972); Emil N. Sunley, Deferral of Tax, in Encyclopedia of Tax and Tax Policy 75, 76 (Cordes, Ebel, and Gravelle, eds. 2d ed.).

⁴ See *infra*, text and notes at notes 13 and 14.

⁵ Paul A. Samuelson, Tax Deductibility of Economic Depreciation to Ensure Invariant Valuations, 1964 J. Pol. Econ. 604 (1964); see Theodore S. Sims, Income Taxation and Asset Valuation (I): Economic Depreciation, Accrual Taxation and the Samuelson Theorem, 66 Tax. L. Rev. 217 (2012).

⁶ Largely for practical reasons, only the preferential capital gains *rate*, not the deferral of taxation until realization, is currently treated as a tax expenditure. See *infra* note 32.

it also turns out to be 0 at a marginal rate of 1 (100% if you prefer). For marginal rates between 0 and 1 the benefit of a timing advantage increases smoothly up to and beyond 50 percent, but at some point achieves a maximum and thereafter smoothly declines.⁷

In Part III, I illustrate these properties, including the norm against which they are measured, using what the late David Bradford referred to as the "canonical" example of a pure discount bond.⁸ Between 1940 and 1982, the U.S. tax system, as a whole, tried out at least three different ways of taxing such instruments, before settling on the current regime, implemented by the "original issue discount" ("OID") rules of I.R.C. § 1272(a).⁹ Those rules implement in taxing discount debt what I take for purposes of this project to be the norm. Conveniently, the three preceding regimes, with one modest modification, provide a complete set of examples with which to illustrate what happens when the system departs from that norm. Part IV elaborates on the implications of the examples in Part III, showing that the smooth, concave variation in value, as a function of an asset holder's marginal rate, is a general property of assets subject to pure timing preferences.

In the current political environment this has to be viewed as an exercise in basic research. As we do not now have marginal rates even approaching 50 percent, the novel property identified in this paper is unlikely to be of immediate relevance to the formulation of U.S. tax policy. It is worth recalling, however, that at times during the past 60 years the republic has without catastrophic consequence deployed marginal rates that exceeded 90 percent; as recently as 32 years ago the top rate on property income was 70 percent. And though the current low-rate mantra still seems to have a near stranglehold on political discourse, it is useful to keep in mind both that even the U.S. economy cannot indefinitely run annual *deficits* exceeding \$1 trillion, and that some basic research into the optimal level of marginal rates suggests that on personal services income they might still range as high as 70 percent, and on property income to in excess of 50

⁷ Although less common the converse is also true: the cost of disadvantageous treatment declines for rates above 0, reaching a minimum at a rate above 50 percent and then rising smoothly to 0 as the marginal rate approaches 1. See *infra* text and notes at notes 67-68.

⁸ David F. Bradford, Fixing Realization Accounting: Symmetry, Consistency, and Correctness in the Taxation of Financial Instruments, 50 Tax L. Rev. 731, 739 (1995).

⁹ See Theodore S. Sims, Long-Term Debt, the Term Structure of Interest, and the Case for Accrual Taxation, 45 Tax L. Rev. 313, 315-321 (1992).

percent.¹⁰ At some point, in some way, triggered by some crisis or another, reality eventually will intrude. When it does, the properties identified in this paper may come to take on some relevance to the policy deliberations that ensue.¹¹ In the interim they may be of intrinsic analytic interest.

II. Preferential Taxation of Capital Income¹²

To describe a feature of the tax system as "preferential" presupposes some baseline norm by comparison with which to identify preferential departures. Although the matter has in fact been contested from the beginning, it would not be unsafe to say that for much of the first 50 years of the now 100-year history of our modern income tax the norm was taken to be some version of a tax on "comprehensive" personal income, as articulated in the work of Robert Murray Haig¹³ and Henry Simons¹⁴, intended roughly to include all individual command over economic resources, at least insofar as they were reflected in market transactions. The usefulness of such an idealized conception of the tax base was thrown into relief by the efforts of Stanley Surrey, during the late 1960's and early 1970's, to highlight as subsidies delivered indirectly through the tax system -- and hence denoted "tax expenditures" -- departures from what in the

¹⁰ Peter A. Diamond & Emmanuel Saez, *The Case for a Progressive Tax: From Basic Research to Policy Recommendations*, 25 *J. Econ. Persp.* 165 (2011); compare N. Gregory Mankiw, Matthew C. Weinzierl & Danny Yagan, *Optimal Taxation in Theory and Practice*, 23 *J. Econ. Persp.* 147 (2009).

¹¹ I cannot resist recounting (with permission) an anecdote involving Professor Daniel Halperin of Harvard, then teaching at Georgetown, not long after passage of the Tax Reform Act of 1986. It occurred at a meeting of a tax policy seminar taught jointly with Professor Stephen Cohen, the enrollment of which included a number of students who worked on capitol hill while pursuing their degrees. After Professor Halperin had gone into some detail about considerations bearing on the propriety under an income tax of deducting personal interest, a student, then working on the staff of the House Committee on Ways and Means, interjected to explain that on the basis of her professional involvement with the problem none of the considerations he had just recounted had played any role in congressional deliberations on that issue. Professor Halperin, who had served for eight years in the Treasury Office of Tax Policy (including three as Deputy Assistant Secretary), leaned back for a moment, and then (to the best of my recollection) replied "Well, if you are asking me a question, I would say that in my experience the correct answer is not always irrelevant to the outcome."

¹² Those with a general familiarity with the Samuelson theorem on economic depreciation and the contours of the Haig-Simons definition of income could safely proceed directly to Part III.

¹³ Robert M. Haig, *The Concept of Income -- Legal and Economic Aspects*, in R.M. Haig (ed.), *The Federal Income Tax* (1921), reprinted in *Readings in the Economics of Taxation* 54, 59-62 (Richard A. Musgrave & Carl A. Shoup eds., 1959)

¹⁴ Henry C. Simons, *Personal Income Taxation: The Definition of Income as Problem of Fiscal Policy* (1938).

abstract was conceived of as the ideal (or at least an idealized) tax base.¹⁵ From its inception, however, Surrey's program provoked efforts both to elaborate on what was properly to be understood as part of an "ideal" personal income tax,¹⁶ and to criticize the very notion of an idealized personal tax base as insufficiently well defined to justify its use as a guide to policy prescriptions.¹⁷ Such disagreements persist to this day.¹⁸

Given the deterioration in its ability to command assent over time,¹⁹ it is hard to view the Haig-Simons definition as providing a useful operational guide to the proper contours of the entire income tax system. Even if it does not, however, it retains analytic value in other respects, among which is that it provides a taxonomy that can be useful in classifying preferential features of the tax system. Formally, Haig and Simons defined personal income as the "money value of the net accretion to one's economic power between two points in time,"²⁰ consisting of the "sum of (1) the market value of rights exercised in consumption and (2) the change in the value of the store of property rights between the beginning and the end of the period in question."²¹ That is, it consists of consumption, plus (or minus) accumulation. In so doing it looks at the uses (rather than the sources) side of the income identity.

¹⁵ The notion was given perhaps its fullest statement by Surrey himself in *Pathways*, supra note 1. The requirement of a formal accounting for tax expenditures was mandated by the Congressional Budget Act of 1974 (P.L. 93-344, 93d cong., 2d sess.), 88 Stat. 297.

¹⁶ E.g., William D. Andrews, *Personal Deductions in an Ideal Income Tax*, 86 Harv. L. Rev. 309 (1972) (hereinafter "*Andrews (Personal Deductions)*"); Mark G. Kelman, *Personal Deductions Revisited: Why They Fit Poorly in an "Ideal" Income Tax and Why They Fit Worse in a Far From Ideal World*, 31 Stan. L. Rev. 831 (1979) (hereinafter "*Kelman*"); {Griffiths}; Stanley A. Koppelman, *Personal Deductions Under An Ideal Income Tax*, 43 Tax L. Rev. 679 (1988) (hereinafter "*Koppelman*").

¹⁷ E.g., Boris I. Bittker, *A "Comprehensive Tax Base" as a Goal of Tax Reform*, 80 Harv. L. Rev. 925 (1967) (hereinafter "*Bittker (CTB)*"); Boris I. Bittker, *Accounting for Federal "Tax Subsidies" in the National Budget*, 22 Nat'l Tax J. 244 (1969); Stanley Surrey & William Hellmuth, *The Tax Expenditure Budget -- Response to Professor Bittker*, 22 Nat'l Tax J. 528 (1969).

¹⁸ See *infra* text and note at note 31.

¹⁹ Even to economists persuaded of the wisdom of taxing capital as well as personal services income, theoretical work on optimal taxation tends not to produce a comprehensive tax base that look like traditional Haig-Simons income. They do tend to find a role for taxing capital as well as personal services income, but for different reasons, and generally at differing marginal rates that are determined by very different modes of analysis. E.g., James Banks & Peter A. Diamond, *The Base for Direct Taxation*, in *Dimensions of Tax Design* 548, 550, 555 (2011); Peter A. Diamond & Emmanuel Saez, *The Case for a Progressive Tax*, supra note 10; *but see, e.g.*, Alvin C. Warren, Jr., *Would a Consumption Tax be Fairer than an Income Tax*, 89 Yale L.J. 1081 (1980).

²⁰ Haig, supra note 13, at 59.

²¹ Simons, supra note 14, at 49-50.

As such, the Haig-Simons definition provides little or no special analytic insight into some of the most self-evidently preferential features of our tax system, such as categorical exclusions from income for various kinds of transfer payments to individuals,²² or credits against tax that are unrelated to any general measure of a person's capacity to bear tax.²³ In contrast, its decomposition of income into consumption and savings is useful in pointing up the distinction between aspects of the tax system that are preferential in conferring favorable tax treatment on things that (at least plausibly) constitute consumption, on the one hand; and preferential ways in which the system treats aspects of accumulation. The former, which include (among others) such quantitatively important items as exclusions for employer provided health care coverage, life insurance premiums and meals and lodging; exclusions for scholarships and tuition benefits; and deductions for non-business state and local taxes, medical expenses, and charitable contributions,²⁴ raise thorny questions about what is truly "consumption" that have been debated since the tax expenditure budget was first formulated, and have never been resolved in a manner that commands widespread agreement.²⁵

The baseline from which to identify preferential aspects of accumulation -- that is, of capital income taxation²⁶ -- on the other hand, has always seemed substantially more well-

²² E.g., I.R.C. §§ 86 (partial exclusion of social security benefits), 104 (exclusion of worker's compensation). See Bittker (*CTB*), *supra* note 2, 17, at 935-938. The treatment of social security is complicated by the fact that the benefit has partly been paid for with tax-paid dollars, which may help to explain why it is partially includible to begin with, though in most instances substantially less than one-half of a recipient's lifetime benefit will have been paid for by them.

²³ Such as the child tax credit, and the Hope and Lifetime Learning credits. I.R.C. §§ 24, 25A.

²⁴ I.R.C. §§ 106(a); 79(a); 119(a); 117(a), 127(a), 164; 170.

²⁵ E.g., Andrews (*Personal Deductions*), *supra* note 16; Kelman, *supra* note 16; David A. Bradford, Untangling the Income Tax 19-21, 30-31 (1986); Koppelman, *supra* note 16.

²⁶ The identification of lifetime consumption with the lifetime earnings of economic agents who start out with no financial endowment has long implied an intimate connection between the "accumulation" component of Haig-Simons income and the taxation of capital income. That connection is reflected in the shibboleth that taxation of capital income amounts to "double taxation of savings." E.g., John Stuart Mill, *The Principles of Political Economy*, Book 5, 936-939 (1848) (Batoche 2001); Irving Fisher, *The Nature of Capital and Income* 252-253 (1906); Irving Fisher, *The Double Taxation of Savings*, 29 *Am. Econ. Rev.* 16 (1939). It is likewise reflected in the equivalence between a tax on consumption, defined as receipts net of additions to (or augmented by withdrawals from) savings, as under a cash-flow personal tax; and a tax that falls only on income from labor, not capital. E.g., U.S. Treasury, *Blueprints for Basic Tax Reform* 110-11 (2d ed. revised 1984); E. Cary Brown, *Business-Income Taxation and Investment Incentives*, in *Income, Employment and Public Policy: Essays in Honor of Alvin H. Hansen* 309-310 (1948), reprinted in *Readings in the Economics of Taxation* (Richard A. Musgrave & Carl A. Shoup eds., 1959); Surrey, *supra* note 1, at 312-314; William D. Andrews, *A Consumption-Type or Cash Flow Personal Income Tax*, 87 *Harv. L. Rev.* 1113 (1974) (hereinafter "*Andrews (Consumption Tax)*"); see also Michael Graetz, *Implementing a*

defined. However impracticable it might seem,²⁷ the accumulation component of Haig-Simons income -- "the change in the value of the store of property rights between the beginning and the end of the period in question" -- read naturally, implies the taxation of property income and gain as it accrues. So, in principle at least, the Haig-Simons definition has historically been identified with what (following Haig) William D. Andrews denoted an "accretion-style" tax,²⁸ understood as a tax that ideally extends not just to current cash-flow and realized gains from property but to changes in the value of assets as they accrue.²⁹ And while that view of Haig-Simons income has itself not always commanded universal assent,³⁰ the specification of preferential aspects of capital income taxation in the tax expenditure budget generally has, at least until recently,³¹

Progressive Consumption Tax, 92 Harv. L. Rev. 1575-1598-1611 (1979) (with an extended analysis of the qualifications to that equivalence). At a practical level that latter relationship is what accounts (with some assumptions) for the {apparent} financial equivalence between the taxation of conventionally tax-favored retirement savings vehicles such as individual retirement accounts (cash-flow treatment under §§ 219, 408) and their so-called "Roth" variants (wage tax treatment, under § 408A). {But see *infra*, text and note at note ____.

²⁷ *E.g.*, Simons, *supra* note 14, at 100; *cf.* David J. Shakow, Taxation Without Realization: A Proposal for Accrual Taxation, 134 U. Penn. L. Rev. 1111 (1986); Andrews (*Consumption Tax*), *supra* note 26, at 1116-17.

²⁸ Haig, *supra* note 13, at 59; Andrews (*Consumption Tax*), *supra* note 26, at 1114-16.

²⁹ It is clear that both Haig and Simons regarded "personal income" as comprehending capital income. Simons' position is underscored by his criticisms of the work of Irving Fisher, an early, fervent proponent of taxing cash-flow -- "realized" income in Fisher's lexicon -- which Fisher was insistent on distinguishing from a tax on accrued gains, which he denoted by "earnings." *E.g.*, Irving Fisher, Capital Income, note 26, at 234-235; Irving Fisher, The Theory of Interest 25-28 (1930); Fisher, Income in Theory and Income Taxation in Practice, 5 *Econometrica* 1, 6 (1937); Simons, *supra* note 14, at 89-100, 225-231. On internal evidence it is likewise clear that both Haig and Simons were independently of the view that capital income properly included accrued but unrealized gain. Haig, *supra* note 13, at 62-65. Simons' view of the matter is reflected in his criticism of the views of E.R.A. Seligman, Simons, *supra* note 14, at 85-88, though Simons seemed to acknowledge the possibility of falling back on realization as a concession to practicality. *Id.* at 100.

³⁰ *See, e.g.*, James Strnad, Taxation of Income From Capital: A Theoretical Reappraisal, 37 *Stan. L. Rev.* 1023 (1985); Strnad, Tax Timing and the Haig-Simons Ideal: A Rejoinder to Professor Popkin, 62 *Ind.L. J.* 73, 77 & n. 16 (1986); Fisher, Income in Theory and Income Taxation in Practice, 5 *Econometrica* 1 (1937). *See infra* note 39.

³¹ In the past 10 years both the Treasury and the Congressional Joint Committee on Taxation have undertaken significant reassessments of the premises and methodologies they use in estimating tax expenditures. In 2003 the Treasury began reporting tax expenditures as determined by reference to a comprehensive consumption tax, as well as a comprehensive income tax baseline. U.S. Office of Management and Budget, Analytical Perspectives, Budget of the United States Government, Fiscal Year 2004, Appendix 130-140. That practice continued through 2009. In 2008 the Joint Committee engaged in a reevaluation that led to a change in the way in which they classified tax expenditures. Joint Committee on Taxation, A Reconsideration of Tax Expenditure Analysis, JCX-37-08 (May 12, 2008). *See, e.g.*, Joint Committee on Taxation, Estimates of Tax Expenditures for Fiscal Years 2009-2013, JCS-1-10 (January 11, 2010). *See also* Leonard E. Burman, Is the Tax Expenditure Concept Still Relevant, 56 *Nat'l Tax J.* 613 (2003); {Carroll, Joulfaian & Mackie, NTJ; NBER conference in NTJ; Clifton-Peroni; denouement?}

been identified with a tax that falls on capital income, in roughly the fashion implied by the Haig-Simons definition.³²

The importance of taxing capital income as it accrues is developed in Paul Samuelson's 1964 paper on economic depreciation. Samuelson showed that taxing capital income while allowing a deduction for economic depreciation leads to a system in which asset valuations are independent of the marginal rates of their holders. In principle at least, the use of economic depreciation induces a system that is free of tax-induced distortions in the values of depreciable assets, and so mitigates clientele effects and the temptation to tax-induced trading in such assets. What is more, while Samuelson's paper was nominally about depreciation, it is actually a far broader formulation: the "depreciation" function that gives rise to Samuelson's result produces in general a system of pure accrual taxation.³³ As such, it offers a possibly comprehensive guide to structuring a personal tax that reaches capital income while leaving asset values unaffected.³⁴

Samuelson's insight thus provides a theoretical justification for the treatment of capital income, arrived at on other grounds by Haig and Simons, in the design of a personal income tax. Whatever its other shortcomings, and in principle at least, a personal income tax that includes accrual taxation of capital income at uniform rates will not produce tax-induced distortions of asset values.³⁵ As far as broad-based taxes are concerned there are, of course, alternatives to an income tax of that sort. Principal among them are cash flow taxes of the sort envisioned by

³² Somewhat ironically, and evidently in a concession to administrative convenience, while the taxation of gains on capital assets such as corporate stock at preferential rates is included among the enumerated tax expenditures, the taxation of such gains on realization, rather than as they accrue, generally is not. See, e.g., Burman, *supra* note 31, at 617 & n. 6. Such a concession might not have surprised Henry Simons. Simons, *supra* note 14, at 100.

³³ Sims, *supra* note 5, at 228-33.

³⁴ Sims, *supra* note 5, at 238-41.

³⁵ By "uniform" I mean only that personal services and capital incomes are taxed at the same rates. Samuelson's result has been shown to hold even when tax rates vary over time, whether because of time-variation in the rate schedule itself or time-variation in the marginal rates faced by individual taxpayers. See Andrew Lyon, *Invariant Valuation When Tax Rates Change Over Time*, 98 J. Pol. Econ. 433 (1990); Sims, *Economic Depreciation and Invariant Valuation: A Constructive Proof of the Samuelson Theorem* (February 26, 2012, available at <http://ssrn.com/abstract=2006557>) (hereinafter "*Sims (Invariant Valuation)*"). Elaborating on more specialized examples offered by Chirelstein and Strnad, Sims shows that what drives Samuelson's result is that, by taxing income as it accrues, the impact of the tax in reducing cash-flow from an asset is exactly offset by its impact on the rate at which those cash-flows are discounted, such that the two exactly balance in every period and at every marginal rate. See *infra* text at notes 49-50. Consequently, the result survives the introduction of time-varying tax and discount rates. *Id.* at 6-7.

Fisher,³⁶ Kaldor,³⁷ and Andrews³⁸; wage taxes, of the general sort used in the United States to finance Social Security and Medicare; and indirect taxes on consumption, such as retail sales taxes or a value added tax. Amongst all of them, however, an accrual tax, along the lines implied by the Haig-Simons definition with the property formalized by Samuelson, stands by itself in seeking to tax capital income as such. And since the objective of this paper is to investigate the impact on asset values of "preferential" features of a tax on capital income, Samuelson's formalization of what Haig and Simons envisioned provides a (perhaps, the) natural norm.³⁹ It will serve as our point of departure. So we start in a world with a capital income tax that has no impact on asset values. From there we proceed to investigate what happens when we introduce departures from that norm.

III. The Impact of Marginal Rates on Preferences

The illustrations that follow explore the impact of various possibilities for taxing a conceptually simple investment, taking the form of a single outlay today, followed by a single receipt at some future date. The amount and date of the receipt may with no loss of generality be fixed arbitrarily. My example consists of a current outlay of \$5,131.58, and a receipt seven years later of \$10,000. This is a 7-year pure discount (or pure OID) bond. The implicit "yield-to-maturity" on the instrument,⁴⁰ on the assumption that interest compounds annually, is 10 percent. That is, \$5,131.58, deposited in an account with a bank that paid interest at 10 percent, compounded each year, would grow to \$10,000 by the end of seven years. That is as illustrated in Table I. By the same token, the same \$5,131.58 invested instead in our 7-year pure discount bond that makes a single payment of \$10,000 at the end of the seven years, on the assumption of annual

³⁶ E.g., Irving Fisher, *Constructive Income Taxation* (1942); *Income in Theory and Income Taxation in Practice*, 5 *Econometrica* 1 (1937).

³⁷ Nicholas Kaldor, *An Expenditures Tax* (1955).

³⁸ Andrews (*Consumption Tax*), *supra* note 26.

³⁹ James Strnad has argued that a tax conforming to Samuelson's model (which he denotes a "Samuelsonian" income tax) is distinct from the tax envisioned by Haig and Simons, and that the tax that in his view most faithfully corresponds to what he calls the "Haig-Simons ideal" is in fact a cash-flow tax. E.g., authorities cited *supra* note 30. That claim is beyond the scope of this article, and deserves to be separately addressed. For present purposes it will suffice to say that the premise of what follows is at odds with that assertion.

⁴⁰ "Yield-to-maturity" of a debt instrument is defined as that interest rate, which if used to discount all payments pursuant to the instrument to present value, produces discounted values that in the aggregate equal the instrument's "issue price." E.g., Sharpe, Alexander & Bailey, *Investments* 112-14 (6th ed. 1999); Brealy, Myers & Allen, *Principles of Corporate Finance* 629-31 (8th ed. 2006); *Income Tax Regulations* § 1.1272-1(b)(1)(i).

compounding, provides an annual return of 10 percent, and the value of the bond would increase in the pattern illustrated in Table I. What is more, as will be illustrated momentarily, it is on just those amounts that the holder would be taxed under I.R.C. § 1272(a).

The regime prescribed by § 1272(a) has effectively been in place since 1982.⁴¹ Before settling there the system had to varying degrees used at least three other methods of taxing pure discount debt. They included (1) deferring taxation of the appreciation until disposition via surrender or sale, and taxing it then as long-term capital gain;⁴² (2) taxing the gain on disposition as ordinary income;⁴³ and (3) accruing the gain periodically over the life of the instrument on a ratable basis and taxing it as ordinary income.⁴⁴ Conveniently enough for our purposes, those three effectively span the space of alternatives to § 1272(a) for taxing discount debt, in that they include one instance each of a pure timing preference (*Midland-Ross*, and pre-1969 §1232(a)(2)(A)) and pure timing disadvantage (1969-82 ratable accrual), and one instance of the use of a differential (in this instance a preferential) rate. In Part IV, I illustrate how almost any instance of preferential (or disadvantageous) taxation may be decomposed into a combination of those basic constituent variations from the treatment now prescribed by § 1272. First, however, I lay out in detail the treatment of our specimen investment under existing law, and then use that

⁴¹ The system now in place was originally found in I.R.C. § 1232A, added by § 232 of the Tax Equity and Fiscal Responsibility Act of 1982, P.L. 97-248. Sections 41-43 of the Revenue Provisions of the Deficit Reduction Act of 1984 (P.L. 98-369) prescribed a general recodification of the statutory provisions governing the treatment of debt instruments, as part of which pre-existing § 1232A became what is now I.R.C. § 1272.

⁴² *Caulkins v Comm'r*, 1 T.C. 656, *acq.*, 1944 Cum. Bull. 5, *aff'd*, 144 F.2d 482 (6th Cir.), *acq. withdrawn*, 1955 Cum. Bull. 7; see *United States v. Midland-Ross Corp.*, 381 U.S. 54, 63-66 (1965).

⁴³ See *United States v. Midland-Ross*, 381 U.S. 54 (1965). *Midland-Ross*, decided under the 1939 Code, involved a 9-month note purchased and sold during the same year but more than six-months apart, at a time when the long-term capital gain holding period was six months. The Supreme Court held only that the earned original issue discount was not entitled to capital gains treatment; it did not have to and expressly refrained from deciding whether the discount was properly to be taxed as it accrued or in the year of sale. *Id* at 58 & note 4. By the time *Midland-Ross* was decided, the 1954 Code had been adopted; it contained a provision (pre-1969 § 1232(a)(2)(A)) pursuant to which gain on the sale or exchange of a debt instrument attributable to earned original issue discount was to be treated as gain from the sale or exchange of property that was not a capital asset. See *Midland-Ross*, at 58-60. Much the same treatment is prescribed today for gain from the sale or exchange of so-called market discount bonds by § 1276. For convenience in what follows, I refer to the taxation as ordinary income of gain on the sale of a debt instrument as "Midland-Ross" treatment, even though *Midland-Ross* did not actually address the timing question.

⁴⁴ Ratable accrual was introduced into the statute by § 413 of the Tax Reform Act of 1969 (P.L. 91-172), in the form of Pre-1982 I.R.C. § 1232A. The ratable accrual regime was replaced by the existing system by § 232 of the Tax Equity and Fiscal Responsibility Act of 1982. Even after recodification in 1984 (see *supra* note 41), the ratable accrual regime has been preserved in I.R.C. § 1272(b) for debt instruments (if by this point any) issued before July 1, 1982.

as a baseline to illustrate the central point of the paper: although the value of a preferential tax rate does strictly increase with an asset holder's otherwise generally applicable marginal rate, the value of a pure timing preference (and conversely for a pure timing disadvantage) does not.

Baseline: Accrual Taxation and Invariant Valuation. That illustration begins with Part A of Table II, which depicts the existing taxation of our illustrative pure discount bond, assuming that interest compounds annually.⁴⁵ The pattern of accrual depicted in Column (1), in effect the sequence of "adjusted issue prices" of the instrument over its seven-year life,⁴⁶ mirrors the account balances at the end of each year if the same \$5,131.58 had instead been deposited in a bank, depicted in Table I. The annual increases in the adjusted issue price, which form the basis for what is includible in the gross income of the holder under I.R.C. § 1272(a)(3), are depicted in Column (2).⁴⁷ The resulting tax liabilities, assuming to begin with a marginal rate of 30 percent, have been calculated in Column 3(a), while the resulting net cash flows from the instrument -- consisting of the initial outlay, six subsequent tax payments, and the amount received at maturity, reduced by the seventh and final tax payment -- are arrayed in Column 4(a). In Column 5(a) each of the cash flows in Column (4a), other than the initial outlay, has been discounted to the date of purchase, using an after-tax discount rate, on these facts taken to be 7 percent.⁴⁸

Observe that the sum of the present values (Column 5(a)) of the 7 net cash flows in Column 4(a) equals the initial outlay. That is, the value of the asset, conceived of as the after-tax discounted value of its constituent after-tax cash flows, is equal to its purchase price. Thus, the asset's value is unaffected by the intervention of the tax. This, then, is a simple illustration of

⁴⁵ The default assumption in § 1272(a)(5) is of semiannual compounding; in effect, however, an instrument subject to the OID rules must compound at least as frequently as it pays interest. Regs. § 1.1272-1(b)(1)(ii). The regulations do, however, effectively permit annual compounding with respect to a pure discount bond. *Ibid.*

⁴⁶ I.R.C. § 1272(a)(4).

⁴⁷ The increase in each period in the adjusted issue price is technically the product of the instrument's yield-to-maturity and the adjusted issue price itself. Since an instrument's compounding period (its statutory "accrual period") need not and rarely does correspond with a taxable year, and since instruments change hands throughout the year, § 1272(a)(3) allocates the increase in the adjusted issue price during any accrual period between taxable years and among holders using a daily ratable methodology. I.R.C. § 1272(a)(1) and (3).

⁴⁸ In a world in which investors face a pre-tax interest rate of 10 percent, a holder taxed at marginal tax rate $z = 30$ percent on (ordinary) interest income would earn interest at an after-tax rate of $r^*(1-z) = 0.10*0.7 = 0.07 = 7$ percent. Hence, in a taxable world, the after-tax cash flows from a taxable investment would be discounted by a 30 percent bracket holder at that after-tax rate.

the Samuelson theorem, according to which the value of an asset subject to "economic depreciation" or (what amounts to the same thing)⁴⁹ pure accrual taxation is unaffected by the imposition of an income tax. Two interacting effects of an income tax drive that result. The first is that taxation reduces future returns to the asset (a "cash-flow effect"); the second is that it reduces the rate at which the after-tax cash flows are discounted (a "discounting effect"), thus *elevating* their present values. As a general matter the cash-flow and discounting effects of an income tax push in opposite directions in their impact on asset values. Under a pure accrual tax, however, they interact in an extraordinary way: the cash-flow and discounting effects *exactly* offset one another, and they do so in every period and at every marginal rate. It is that exact counterbalancing that leaves the asset's *value* unchanged.⁵⁰ That is illustrated in the balance of Table II-A. The five subsequent panels (labelled (3b)-5(b) through 3(f)-5(f)) depict tax, net cash flows, and present values, assuming marginal rates that range from 40-70%. At each marginal rate both the after-tax cash flows and discount rate are reduced,⁵¹ and the after-tax value of the asset remains \$5,131.58.⁵² That is the Samuelson theorem in a nutshell. And a tax that is neutral in its impact on asset values forms a natural baseline by reference to which to explore the impact of variations in income taxation that are not.

Preferential Rates. The first species of preferential taxation is illustrated in Part B of Table II, where the appreciation in value of that same asset is taxed entirely on surrender in year 7, at the rate that applies to realized long-term capital gain, assumed here to be 15 percent. The tax liability in year 7 is \$730.26; the after-tax cash flow from the asset is \$9,269.74, realized entirely in year 7; and so the entire set of cash flows consists of the initial outlay of \$5,131.58 offset by \$9,269.74 in year 7. Again discounted (in Column 5(a)) at 7 percent, on the assumption that the holder's marginal rate on ordinary income is 30 percent, the asset now has a present value of \$5,772.73, substantially in excess of its value if taxed under I.R.C. § 1272(a), as well

⁴⁹ Sims, *supra* note 5, at 230-33.

⁵⁰ See *supra* notes 35 and 48; Sims, *supra* note 5, at 246-48; Sims (*Invariant Valuation*), *supra* note 35.

⁵¹ For a discount bond the after-tax cash flows, consisting (except in the final period) entirely of tax liabilities, are reduced in that they become more negative. The discount rate in each instance is $r^*(1-z)$. See *supra* note 48.

⁵² This result holds in the presence of time-varying r and z . See *supra* note 35; Lyon, *supra* note 35; Sims, *supra* note 35.

as its nominal purchase price, of \$5,131.58. The difference, \$641.15, is the value to a holder otherwise taxed at 30 percent of the preferential treatment illustrated in Table II-B.⁵³

That treatment, however, is a composite of both a preferential tax rate and preferential timing, the latter due to the realization requirement, which defers until disposition the taxation of accrued gain. Some rough sense for the relative contribution of the two can be obtained by a comparison to the first panel of Table II-C, which illustrates deferred taxation of the same asset as ordinary income -- a pure timing preference -- as was the case (between 1954 and 1969) under (then) I.R.C. § 1232(a)(2)(A).⁵⁴ The only difference between Tables II-B and II-C is that, in the latter, tax is (initially) imposed at 30 rather than 15 percent, so that the tax liability (Column 3(a)) is \$1,460.53 rather than \$720.26. Consequently, after-cash flow (again entirely in year 7, in Column 4(a)) is \$8,539.47, and its present value (again discounted at 7 percent, in Column 5(a)) is \$5,317.96, so that the value of the pure timing preference is \$186.38. Thus, by comparison with Table II-C, the bulk of the value of the preference illustrated in Table II-B appears in these particular examples to be due to the preferential rate.⁵⁵

The balance of Table II-B illustrates the value of the asset, benefitting from both preferential timing and the favorable long-term capital gain rate, when the holder is otherwise taxed on ordinary income at rates of 40, 50, 55, 60 and 70 percent. In each case, the undiscounted after-tax value of the asset, fixed by the long-term capital gain rate, is unchanged. Variations in the holder's marginal tax rate on ordinary income, in other words, have no effect on the asset's

⁵³ These are partial equilibrium effects; they do not take into account the possible impact of preferential taxation on such economic variables as interest rates (and other rates of return). They also leave undetermined exactly how the asset would finally be priced in the presence of preferential taxation and graduated marginal rates, and to what extent the benefit of preferential taxation would be "capitalized" into the price of the asset. *See, e.g.,* Sims, Debt, Accelerated Depreciation, and the Tale of a Teakettle: Tax Shelter Abuse Reconsidered, 42 U.C.L.A. L. Rev. 263 298-309 (1994). If, for example, because of its favorable treatment, the asset were repriced to \$5773, its after tax return to a 30 percent marginal rate holder would be 7 percent. On that assumption the favorable treatment would have been "fully capitalized," at least to a 30 percent bracket holder. To holders taxed at higher marginal rates it would continue to be preferentially treated.

⁵⁴ *See supra* notes 41 and 44; *see also* United States v. Midland-Ross, 381 U.S. 54, 58 (1965).

⁵⁵ Any inference drawn from this example about the sources of the value of the overall preference must be taken with a grain of salt. The relative contributions of a preferential rate and preferential timing will be affected by (1) the magnitude of the discount of the preferential rate from the otherwise applicable marginal rate, and (2) the duration of the deferral. For mildly preferential rates and long deferrals, the contribution of the timing advantage would be more pronounced. It is also to be noted that the functions that produce these effects are non-linear, so that one cannot in general decompose the value of a treatment that involves both a preferential rate and preferential timing into a simple linear combination of the constituent effects.

undiscounted after-tax cash flow. They influence the value of the asset only through the discounting effect. As the holder's marginal rate rises, the after-tax discount rate falls, elevating the discounted value of the cash flow realized in year 7,⁵⁶ as illustrated in the final line of Table II-B. The present values rise steadily from \$5,772.73 at 30 percent to \$7,537.14 at 70 percent. As the holder's marginal rate approached 100 percent, the factor used to discount the cash flow would approach 1,⁵⁷ so that the present value of the preferentially taxed asset would approach its undiscounted after (capital gains) tax value, \$9,269.74. Table II-B is thus consistent with the standard intuition that the value of a preferentially taxed asset increases with the holder's marginal rate.

Pure Timing Preferences. Returning to Table II-C, we come to the point of this paper. The remainder of the panels ((b)-(e)) in Table II-C replicate the computation of the asset's after-tax present value for the same increasing sequence of marginal rates used in Tables II-A and -B. In contrast with Table II-B, however, two things change from panel to panel: in addition to a tax-induced reduction in the discount rate, the amount of the tax liability in year 7 increases with the marginal rate, reducing the after-tax cash flow. So, as with economic accrual, the undiscounted after-tax cash flows *and* the discount rate both fall, with the "cash-flow" and "discounting" effects again pushing in opposite directions in determining the (after-tax) present values of the after-tax cash flows.⁵⁸ In contrast with economic accrual, however, those two effects *do not* exactly offset one another in every period. Collectively, then, they do have an influence on the asset's after-tax value. At a 30 percent rate the discounting effect evidently dominates the cash-flow effect. Hence, as with a preferential rate, preferential timing alone elevates that value.

In this case, however, the impact proves to be more complex. As the marginal rate rises above 30 percent, the after-tax value likewise rises above \$5,317.96. At a marginal rate of about 55 percent, however, the asset value achieves a maximum of about \$5,380.68.⁵⁹ Thereafter it

⁵⁶ In discrete time the discount factor takes the form $1/(1+r(1-z))^n$, where r is the pre-tax discount rate and z is the marginal tax rate, so that as z rises, the after-tax discount rate $r(1-z)$ falls, the denominator falls, and the resulting present value rises, consistent with the insight of introductory economics or finance courses that the value of debt rises with falling interest rates (and conversely).

⁵⁷ See supra note 56. The value of the expression $1/(1+r(1-z))^n$ approaches 1 as z approaches 1.

⁵⁸ See text and notes at notes 35 and 49-50, supra.

⁵⁹ For this instrument, the maximum value is actually \$5,380.86, achieved at a marginal rate of about 55.8%, but the illustrated values are sufficiently accurate for our purposes. See infra note 64.

falls as the marginal rate continues to rise, declining to \$5,379.42 at 60 percent, and to \$5,359.98 at 70 percent, reported in the last line of Columns 5(e) and (f). Although not reported in the Table, the asset value continues to decline, toward a limiting value of \$5,131.58, its pre-tax present value, as the marginal tax rate approaches 100 percent.

Why? There are several ways of getting at it, but in the setting of this example perhaps the most intuitively accessible is this.⁶⁰ Any of the Column 5 computations in Table II-C establishes that, in the presence of a positive marginal rate and a pure timing preference, this asset takes on a value in excess of \$5,131.58. Intuitively, on the other hand, at a marginal rate of 0 the asset would be worth just its pre-tax surrender value of \$10,000, discounted for 7 years with annual compounding at the pre-tax discount rate of 10 percent, or \$5,131.58. A moment's reflection should likewise suggest that the asset would take on that *same* value at a marginal rate approaching 100 percent. At that rate the \$4,868.42 in appreciation realized at maturity would be entirely taxed away, leaving the holder with just their original purchase price (and adjusted basis) of \$5,131.58. At a marginal tax rate of 100 percent, however, the after-tax *discount rate* would be 0.⁶¹ So the \$5,131.58 that remained at maturity after confiscatory taxation would also have a *present value* of \$5,131.58. In sum, the after-tax present values of the amounts remaining at maturity are \$5,131.58 at marginal rates of *both* 0 and 100 percent. But, as reflected in Table II-C, the after-tax present value is more than \$5,131.58 at positive marginal rates in between. Since, moreover, the value of the asset is a continuous function of the marginal tax rate,⁶² it follows as a mathematical matter that it *must* take on a maximum value somewhere between the rates 0 and 1,⁶³ just as Table II-C confirms. Thus, the value of our pure timing preference, in

⁶⁰ A more detailed account will be found *infra*, text at note 79.

⁶¹ See *supra* notes 56-57.

⁶² That is, although in the discounting function $1/(1+r(1-z))^n$ time (n) is discrete, the function itself varies smoothly with variations in either of the parameters z or r on the interval $[0, 1]$; the function takes on no undefined values on those intervals, nor does it exhibit any "kinks" in the graph of its values. Mathematically, the discounting function is "differentiable" in its parameters (the same is true of the continuous discounting function $e^{-rt(1-z)}$).

⁶³ The foundation for this assertion is an elementary proposition of the differential calculus called "Rolle's theorem." *E.g.*, Earl Swokowski, *Calculus with Analytic Geometry* (4th ed. 1984) 132-33. It requires (1) that the function be continuous and smooth (that is to say, "differentiable") in its parameters (here r and z), in this instance in the interval between 0 and 1, a condition that the discounting functions satisfy, and (2) that the function take on the same value at 0 and 1. With those conditions the theorem says that, unless the function takes on the value at its endpoints *throughout* the interval -- *i.e.*, it is constant -- there must be at least one point in the interval at which it achieves a maximum (or minimum). Roughly translated, Rolle's theorem says, for a differentiable function, that "What goes up and then comes down again must reach a (at least one) peak somewhere along the way."

contrast with a preferential rate, does not (as in Table II-B) strictly increase with the holder's marginal tax rate. It behaves instead as illustrated in Table II-C, rising first up to some marginal rate in excess of 50 percent, and thereafter falling to 0.

The identification of that property, which to the best of my knowledge has not previously been recognized, is the principal contribution of this paper. In Part IV, I show that what is illustrated here using the example of a pure discount bond is characteristic of pure timing preferences in general. In this instance, moreover, it is possible to be considerably more precise. As a formal matter what is illustrated in Table II-C is a solution to the problem of maximizing the value of a preferentially taxed pure discount bond by varying the holder's marginal rate. As developed in the Appendix, it is possible to obtain an explicit expression for the value-maximizing marginal rate as a function of the discounting parameters, consisting of the pre-tax interest rate (here assumed to be 10 percent) and the period of deferral (here 7 years). For that combination the value-maximizing marginal rate is around 56 percent. It can also be confirmed (it comes as no surprise) that the value of deferral is strictly increasing in those parameters; that is, everything else held constant, deferral is more valuable at higher interest rates and/or for longer periods of deferral. Perhaps most importantly, the value-maximizing marginal rate is likewise increasing in the discounting parameters, but it always lies strictly between 50 and 100 percent.⁶⁴ However low the interest rate, or however short the discounting period, the value-maximizing marginal rate never falls below 50 percent.

⁶⁴ These properties are established explicitly for a pure discount bond in the Appendix, which also illustrates the offsetting contributions of the cash-flow and discounting effects to this phenomenon. See text at notes 35 and 58. In this example, the impact of the discounting effect on the rate of change of the value of deferral is positive, and relatively stable as the marginal rate rises. The cash-flow effect, in contrast, is negative, but at low marginal rates smaller in value than the discounting effect; as the marginal rate rises, however, the negative impact of the cash-flow effect on the rate of change grows steadily in magnitude, eventually offsetting the discounting effect. In the more general illustration of deferral developed in Part IV, the signs of the cash-flow and discounting effects are the opposite of those found here with respect to the pure discount bond. See *infra* notes 74-75. In that setting it is possible to develop a more detailed intuitive account. See *infra* text following note 79.

FIGURE I: Deferral Surface for 10% Pure Discount Bond Taxed On Surrender

Most of these properties are illustrated in Figure I, a 3-dimensional surface plot of the value of deferral to the holder of a 10 percent pure discount bond, as a function of the maturity of the instrument and the marginal rate of the holder. As can be seen from the plot, the (proportionate)⁶⁵ value of deferral is strictly increasing in the maturity of the bond. A careful inspection of the plot shows that at low maturities the value-maximizing rate is modestly in excess of 50 percent, and increases as the maturity of the instrument (and the period of deferral) grows longer.⁶⁶ In sum, as has long been assumed, the benefit of deferral increases with the duration of deferral; but, at every maturity, contrary to conventional wisdom, that benefit first rises with the taxpayer's marginal rate, before reaching a maximum and then declining to 0 as the marginal rate approaches 100 percent. In Part IV, I illustrate that these properties carry over to the more general case.

Pure Timing Disadvantages. But the value of deferral is not quite the end of the story. What Table II-C suggests about the behavior of pure timing preferences leads naturally to the question whether the opposite is also true: do the effects of a pure timing *disadvantage* mirror those of a pure timing preference? Given the natural tendency of the tax legislative process to weed out treatment that is disadvantageous and proliferate what is preferential, it is probably not a matter of pressing practical concern. As long as we are here it is worth exploring nevertheless. Happily, the taxation of discount debt between 1969 and 1982 provides an historical example. Congress's first attempt at requiring year-by-year accrual of original issue discount took the form of the ratable inclusion rule, now found in I.R.C. § 1272(b) for debt instruments issued before 1982.⁶⁷ Given that, as illustrated in Tables I and II-A, economic accrual provides for an *increasing* sequence of accruals of interest, ratable inclusion, which taxes the same aggregate

⁶⁵ In the special case of a pure discount bond the period of deferral is inextricably tied to the maturity of the instrument, so that the value of the instrument as depicted in a plot like Figure 1 would appear to be strictly *decreasing* in the period of deferral. To avoid that possible confusion in constructing Figure I, the value of deferral as a function of the marginal rate is at each maturity expressed as the increase in the value of the instrument, *normalized* by its value in the absence of a tax. That is, what is presented at each maturity is the value at marginal rates $z \in (0, 1)$, divided by the value at $z = 0$. In effect what is plotted is the benefit of deferral in percentage terms, which increases as maturity gets longer.

⁶⁶ Although the impact of the discount rate is not shown, the period of deferral (the instrument's maturity) and the discount rate enter into the discounting functions symmetrically, a property that is easiest to say with the continuous discounting function $e^{-r(1-z)t}$. Hence, the value of deferral changes in the same way with increasing discount rates (r) as it does with increasing maturity (t). See the Appendix.

⁶⁷ See *supra* notes 41 and 44.

amount of OID in equal annual installments, accelerates the timing of income inclusion and by comparison is disadvantageous.

The resulting disadvantage is illustrated in Table II-D. At a 30 percent rate, as illustrated in Column 5(a), the value of the asset is indeed depressed, from \$5,131.58 to \$5,103.04. As the marginal rate rises the asset's value continues to fall, reaching a minimum of \$5,094.22, again in the vicinity of a marginal rate of about 55%.⁶⁸ Thereafter it rises to \$5,094.56 and \$5,097.75 at marginal rates of 60 and 70 percent, eventually approaching \$5,131.58 from below as the marginal rate approaches 100 percent. So, as was probably to be expected, the effects of pure timing variations from economic accrual are indeed symmetric: preferential timing increases an asset's value up to some marginal tax rate in excess of 50 percent; thereafter it declines to its pre-tax value as the marginal rate continues to rise; disadvantageous timing has exactly the opposite effect.⁶⁹

The net effect of Table II in its entirety is summarized in Table III, which reports the values of our illustrative asset at marginal rates ranging from 30 to 99.99 percent, for all four tax regimes that we have studied. For this example, the preferential rate leads to asset values that are strictly increasing in the holder's marginal rate. Pure timing changes, in contrast -- whether preferential or disadvantageous -- do not.

IV. Preferential Taxation of Capital Income in General.

Table II, however, is just a set of illustrations, based on the somewhat specialized example of a pure discount bond. Do the phenomena it illustrates hold generally? The answer is that they do. In this section I provide intuitive arguments for and illustrations of those assertions. An outline of the proofs will be found in the Appendix.

Preferential Rates. With respect to preferential rates, a simple thought experiment, in the context of Table II-A, will provide an intuitive foundation for that conclusion. The invariance

⁶⁸ The value-minimizing marginal rate in Table II-D, about 55.3 percent, is not identical to the value-maximizing rate in Table II-C. Compare note 59. The rates are similar, however, in that they both vary with the discounting parameters, approaching 50 percent as a limiting value as those parameters become small, and 100 percent as they become large. See *supra* note 64; {see the Appendix.}

⁶⁹ [It is to be noted that this symmetry does not carry over to preferential and disadvantageous rates. The burden of a disadvantageous rate turns out to be declining as the taxpayer's generally applicable marginal rate goes up. That is, the benefit of a preferential rate goes up with the taxpayer's marginal rate, while the burden of a disadvantageous rate goes down. Stated more technically but more simply, the impact on asset value of rate departures, favorable or not, is strictly monotone in the holder's marginal rate. The value of a rate departure is always increasing in the marginal rate.]

property illustrated in that Table is a general characteristic of assets taxed using economic accrual. That example may therefore be taken as a proxy for the discrete-time representation of *any* asset taxed using economic accrual. In the abstract, the pre-tax value of any such asset can be represented algebraically as the sum of the discounted values of its constituent cash flows. If, in addition, the asset is taxed using economic accrual, its after tax value -- that is, its after-tax cash flows, discounted at an after-tax rate -- will exhibit the invariance property illustrated in Table II-A.

Now imagine that any part of any one of those cash flows is for any reason taxed at a preferential rate, defined as a rate that is to any extent lower than the holder's otherwise applicable marginal rate, with everything else held constant. The result will be to introduce an additional, positive cash flow into a set of after-tax discounted values that had hitherto exhibited the invariance property of the Samuelson theorem. For those latter values, in isolation, however, that property will not be altered by the introduction of the additional cash flow. That is, the value of the asset -- after a rate preference has been introduced in the form of the additional cash flow described above -- may be decomposed into its tax-rate-invariant value, *plus* the discounted value of the additional preference-induced cash flow. As such, the resulting *variation* in the value of the asset will just be the variation in the preference-induced cash flow. As with any such cash flow, the after-tax present value of the latter will rise with the holder's marginal tax rate, as the increasing tax rate reduces the after-tax discount rate. Since that is the *only* element of the asset's value that varies with the tax rate, the value of the asset as a whole, following that of the preference-induced cash flow, will increase with the holder's marginal rate.

To be concrete, imagine that \$100 of the accrued discount in Table II-A had been taxed at 25 instead of 30 percent, just in (say) period 2. The effect would be to reduce tax expense, thereby raising net revenue, by \$5 in period 2, *leaving all other cash flows, and their present values*, unchanged. This, then, is a pure rate preference, uncontaminated by any preferential timing. The value of the asset will now be just the sum of the after-tax present value of the additional \$5 realized in year 2, plus the sum of the after-tax present values of the original cash flows, which are already known to be tax-rate invariant. Since they are, any variation in the value of the asset as a whole will just be the variation in the present value of the additional cash flow. Like any other discounted cash flow that value would increase with the tax rate, as the latter depressed the rate at which the cash flow was discounted.

The net effect is illustrated in Table IV-AR. That Table is identical to Table II-A, except for our 5 percent rate preference applied to \$100 of accrued income in Period 2, reducing tax expense (and increasing revenue) by \$5, in the cell marked by an asterisk in Column (3a). What then also change are the asset's Period 2 after-tax cash flow and its discounted value, depicted in Columns (4a) and (5a), likewise marked with asterisks. As a result, the asset's aggregate present value, reported in the final row of Column (5a), increases from its tax-invariant value of \$5,131.58 to \$5,135.95, or by \$4.37. But that \$4.37 is just the present value, discounted for two years at 7 percent, of the additional \$5 of tax-induced cash flow in Period 2. Now observe that, while the effects are modest, the value of the asset increases steadily as the holder's marginal rate increases from 30 to 70 percent, as reported in the final rows of Columns (5b)-(5f), just as it did in Table II-B. At 70 percent, the increase in value is \$4.71, which is also the present value of \$5, discounted for two periods, this time at 3 percent. At each intermediate rate between 30 and 70, the increase in the value of the asset is just the \$5 of Period 2 cash flow, discounted for two periods at the resulting sequence of after-tax discount rates.⁷⁰ Thus, just as predicted by the thought experiment above, the observed change in the asset's value in Table II-AR is just the change in the present value of the \$5 cash flow, introduced by the 5 percent rate preference applied to \$100 of income in Period 2, as the marginal rate rises and the discount rate falls. The same experiment is carried out algebraically in the Appendix, where it is shown that *any* such rate-induced preference leads to an asset value that is strictly increasing in the holder's marginal rate. The changes induced by a preferential rate are consistent with conventional wisdom.

Preferential Timing. Preferential timing again proves to be more complex. Once again, we can begin with a thought experiment, set again in the context of Table II-A. This time however, with no variation in rates, the experiment is simply to retard the timing of the taxation of some element of income from an earlier to a later period. That, after all, is what a timing preference does. With no change in rates, the impact of that change would be to reduce tax expense and increase revenue in some earlier period, and then to make an adjustment in a later period that is *exactly* offsetting on an undiscounted basis. It should intuitively be clear, as long as the marginal rate remains constant, as we have been assuming throughout this analysis, that the deferral

⁷⁰ The changes in value are reported in the final lines of Table IV-AR. For comparison, the present values of \$5 discounted for 2 years at the after-tax discount rates implied by that Table are reported in the top of Table V.

of tax to a later and acceleration of revenue to an earlier period should have a positive impact on the value of the asset. On a discounted basis the accelerated revenue, net of the later offsetting adjustment, is more valuable than if the timing had remained unchanged. What is more, just as with our illustrative preferential rate, the impact of the timing change may be isolated from the present values of the constituent after-tax cash flows from the underlying asset, which continue to exhibit tax-rate invariance. Hence, the variation in the value of the asset as a whole should again be captured by the variation in the present values of the cash flows induced by the timing change.

A concrete version of that experiment will be found in Table IV-AT, in which Table II-A has been subjected to a single pair of changes, consisting of a shift of \$100 of *taxable income* from period 2 to period 6. Again assuming initially a 30 percent marginal rate, that change reduces tax expense (and increases cash flow) in period 2 by \$30, offset by a corresponding \$30 increase in tax liability (and decrease in cash flow) in period 6. (The cells that differ from Table II-A are again marked with an asterisk.) At 30 percent, that elementary timing change elevates the asset's value from \$5,131.58 to \$5,137.79. But the difference, \$6.21, is just the net value of a \$30 cash inflow two periods from now offset by a \$30 cash outlay four periods later, each discounted at 7 percent. Thus, as in Table IV-AR, the value of the asset in Table IV-AT is just its tax-invariant value, plus the net present value of the timing change. Now observe that, while the effects are again modest, as the tax rate increases, the asset value continues to rise, to \$5,139.71 at a marginal rate of 55 percent, after which it begins to decline, reaching \$5,138.94 at 70 percent.⁷¹ The observed changes are consistent with the inference that the value of an asset subject to a single (arbitrary) preferential timing change exhibits properties qualitatively similar to those observed with the preferentially taxed discount bond in Table II-C, taking on a maximum value at a marginal rate in excess of 50 percent. Note, too, that as with the simple rate preference, the observed change in the asset's value is simply the change in the net present value of the timing preference by itself, reported in the last lines of Table IV-AT.⁷²

⁷¹ As usual, the decline is toward the original value of \$5,131.58 as the tax rate approaches 100 percent. Compare, *supra*, Table III.

⁷² For comparison, the present values of the timing change in Table IV-AT in isolation are reported in the bottom half of Table V.

This abstract representation of a pure timing preference is, from both an intuitive and mathematical perspective, somewhat less accessible than the version of our discount bond illustrated in Table II-C.⁷³ It is similar to the latter in that varying the tax rate again has both a cash-flow and a discounting effect; it is once again the interaction of those two that produces the variations in value as the tax rate changes; and, as in that example, and in contrast with the valuation-neutral after-tax cash flows from the underlying asset, those effects do not exactly offset one another. That interaction, however, is intuitively different and mathematically less tractable than with the preferentially taxed bond in Table II-C. Here, the "cash-flow" effect of deferral is increasing in the marginal rate: at higher tax rates more is deferred, and the more valuable the resulting deferral. In contrast, an increase in the tax rate reduces the present value of that deferral.⁷⁴ Compared to the prior example, the signs of the cash-flow and discounting effects of taxation are reversed.⁷⁵

From a mathematical perspective, though it is easy to represent this model of deferral algebraically,⁷⁶ it is not possible to obtain an explicit solution for the value-maximizing marginal

⁷³ See *supra*, text and notes at notes 35 and 58.

⁷⁴ This property is naturally consistent with the fact that the value of "deferral" is entirely a creature of the discounting process. If the discount rate is effectively zero -- whether because the pre-tax interest rate is zero, or the marginal tax rate is 100 percent, so that the after-tax discount rate is zero -- tax deferral confers no advantage. It is almost surely in part for that reason that tax deferral has seemed a less pressing issue in recent years, when nominal interest rates have been hovering near zero.

⁷⁵ That is because preferential taxation was effectively embedded in the model of the pure discount bond in Table II-C. The only taxes observed in that example (and the mathematical abstractions of it in the Appendix) were those actually imposed at the time of surrender. So all that was observed as tax rates rose was the resulting decrease in after-tax cash flow, a decrease that was (more, or less, in a literal sense) offset by the tax-induced change in the discount rate. For the preferentially taxed bond to be represented consistently with the abstract model of deferral under discussion here, we would have to start with the bond as now taxed under I.R.C. § 1272(a), as in Table II-A, and then introduce a (linear) *combination* of adjustments like that in Table IV-AT, each of which reduced to zero the income accrued during one statutory accrual period, with each then mirrored by a compensating increase in income taxed at maturity. If modelled in that fashion the bond would look like a (more than seven times as) complicated a version of what is presented in Table IV-AT. Compare note 64, *supra*.

⁷⁶ As set out in the Appendix, a general expression for the deferral illustrated in Table IV-AT is

$$(1) \quad D(r, z) = z e \left([1 + r(1 - z)]^{-j} - [1 + r(1 - z)]^{-(j+k)} \right),$$

where z is the tax rate, r is the interest rate, and income of ϵ is deferred in period j and restored in period $j + k$. See *supra* note 56. (In Table IV-AT, $\epsilon = \$100$, $j = 2$, and $k = 4$, so that $j + k = 6$.)

In a cooperative world one would differentiate that expression, set the derivative equal to zero, and solve the resulting equation to obtain an explicit expression for the value-maximizing marginal rate z^* . That is how we proceeded with the pure discount bond. (See *supra*, text and note at note 64; Appendix.) Here, however, the resulting equation admits of no exact solution, so one is left to draw more indirect inferences about its behavior as the tax rate changes, as developed in the Appendix and described in the text.

rate. One can, however, show formally that pure deferral (as captured by our \$100 timing change in Table IV-AT) has a value of \$0 at marginal rates of both 0 and 1. It can also be shown that the value of deferral is increasing in the marginal rate in the neighborhood of 0 and decreasing when that rate is near 1.⁷⁷ Just those properties, combined with the fact that the discounting function is differentiable in the tax rate, are sufficient (again via Rolle's theorem) to establish that the value of deferral is maximized at a marginal rate between 0 and 1.⁷⁸ Indeed, it is possible to verify, indirectly but explicitly, that the value-maximizing marginal rate lies strictly between 0.50 and 1 -- consistent with the properties of the value-maximizing marginal rates exhibited in both Tables II-C and IV-AT.

From an intuitive perspective what appears to be happening is this.⁷⁹ The *undiscounted* value of deferring \$1 of income will depend in a linear fashion on the taxpayer's marginal rate, will be small at very low marginal rates, and can never be more than \$1 (at a marginal rate of 1). At the same time, the impact of discounting on the tax savings from deferring \$1 of income is also constrained to lie between 1 and 0: \$1 (typically less) of tax savings in hand now and repayable infinitely far into the future will be worth the undiscounted amount of the tax savings *at most*; with very *little* discounting -- at low interest rates and/or short periods of deferral -- it will be worth approximately \$0. What is more, for any given pre-tax discounting assumptions, the impact of deferring \$1, already constrained to be no greater than 1, is decreasing in the marginal tax rate, approaching zero as that rate approaches 1. The net effect is this: for \$1 of deferral, the cash-flow effect of taxation will rise from \$0 to \$1 as a linear function of the marginal rate; the discounting effect will fall from (at most) \$1 to \$0 as a *non*-linear function of that rate. Since each function takes on a finite value at one end point or the other of the interval (0, 1), while the other approaches 0 at that point, their product -- the pure timing function illustrated in Table IV-AT -- must approach 0 at each end of the interval. Since, moreover, the two functions do not change at the same *rate* -- one is linear while the other is not -- the different

⁷⁷ See the Appendix.

⁷⁸ See *supra*, text and note at note 63. In this case it is clear that the value of deferral, isolated from the tax-invariant value of the underlying asset, is 0 at marginal rates of both 0 and 1. See *supra* note 76. But Table V likewise establishes that it has positive value at rates in between. So by the same argument used with respect to the Example in Table II-C, it must attain a local maximum at at least one marginal rate in between.

⁷⁹ Special thanks to Robert Guth for persevering on this point. Reader: take a deep breath.

ways in which they vary with the marginal tax rate interact to produce the concave variations in value observed in Table IV-AT. That is the heart of what is going on.

Those properties are illustrated in Figures II and III. Figure II decomposes \$1 of deferral of the sort illustrated in Table IV-AT into its cash flow and discounting components,⁸⁰ and plots each component, as well as their product, as functions of the marginal tax rate. The cash flow effect increases dollar for dollar with the tax rate, while the effect of discounting \$1, about \$0.70 to begin with, declines, initially more slowly than the cash flow effect is increasing, but then at an increasing rate, reaching \$0 just as the cash-flow effect reaches \$1. The *product* of the two produces the overall behavior of deferral that we have observed, depicted by the concave green curve anchored at 0 at both ends in the plot. More specifically, it is the fact that the cash-flow effect initially increases more quickly in the tax rate than the discounting effect declines that initially elevates the value of the asset.

Figure III is another 3-dimensional surface plot, this time of the value (at 10%) of deferring \$100 of income, at marginal rates ranging from 0 to 1. What is more, in contrast with Table IV-AT (or indeed Figure II), which depict the value of deferral for fixed numbers of years, Figure III illustrates that property for periods of deferral ranging from 0 to 100 years. As with the pure discount bond illustrated in Figure I, the value of deferral is strictly increasing in the period of deferral.⁸¹ And both surface plots exhibit the same basic properties. At very low discounting the asset's value is maximized at a marginal rate modestly over 50 percent. As the deferral period grows, however, the marginal rate at which value is maximized increases towards a limiting value of 1.

All three plots, especially the last two, tell essentially the same story. Figure II depicts the interaction of the cash-flow and discounting effects that produces the smooth concave variation in the value of deferral; it also serves to explain, graphically if not analytically, why the maximum value occurs beyond 50 percent. Figure III amplifies on the composite plot of deferral in Figure II, illustrating how it varies with discount rate and the duration of deferral.

⁸⁰ That is, we have factored the expression for $D(r, z)$, in note 76 supra, into its cash-flow effect ($z\varepsilon$, with $\varepsilon = 1$) and its discounting component (the material in brackets). To obtain a suitable scaling and a sufficiently pronounced plot, Figure II uses deferral of \$1 rather than \$100, as in Table IV-AT, and from period 2 to period 22.

⁸¹ Although not illustrated, it is also increasing in the interest rate. See supra note 66.

**FIGURE II: Value of Deferral (\$1, @ 10%, Y2-Y20)
= Cash-Flow Effect X Discounting Effect**

FIGURE III: Deferral Surface (\$100, @ 10%)

Both Tables II-C and IV-AT, then, and their associated plots, illustrate the basic properties of preferential timing identified in this paper. It may also be shown, as suggested by Table II-D, that these properties are symmetric, in that an asset subject to a single *disadvantageous* timing change behaves in the opposite way, declining below its pre-tax (or tax-invariant) value as the tax rate rises above 0, eventually returning to 0 as the marginal rate approaches 1.⁸² This, then, is the essence of what happens when the timing of income from an asset varies from accrual, as the Samuelson theorem otherwise would require. If the timing change is preferential, the value of the asset will increase up to a marginal rate in excess of 50 percent (and for normal discounting parameters, typically around 55 percent). After that it begins to decline. Disadvantageous timing works in what for all practical purposes is exactly the opposite way.

A question that remains is why, with all the attention that tax deferral has commanded,⁸³ this facet of the problem has remained obscure for so long.⁸⁴ The answer is probably multifaceted. One aspect of it may have been the unexamined extension of what is undeniably true of tax rate preferences, that they increase with the holder's marginal rate, to the conclusion that deferral alone was comparably advantageous.⁸⁵ Beyond that, in analyzing the problem, most analysts have taken as a starting point some given amount of tax, payment of which is deferred, and analyzed the impact using a given discount rate.⁸⁶ While that seems like a natural point of departure, it leaves out of the equation the role of the taxpayer's marginal rate in determining jointly *both* the amount to be deferred *and* the rate at which that amount is discounted to present value. What emerges, however, from studying the operation of the Samuelson theorem is that it is the *interaction* between the offsetting effects of a taxpayer's marginal rate on their after-tax cash flow and their after-tax discount rate that produces Samuelson's original result, at least when income is taxed as it currently accrues, as that theorem effectively requires.⁸⁷ But even when

⁸² Mathematically, disadvantageous timing would be represented by the same function found in note 76, with the signs on the two terms reversed.

⁸³ See authorities cited *supra*, notes 1 and 26. See, e.g., Mitchell Engler, Partial Basis Indexation: An Implicit Response to Tax Deferral, 53 Tax. L. Rev. 177 (2000); Mitchell Engler, Partial Basis Indexation: Tax Arbitrage and Related Issues, 55 Tax L. Rev. 69 (2001); Stephen Land, Defeating Deferral: A Proposal for Retrospective Taxation, 52 Tax L. Rev. 45 (1996). {more.}

⁸⁴ This paragraph is being written in the evening of February 28, 2013.

⁸⁵ E.g., Surrey (*Tax Incentives*), *supra* note 1, at 720-723; Surrey (*Tax Deferral*), *supra* note 1, at 317-318.

⁸⁶ [Citations.]

⁸⁷ Sims, *supra* note 35, at 4.

the conditions for application of the theorem are not satisfied, it is still the case, at least with respect to pure timing preferences, that the marginal rate influences value both through its impact on after-tax cash flows and on the rate at which they are discounted. So it appears to be a failure to have modelled the problem in a manner that preserves both of those effects that has left the property highlighted by this paper undetected.

V. Conclusion

As I began by acknowledging, with the top U.S. marginal income tax rate currently below 40 percent, the characteristic of timing preferences identified in this paper will for the foreseeable future be of largely theoretical interest. What is more, with *pre-tax* discount rates at historic lows, deferral feels like a less pressing topic in general than it did when interest rates were higher. It seems unlikely, however, that at current levels of outstanding U.S. Government debt, and annual U.S. Government deficits in the trillions, that we will enjoy such historically low interest (or marginal tax) rates indefinitely. Eventually, something will give, or at least one is inclined to suppose. When that happens it is conceivable at least that the aspect of deferral identified in this paper may take on some relevance in the deliberations about the contours of our tax system that ensue.

TABLES

I. Bank Account (@ 10%)	
Balance	Interest
5131.58	513.16
5644.74	564.47
6209.21	620.92
6830.13	683.01
7513.15	751.31
8264.46	826.45
9090.91	909.09
10000.00	

II-A EXPONENTIAL ACCRUAL (I.R.C. § 1272(a))											
	(1)	(2)	(3a)	(4a)	(5a)	(3b)	(4b)	(5b)	(3c)	(4c)	(5c)
n =	Accrual of Value	Income	Tax (@30%)	Cash Flow	PV (@7%)	Tax (@40%)	Cash Flow	PV (@6%)	Tax (@50%)	Cash Flow	PV (@5%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5644.74	513.16	-153.95	-153.95	-143.88	-205.26	-205.26	-193.64	-256.58	-256.58	-244.36
2	6209.21	564.47	-169.34	-169.34	-147.91	-225.79	-225.79	-200.95	-282.24	-282.24	-256.00
3	6830.13	620.92	-186.28	-186.28	-152.06	-248.37	-248.37	-208.53	-310.46	-310.46	-268.19
4	7513.15	683.01	-204.90	-204.90	-156.32	-273.21	-273.21	-216.40	-341.51	-341.51	-280.96
5	8264.46	751.31	-225.39	-225.39	-160.70	-300.53	-300.53	-224.57	-375.66	-375.66	-294.34
6	9090.91	826.45	-247.93	-247.93	-165.21	-330.58	-330.58	-233.04	-413.22	-413.22	-308.35
7	10000.00	909.09	-272.73	9727.27	6057.66	-363.64	9636.36	6408.73	-454.55	9545.45	6783.77
		Totals =	-1460.53	8539.47	5131.58	-1947.37	8052.63	5131.58	-2434.21	7565.79	5131.58

EXPONENTIAL ACCRUAL (con't)											
	(1)	(2)	(3d)	(4d)	(5d)	(3e)	(4e)	(5e)	(3f)	(4f)	(5f)
n =	Accrual of Value	Income	Tax (@55%)	Cash Flow	PV (@4.5%)	Tax (@60%)	Cash Flow	PV (@4%)	Tax (@70%)	Cash Flow	PV (@3%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5644.74	513.16	-282.24	-282.24	-270.08	-307.89	-307.89	-296.05	-359.21	-359.21	-348.75
2	6209.21	564.47	-310.46	-310.46	-284.30	-338.68	-338.68	-313.13	-395.13	-395.13	-372.45
3	6830.13	620.92	-341.51	-341.51	-299.26	-372.55	-372.55	-331.20	-434.64	-434.64	-397.76
4	7513.15	683.01	-375.66	-375.66	-315.01	-409.81	-409.81	-350.31	-478.11	-478.11	-424.79
5	8264.46	751.31	-413.22	-413.22	-331.59	-450.79	-450.79	-370.52	-525.92	-525.92	-453.66
6	9090.91	826.45	-454.55	-454.55	-349.04	-495.87	-495.87	-391.89	-578.51	-578.51	-484.49
7	10000.00	909.09	-500.00	9500.00	6980.87	-545.45	9454.54	7184.68	-636.36	9363.63	7613.49
		Totals =	-2677.63	7322.37	5131.58	-2921.05	7078.95	5131.58	-3407.89	6592.11	5131.58

II-B CAULKINS (long-term capital gain on surrender)										
	(1)	(2)	(3)	(4)	(5a)	(5b)	(5c)	(5d)	(5e)	(5f)
n =	Accrual of Value	Income	Tax (@15%)	Cash Flow	PV (@7% t=30%)	PV (@6% t=40%)	PV (@5% t=50%)	PV (@4.5% t=55%)	PV (@4% t=60%)	PV (@3% t=70%)
0	5131.58			-5131.58	-5131.58	-5131.58	-5131.58	-5131.58	-5131.58	-5131.58
1	5644.74									
2	6209.21									
3	6830.13									
4	7513.15									
5	8264.46									
6	9090.91									
7	10000.00	4868.42	-730.26	9269.74	5772.73	6164.90	6587.83	6811.67	7044.24	7537.14

II-C MIDLAND-ROSS (ordinary income on surrender)											
	(1)	(2)	(3a)	(4a)	(5a)	(3b)	(4b)	(5b)	(3c)	(4c)	(5c)
n =	Accrual of Value	Income	Tax (@30%)	Cash Flow	PV (@7%)	Tax (@40%)	CashFlow	PV (@6%)	Tax (@50%)	Cash Flow	PV (@5%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5644.74										
2	6209.21										
3	6830.13										
4	7513.15										
5	8264.46										
6	9090.91										
7	10000.00	4868.42	-1460.53	8539.47	5317.96	-1947.37	8052.63	5355.46	-2434.21	7565.79	5376.86

MIDLAND-ROSS (con't)											
	(1)	(2)	(3d)	(4d)	(5d)	(3e)	(4e)	(5e)	(3f)	(4f)	(5f)
n =	Value	Income	Tax (@55%)	Cash Flow	PV (@4.5%)	Tax (@60%)	Cash Flow	PV (@4%)	Tax (@70%)	Cash Flow	PV (@3%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5644.74										
2	6209.21										
3	6830.13										
4	7513.15										
5	8264.46										
6	9090.91										
7	10000.00	4868.42	-2677.63	7322.37	5380.68	-2921.05	7078.95	5379.42	-3407.89	6592.11	5359.98

II-D RATABLE ACCRUAL (I.R.C. § 1272(b))											
	(1)	(2)	(3a)	(4a)	(5a)	(3b)	(4b)	(5b)	(3c)	(4c)	(5c)
n =	Accrued Value	Income	Tax (@30%)	Cash Flow	PV (@7%)	Tax (@40%)	Cash Flow	PV (@6%)	Tax (@50%)	Cash Flow	PV (@5%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5827.07	695.49	-208.65	-208.65	-195.00	-278.20	-278.20	-262.45	-347.74	-347.74	-331.18
2	6522.56	695.49	-208.65	-208.65	-182.24	-278.20	-278.20	-247.59	-347.74	-347.74	-315.41
3	7218.05	695.49	-208.65	-208.65	-170.32	-278.20	-278.20	-233.58	-347.74	-347.74	-300.39
4	7913.54	695.49	-208.65	-208.65	-159.18	-278.20	-278.20	-220.36	-347.74	-347.74	-286.09
5	8609.03	695.49	-208.65	-208.65	-148.76	-278.20	-278.20	-207.88	-347.74	-347.74	-272.47
6	9304.52	695.49	-208.65	-208.65	-139.03	-278.20	-278.20	-196.12	-347.74	-347.74	-259.49
7	10000.00	695.49	-208.65	9791.35	6097.56	-278.20	9721.80	6465.55	-347.74	9652.25	6859.68
		Totals =	-1460.53	8539.47	5103.04	-1947.37	8052.63	5097.58	-2434.21	7565.79	5094.63

RATABLE ACCRUAL (con't)											
	(1)	(2)	(3d)	(4d)	(5d)	(3e)	(4e)	(5e)	(3f)	(4f)	(5f)
n =	Accrue Value	Income	Tax (@55%)	Cash Flow	PV (@4.5%)	Tax (@60%)	Cash Flow	PV (@4%)	Tax (@70%)	Cash Flow	PV (@3%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5827.07	695.49	-382.52	-382.52	-366.05	-417.29	-417.29	-401.24	-486.84	-486.84	-472.66
2	6522.56	695.49	-382.52	-382.52	-350.28	-417.29	-417.29	-385.81	-486.84	-486.84	-458.90
3	7218.05	695.49	-382.52	-382.52	-335.20	-417.29	-417.29	-370.97	-486.84	-486.84	-445.53
4	7913.54	695.49	-382.52	-382.52	-320.77	-417.29	-417.29	-356.70	-486.84	-486.84	-432.55
5	8609.03	695.49	-382.52	-382.52	-306.95	-417.29	-417.29	-342.98	-486.84	-486.84	-419.95
6	9304.52	695.49	-382.52	-382.52	-293.73	-417.29	-417.29	-329.79	-486.84	-486.84	-407.72
7	10000.00	695.49	-382.52	9617.48	7067.20	-417.29	9582.70	7282.07	-486.84	9513.16	7735.07
		Totals =	-2677.63	7322.37	5094.22	-2921.05	7078.95	5094.56	-3407.89	6592.11	5097.75

III COMPARISON OF VALUES AS A FUNCTION OF MARGINAL RATE

Treatment	t = 0.15	0.20	0.30	0.40	0.50	0.55	0.60	0.70	0.80	0.90	0.95	0.9999
Caulkins	5236.72	5408.80	5772.73	6164.90	6587.83	6811.67	7044.24	7537.14	8069.86	8646.05	8951.69	9269.09
Mid-Ross	5236.72	5266.77	5317.95	5355.46	5376.86	5380.68	5379.42	5359.98	5315.00	5240.40	5190.58	5131.71
1272(a)	5131.58	5131.58	5131.58	5131.58	5131.58	5131.58	5131.58	5131.58	5131.58	5131.58	5131.58	5131.58
1272(b)	5115.28	5110.71	5103.04	5097.58	5094.63	5094.22	5094.56	5097.75	5104.64	5115.74	5123.03	5131.56

IV-AR EXPONENTIAL ACCRUAL WITH \$100 TAXED @ 5% RATE PREFERENCE IN PERIOD 2											(Cells marked * differ from Table II-A)
	(1)	(2)	(3a)	(4a)	(5a)	(3b)	(4b)	(5b)	(3c)	(4c)	(5c)
	Accrual of Value	Income	Tax (@30%)	Cash Flow	PV (@7%)	Tax (@40%)	Cash Flow	PV (@6%)	Tax (@50%)	Cash Flow	PV (@5%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5644.74	513.16	-153.95	-153.95	-143.88	-205.26	-205.26	-193.64	-256.58	-256.58	-244.36
2	6209.21	564.47	-164.34*	-164.34*	-143.54*	-220.79*	-220.79*	-196.50*	-277.24*	-277.24*	-251.46*
3	6830.13	620.92	-186.28	-186.28	-152.06	-248.37	-248.37	-208.53	-310.46	-310.46	-268.19
4	7513.15	683.01	-204.90	-204.90	-156.32	-273.21	-273.21	-216.40	-341.51	-341.51	-280.96
5	8264.46	751.31	-225.39	-225.39	-160.70	-300.53	-300.53	-224.57	-375.66	-375.66	-294.34
6	9090.91	826.45	-247.93	-247.93	-165.21	-330.58	-330.58	-233.04	-413.22	-413.22	-308.35
7	10000.00	909.09	-272.73	9727.27	6057.66	-363.64	9636.36	6408.73	-454.55	9545.45	6783.77
		Totals = Increase =	-1455.53*	8544.47*	5135.95* 4.37	-1942.37*	8057.63*	5136.03* 4.45	-2429.21*	7570.79*	5136.12* 4.54

EXPONENTIAL ACCRUAL WITH \$100 TAXED @ 5% RATE PREFERENCE IN PERIOD 2 (con't)											(Cells marked * differ from Table II-A)		
	(1)	(2)	(3d)	(4d)	(5d)	(3e)	(4e)	(5e)	(3f)	(4f)	(5f)		
n =	Accrual of Value	Income	Tax (@55%)	Cash Flow	PV (@4.5%)	Tax (@60%)	Cash Flow	PV (@4%)	Tax (@70%)	Cash Flow	PV (@3%)		
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58		
1	5644.74	513.16	-282.24	-282.24	-270.08	-307.89	-307.89	-296.05	-359.21	-359.21	-348.75		
2	6209.21	564.47	-305.46*	-305.46*	-279.72*	-333.68*	-333.68*	-308.51*	-390.13*	-390.13*	-367.74*		
3	6830.13	620.92	-341.51	-341.51	-299.26	-372.55	-372.55	-331.20	-434.64	-434.64	-397.76		
4	7513.15	683.01	-375.66	-375.66	-315.01	-409.81	-409.81	-350.31	-478.11	-478.11	-424.79		
5	8264.46	751.31	-413.22	-413.22	-331.59	-450.79	-450.79	-370.52	-525.92	-525.92	-453.66		
6	9090.91	826.45	-454.55	-454.55	-349.04	-495.87	-495.87	-391.89	-578.51	-578.51	-484.49		
7	10000.00	909.09	-500.00	9500.00	6980.87	-545.45	9454.54	7184.68	-636.36	9363.63	7613.49		
		Totals = Increase =	-2672.63*	7327.37*	5136.16* 4.58	-2916.05*	7083.95*	5136.20* 4.62	-3402.89*	6597.11*	5136.29* 4.71		

IV-AT	EXPONENTIAL ACCRUAL WITH \$100 DEFERRED FROM PERIOD 2 TO PERIOD 6						(Cells marked * differ from Table II-A)				
	(1)	(2)	(3a)	(4a)	(5a)	(3b)	(4b)	(5b)	(3c)	(4c)	(5c)
	Accrual of Value	Income	Tax (@30%)	Cash Flow	PV (@7%)	Tax (@40%)	Cash Flow	PV (@6%)	Tax (@50%)	Cash Flow	PV (@5%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5644.74	513.16	-153.95	-153.95	-143.88	-205.26	-205.26	-193.64	-256.58	-256.58	-244.36
2	6209.21	464.47*	-139.34*	-139.34*	-121.71*	-185.79*	-185.79*	-165.35*	-232.24*	-232.24*	-210.65*
3	6830.13	620.92	-186.28	-186.28	-152.06	-248.37	-248.37	-208.53	-310.46	-310.46	-268.19
4	7513.15	683.01	-204.90	-204.90	-156.32	-273.21	-273.21	-216.40	-341.51	-341.51	-280.96
5	8264.46	751.31	-225.39	-225.39	-160.70	-300.53	-300.53	-224.57	-375.66	-375.66	-294.34
6	9090.91	926.45*	-277.93*	-277.93*	-185.20*	-370.58*	-370.58*	-261.24*	-463.22*	-463.22*	-345.66*
7	10000.00	909.09	-272.73	9727.27	6057.66	-363.64	9636.36	6408.73	-454.55	9545.45	6783.77
		Total = Increase =	-1460.53	8539.47	5137.79* 6.21	-1947.37	8052.63	5138.98* 7.40	-2434.21	7565.79	5139.62* 8.04

	EXPONENTIAL ACCRUAL WITH \$100 DEFERRED FROM PERIOD 2 TO PERIOD 6 (con't)						(Cells marked * differ from Table II-A)				
	(1)	(2)	(3d)	(4d)	(5d)	(3e)	(4e)	(5e)	(3f)	(4f)	(5f)
	Accrual of Value	Income	Tax (@55%)	Cash Flow	PV (@4.5%)	Tax (@60%)	Cash Flow	PV (@4%)	Tax (@70%)	Cash Flow	PV (@3%)
0	-5131.58			-5131.58	-5131.58		-5131.58	-5131.58		-5131.58	-5131.58
1	5644.74	513.16	-282.24	-282.24	-270.08	-307.89	-307.89	-296.05	-359.21	-359.21	-348.75
2	6209.21	464.47*	-255.46*	-255.46*	-233.93*	-278.68*	-278.68*	-257.66*	-325.13*	-325.13*	-306.47*
3	6830.13	620.92	-341.51	-341.51	-299.26	-372.55	-372.55	-331.20	-434.64	-434.64	-397.76
4	7513.15	683.01	-375.66	-375.66	-315.01	-409.81	-409.81	-350.31	-478.11	-478.11	-424.79
5	8264.46	751.31	-413.22	-413.22	-331.59	-450.79	-450.79	-370.52	-525.92	-525.92	-453.66
6	9090.91	926.45*	-509.55*	-509.55*	-391.28*	-555.87*	-555.87*	-439.31*	-648.51*	-648.51*	-543.12*
7	10000.00	909.09	-500.00	9500.00	6980.87	-545.45	9454.54	7184.68	-636.36	9363.63	7613.49
		Total = Increase =	-2677.63	7322.37	5139.71* 8.13	-2921.05	7078.95	5139.63* 8.05	-3407.89	6592.11	5138.94* 7.36

V: PREFERENTIAL ELEMENTS OF TAXATION IN TABLES IV-AR AND IV-AT
 ($r = 10\%$, $z = \{0.30, 0.40, 0.50, 0.55, 0.60, 0.70\}$)

$r =$ 0.10

$z =$ 0.30 0.40 0.50 0.55 0.60 0.70

Present Value of 5 Percent Rate Preference in Period 2

$n =$	Cash Flow	<i>Present Value</i>	<i>Present Value</i>	<i>Present Value</i>	<i>Present Value</i>	<i>Present Value</i>	<i>Present Value</i>
2	5.00	4.37	4.45	4.54	4.58	4.62	4.71

Present Value of Deferring \$100 from Period 2 to Period 6

$n =$	Cash Flow	<i>Present Value</i>	Cash Flow	<i>Present Value</i>	Cash Flow	<i>Present Value</i>	Cash Flow	<i>Present Value</i>	Cash Flow	<i>Present Value</i>	Cash Flow	<i>Present Value</i>
2	30.00	26.20	40.00	35.60	50.00	45.35	55.00	50.37	60.00	55.47	70.00	65.98
6	-30.00	-19.99	-40.00	-28.20	-50.00	-37.31	-55.00	-42.23	-60.00	-47.42	-70.00	-58.62
<i>Value =</i>		6.21		7.40		8.04		8.13		8.05		7.36

APPENDIX¹

This appendix develops several of the propositions touched on in the text. For tractability continuous discounting is used in the representation of the preferentially taxed discount bond illustrated in Table II-C. For simplicity of formulation, discrete time is used in the representation of the pure rate and timing preferences illustrated in Part IV.

Discount Bond Taxed as Ordinary Income at Maturity. The present value (issue price) of a discount paying \$1 at maturity t , discounted at pre-tax rate r (its yield-to-maturity) is

$$(1) \quad P = e^{-rt}.$$

Since r and t enter the discounting function symmetrically, we can simplify matters by setting $s = rt$ and writing (1) as a function of a single discounting parameter, $P = e^{-s}$.

Let $z \in [0, 1]$ be the marginal tax rate, and assume that the gain -- $(1 - e^{-s})$ -- were to be taxed on surrender at maturity as ordinary income, as under pre-1969 § 1232A, or as now prescribed by § 1276 for accrued market discount on disposition. Then the instrument's after-tax proceeds, discounted to present value at an after-tax discount rate $r^*(1-z)$, are given by

$$(2) \quad V(z, s) = [1 - z(1 - e^{-s})] e^{-s(1-z)}.$$

At $z = 0$ and $z = 1$ the instrument's value is just its pre-tax present value, e^{-s} , in the latter case because, although the pre-tax gain is entirely taxed away, the after-tax discount rate is simultaneously reduced to zero, so that the instrument's present value on surrender is identical to its cost. Since (2) is continuously differentiable on $(0, 1)$, if at any point $0 < z < 1$ it takes on value $V > e^{-s}$, by Rolle's theorem it attains a local maximum somewhere in that interval.

Differentiating (2) with respect to z yields

$$(2') \quad \frac{\partial V}{\partial z} = [- (1 - e^{-s}) + s(1 - z + ze^{-s})] e^{-s(1-z)}.$$

The first (negative) term in brackets is the reduction in the instrument's value due to the impact of the tax in reducing its constituent (including tax-induced) cash flows; the second term captures the increase in value due to a reduction in the (after-tax) discount rate. Note that

$$(2'a) \quad \begin{aligned} \left. \frac{\partial V}{\partial z} \right|_{s=0} &= 0; \text{ and} \\ \left. \frac{\partial V}{\partial z} \right|_{s \rightarrow \infty} &= \lim_{s \rightarrow \infty} [-1 + s(1 - z)] e^{-s(1-z)} \\ &\approx \lim_{s \rightarrow \infty} s(1 - z) e^{-s(1-z)}; \end{aligned}$$

¹ Special thanks to AnTon Bui.

as $s \rightarrow \infty$ the latter takes on the indeterminate form $f(s)/g(s) = \infty/\infty$, with $f(s) = (s^*[1 - z])$ and $g(s) = e^{s(1-z)}$. By l'Hopital's rule²

$$\left. \frac{f(s)}{g(s)} \right|_{s \rightarrow \infty} = \left. \frac{f'(s)}{g'(s)} \right|_{s \rightarrow \infty} = \lim_{s \rightarrow \infty} \frac{1 - z}{(1 - z)e^{s(1-z)}} = 0,$$

So the derivative (2') takes on the value 0 at $s = 0$ and as $s \rightarrow \infty$. It is also straightforward that

$$(3) \quad \frac{\partial V}{\partial z} \geq 0 \Leftrightarrow$$

$$z \leq z^* = (1 - e^{-s})^{-1} - (s)^{-1}.$$

Since $\partial V/\partial z$ is 0 at $s = 0$, and for $s > 0$ is non-decreasing in z on $(0, z^*)$, it follows that z^* is a local maximum of (2), confirmed by the fact that the second derivative of (2),

$$\begin{aligned} \frac{\partial^2 V}{\partial z^2} &= \frac{\partial}{\partial z} \left\{ \left[- (1 - e^{-s}) + s(1 - z + ze^{-s}) \right] e^{-s(1-z)} \right\} \\ &= \left\{ -2s(1 - e^{-s}) + (s)^2(1 - z + ze^{-s}) \right\} e^{-s(1-z)}, \end{aligned}$$

evaluated at $z = z^*$ is

$$\left\{ -s(1 - e^{-s}) \right\} e^{-s(1-z^*)} < 0.$$

So for rates below z^* the value of the instrument is increasing in the holder's marginal rate, taking on values in excess of its pre-tax present value, attaining a maximum at z^* and thereafter declining, approaching its pre-tax present value again as $z \rightarrow 1$.

It is possible to characterize the value-maximizing marginal tax rate in terms of the discounting parameter s . Note, first, that

$$\begin{aligned} \lim_{s \rightarrow 0} z^* &= \lim_{s \rightarrow 0} \left\{ (1 - e^{-s})^{-1} - (s)^{-1} \right\} \\ (3a) \quad &= \lim_{s \rightarrow 0} \left\{ \frac{s - 1 + e^{-s}}{s(1 - e^{-s})} \right\} \end{aligned}$$

has the indeterminate form 0/0; setting $f(s) = (s - 1 - e^{-s})$, and $g(s) = (s(1 - e^{-s}))$, and differentiating the numerator and denominator each twice yields

$$\begin{aligned} \lim_{s \rightarrow 0} \frac{f'(s)}{g'(s)} &= \lim_{s \rightarrow 0} \left\{ \frac{1 - e^{-s}}{1 - e^{-s} + se^{-s}} \right\} = \frac{0}{0} = \\ \lim_{s \rightarrow 0} \frac{f''(s)}{g''(s)} &= \lim_{s \rightarrow 0} \left\{ \frac{e^{-s}}{e^{-s} + e^{-s} - se^{-s}} \right\} = \frac{1}{2}. \end{aligned}$$

² E.g., Swokowski, Calculus With Analytic Geometry (3rd ed. 1984) 453-456.

Hence $z^* > 1/2$ for $s > 0$. Next we show that z^* is strictly increasing in s . To do so, differentiate (3) to find that

$$\begin{aligned}
 z^{*/'}(s) &= -e^{-s}(1 - e^{-s})^{-2} + (s)^{-2} \\
 (3b) \quad &= \frac{(1 - e^{-s})^2 - e^{-s}s^2}{(1 - e^{-s})^2 s^2} > 0 \text{ if} \\
 F(s) &= (1 - e^{-s})^2 - e^{-s}s^2 > 0.
 \end{aligned}$$

To show that $F(s) > 0$ for $s > 0$, note that $F(0) = 0$ and that

$$\begin{aligned}
 F'(s) &= 2(1 - e^{-s})e^{-s} - 2se^{-s} + s^2e^{-s}, \\
 &= e^{-s}\{2 - 2e^{-s} - 2s + s^2\} \\
 &\equiv e^{-s}G(s),
 \end{aligned}$$

so that $F'(s) > 0$ if $G(s) > 0$ for $s > 0$. But $G(0) = 0$, and

$$G'(s) = 2e^{-s} - 2 + 2s, \text{ with } G'(0) = 0,$$

and

$$G''(s) = -2e^{-s} + 2 > 0.$$

Hence, $G'(s) > 0$ for all $s > 0$, and with $G(0) = 0$, it follows that $G(s) > 0$ for $s > 0$. It then also follows from $G(s) = F'(s) > 0$ for $s > 0$ and $F(0) = 0$ that $F(s) > 0$. So $\partial z^*(s)/\partial s > 0$. Thus, $1/2 < z^* < 1$ and z^* is strictly increasing in the discounting parameter s .

A General Formulation. To fashion an abstract but transparent formalization of a rate or timing preference is simpler in discrete than continuous time. In that setting, the value of an asset consisting of a collection of cash flows $\{C_1, \dots, C_N\}$, after taxation at rate z , with depreciation (equal to original cost) allowed in computing taxable income, can be represented by

$$(4) \quad V_0(z, r) = \sum_{j=1}^N \frac{C_j - z(C_j - \delta_j)}{[1 + r(1 - z)]^j},$$

where δ_j is the (not necessarily economic) depreciation allowance in period j . As implied by Samuelson (1964) and illustrated in Sims (2012), "economic" depreciation of this asset in period j is given by

$$(5) \quad \Delta_j = -rV_{j-1} + C_j,$$

where V_{j-1} is the (pre-tax) discounted value of the cash-flows remaining at the end of period $j-1$. Substituting (5) into (4), the after tax present value of the asset with economic depreciation is given by

$$(6) \quad V_0(z, r) = \sum_{j=1}^N \frac{C_j - zrV_{j-1}}{[1 + r(1-z)]^j}.$$

Samuelson's finding was that the value of the continuous time version of this asset, in the presence of income taxation with economic depreciation, is independent of the marginal rate of the holder, as corroborated for (6) by Sims (2012). It is against that background that the impact of "preferential" departures from valuation-neutral income taxation should be evaluated.

A moment's reflection should suggest that, in the context of (6), preferential taxation can take (some combination of) one of two forms: (1) one or more of the constituent cash flows can be taxed at a preferential rate $\tau < z$, or (2) the timing of taxation of one or more of the constituent cash flows can be retarded (deferred) to some later moment in time. In either case, the overall impact of taxation on the value of the asset may be decomposed into (a) the valuation-neutral cash flows in (6), themselves, and (b) the preferential feature. When separated from the latter, the former will remain valuation neutral. Hence, the impact of preferential taxation on the asset's value will be entirely captured by (b). So the effects of preferential taxation may easily be elucidated by modelling each preferential possibility in isolation, and studying how they respond to changes in the marginal rate.

Preferential rates. The asset in (6) will be subject to a preferential rate if, everything else held constant, any cash flow in any period $j \in \{1, \dots, n\}$ is taxed at rate $\tau < z$. In that event, the undiscounted benefit of preferential taxation of, say, \$1 in period j will just be $(z - \tau) > 0$, and the contribution of the resulting rate preference to the value of the asset, discounted to the date of purchase, will be

$$(7) \quad R(z, \tau) = \frac{z - \tau}{[1 + r(1-z)]^j}.$$

Differentiating (7) with respect to z produces

$$(7a) \quad \frac{\partial R(z, \tau)}{\partial z} = \frac{1}{[1 + r(1-z)]^j} + \frac{rj(z - \tau)}{[1 + r(1-z)]^{j+1}} > 0,$$

confirming the standard intuition that the benefit of preferential taxation of an asset, in this instance taking the form of a preferential rate, is strictly increasing in the holder's marginal rate. And since any collection of preferential rates can in principle be expressed as a linear combination of R 's, it follows that value of an asset subject to preferential rates will be strictly increasing in the holder's otherwise generally applicable marginal rate.

Preferential Timing. It is preferential timing that produces the surprise. Everything else in (6) held constant, define preferential timing as taxing \$1 less in period j , offset by taxing \$1 more in some later period $j+k$. Those offsetting changes produce an undiscounted benefit of \$ z in j at a cost of -\$ z in $j+k$, with values (again discounted to the date of purchase) of

$$(8) \quad D(z, r) = \frac{z}{[1 + r(1-z)]^j} - \frac{z}{[1 + r(1-z)]^{j+k}}.$$

As with (2), the tax rate influences the value of deferral by its impact on the amount deferred and by its impact on the discount rate. Note that

$$D(0, r) = 0 = D(1, r),$$

that is, deferral has no value at tax rates of *either* 0 or 1. Differentiating (8) with respect to z produces

$$(8a) \quad \begin{aligned} \frac{\partial D(z, r)}{\partial z} = & \frac{1}{[1 + r(1-z)]^j} - \frac{1}{[1 + r(1-z)]^{j+k}} \\ & + \frac{jrz}{[1 + r(1-z)]^{j+1}} - \frac{(j+k)rz}{[1 + r(1-z)]^{j+k+1}}. \end{aligned}$$

Evaluating (8a) at $z = 0$ yields

$$\frac{\partial D(0, r)}{\partial z} = \frac{1}{[1 + r]^j} - \frac{1}{[1 + r]^{j+k}} > 0,$$

while evaluating it at $z = 1$ produces

$$\frac{\partial D(1, r)}{\partial z} = jrz - (j+k)rz < 0.$$

Thus, the pure timing preference $D(z, r)$ is increasing at $z = 0$ and decreasing at $z = 1$, and takes on the same value (zero in this instance) at either extreme. So, again by Rolle's theorem, it takes on some maximum value at a marginal rate between 0 and 1. In contrast with a preferential rate, preferential timing is not strictly increasing in z . Note that *disadvantageous* timing, though infrequently encountered, is defined exactly as in (8), with the signs on the two terms in $D(z, r)$ reversed. So it will have exactly the opposite properties as $D(z, r)$ itself.

Although (8a) cannot be solved explicitly for the z^* that maximizes the value of the asset, it may be used to show that $z^* \in (1/2, 1)$ and that it is a global maximum. To see this, set (8a) equal to zero, multiply through by $[1+r(1-z)]^{j+1}$, and rewrite it as:

$$(8b) \quad 1 + r + r(j-1)z = \frac{1 + r + r(j+k-1)z}{[1 + r(1-z)]^k}.$$

It may readily be shown that the linear function on the left-hand side of (8b) and the curve on the right intersect at a unique $z^* \in (1/2, 1)$. Thus, the pure timing preference defined by $D(z, r)$ has a unique maximum at $z^* > 50$ percent. It may also be shown, though it is easier in continuous time and still somewhat laborious, that z^* approaches 1/2 as the discount factor approaches 1, and approaches 1 as the discount factor increases without limit. As our discount bond example is a special case of (8), this should not be surprising. Details may be found in Sims (2013).